Cambridge Assessment

A Level

Mathematics

Session: 2010 June
Type: Question paper
Code: 3890-7890; 3892-7892
Unit: 4721, 4722, 4723, 4724

© UCLES

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS **4721**

Core Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book

OCR Supplied Materials:
- Printed Answer Book 4721
- List of Formulae (MF1)

Other Materials Required:
None

Monday 24 May 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- **The questions are on the inserted Question Paper.**
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or destroyed.

No calculator can be used for this paper

UCLES

© OCR 2010 [Y/102/2693]
3R–0B10
OCR is an exempt Charity

1 **(i)** Evaluate 9^0. **[1]**

(ii) Express $9^{-\frac{1}{2}}$ as a fraction. **[2]**

2 **(i)** Sketch the curve $y = -\frac{1}{x^2}$. **[2]**

(ii) Sketch the curve $y = 3 - \frac{1}{x^2}$. **[2]**

(iii) The curve $y = -\frac{1}{x^2}$ is stretched parallel to the y-axis with scale factor 2. State the equation of the transformed curve. **[1]**

3 **(i)** Express $\frac{12}{3+\sqrt{5}}$ in the form $a - b\sqrt{5}$, where a and b are positive integers. **[3]**

(ii) Express $\sqrt{18} - \sqrt{2}$ in simplified surd form. **[2]**

4 **(i)** Expand $(x-2)^2(x+1)$, simplifying your answer. **[3]**

(ii) Sketch the curve $y = (x-2)^2(x+1)$, indicating the coordinates of all intercepts with the axes. **[3]**

5 Find the real roots of the equation $4x^4 + 3x^2 - 1 = 0$. **[5]**

6 Find the gradient of the curve $y = 2x + \frac{6}{\sqrt{x}}$ at the point where $x = 4$. **[5]**

7 Solve the simultaneous equations

$$x + 2y - 6 = 0, \qquad 2x^2 + y^2 = 57.$$ **[6]**

8 **(i)** Express $2x^2 + 5x$ in the form $2(x+p)^2 + q$. **[3]**

(ii) State the coordinates of the minimum point of the curve $y = 2x^2 + 5x$. **[2]**

(iii) State the equation of the normal to the curve at its minimum point. **[1]**

(iv) Solve the inequality $2x^2 + 5x > 0$. **[4]**

UCLES

9 **(i)** The line joining the points $A\,(4,\,5)$ and $B\,(p,\,q)$ has mid-point $M\,(-1,\,3)$. Find p and q. **[3]**

AB is the diameter of a circle.

(ii) Find the radius of the circle. **[2]**

(iii) Find the equation of the circle, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$. **[3]**

(iv) Find an equation of the tangent to the circle at the point $(4,\,5)$. **[5]**

10 **(i)** Find the coordinates of the stationary points of the curve $y = 2x^3 + 5x^2 - 4x$. **[6]**

(ii) State the set of values for x for which $2x^3 + 5x^2 - 4x$ is a decreasing function. **[2]**

(iii) Show that the equation of the tangent to the curve at the point where $x = \frac{1}{2}$ is $10x - 4y - 7 = 0$. **[4]**

(iv) Hence, with the aid of a sketch, show that the equation $2x^3 + 5x^2 - 4x = \frac{5}{2}x - \frac{7}{4}$ has two distinct real roots. **[2]**

UCLES

UCLES

OCR
RECOGNISING ACHIEVEMENT

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4722

Core Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book

OCR Supplied Materials:
- Printed Answer Book 4722
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Thursday 27 May 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- **The questions are on the inserted Question Paper.**
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or destroyed.

UCLES

© OCR 2010 [D/102/2694] OCR is an exempt Charity

1 The cubic polynomial $f(x)$ is defined by $f(x) = x^3 + ax^2 - ax - 14$, where a is a constant.

(i) Given that $(x - 2)$ is a factor of $f(x)$, find the value of a. **[3]**

(ii) Using this value of a, find the remainder when $f(x)$ is divided by $(x + 1)$. **[2]**

2 **(i)** Use the trapezium rule, with 3 strips each of width 3, to estimate the area of the region bounded by the curve $y = \sqrt[3]{7 + x}$, the x-axis, and the lines $x = 1$ and $x = 10$. Give your answer correct to 3 significant figures. **[4]**

(ii) Explain how the trapezium rule could be used to obtain a more accurate estimate of the area. **[1]**

3 **(i)** Find and simplify the first four terms in the binomial expansion of $(1 + \frac{1}{2}x)^{10}$ in ascending powers of x. **[4]**

(ii) Hence find the coefficient of x^3 in the expansion of $(3 + 4x + 2x^2)(1 + \frac{1}{2}x)^{10}$. **[3]**

4 A sequence $u_1, u_2, u_3, \ldots$ is defined by $u_n = 5n + 1$.

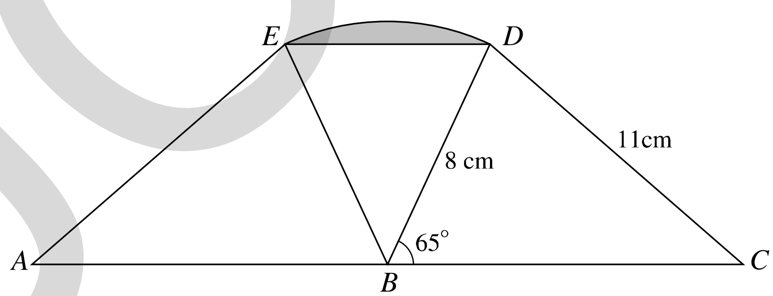
(i) State the values of u_1, u_2 and u_3. **[1]**

(ii) Evaluate $\sum_{n=1}^{40} u_n$. **[3]**

Another sequence $w_1, w_2, w_3, \ldots$ is defined by $w_1 = 2$ and $w_{n+1} = 5w_n + 1$.

(iii) Find the value of p such that $u_p = w_3$. **[3]**

5



The diagram shows two congruent triangles, BCD and BAE, where ABC is a straight line. In triangle BCD, $BD = 8$ cm, $CD = 11$ cm and angle $CBD = 65°$. The points E and D are joined by an arc of a circle with centre B and radius 8 cm.

(i) Find angle BCD. **[2]**

(ii) **(a)** Show that angle EBD is 0.873 radians, correct to 3 significant figures. **[2]**

(b) Hence find the area of the shaded segment bounded by the chord ED and the arc ED, giving your answer correct to 3 significant figures. **[4]**

UCLES

6 **(a)** Use integration to find the exact area of the region enclosed by the curve $y = x^2 + 4x$, the x-axis and the lines $x = 3$ and $x = 5$. **[4]**

(b) Find $\int (2 - 6\sqrt{y})\, dy$. **[3]**

(c) Evaluate $\int_1^{\infty} \frac{8}{x^3}\, dx$. **[4]**

7 **(i)** Show that $\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} \equiv \tan^2 x - 1$. **[2]**

(ii) Hence solve the equation

$$\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = 5 - \tan x,$$

for $0° \leqslant x \leqslant 360°$. **[6]**

8 **(a)** Use logarithms to solve the equation $5^{3w-1} = 4^{250}$, giving the value of w correct to 3 significant figures. **[5]**

(b) Given that $\log_x(5y + 1) - \log_x 3 = 4$, express y in terms of x. **[4]**

9 A geometric progression has first term a and common ratio r, and the terms are all different. The first, second and fourth terms of the geometric progression form the first three terms of an arithmetic progression.

(i) Show that $r^3 - 2r + 1 = 0$. **[3]**

(ii) Given that the geometric progression converges, find the exact value of r. **[5]**

(iii) Given also that the sum to infinity of this geometric progression is $3 + \sqrt{5}$, find the value of the integer a. **[4]**

UCLES

UCLES

OCR
RECOGNISING ACHIEVEMENT

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4723

Core Mathematics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes

4723

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

UCLES

© OCR 2010 [L/102/2710] OCR is an exempt Charity
2R–9K03

1 Find $\frac{dy}{dx}$ in each of the following cases:

(i) $y = x^3 e^{2x}$, **[2]**

(ii) $y = \ln(3 + 2x^2)$, **[2]**

(iii) $y = \frac{x}{2x+1}$. **[2]**

2 The transformations R, S and T are defined as follows.

R : reflection in the x-axis
S : stretch in the x-direction with scale factor 3
T : translation in the positive x-direction by 4 units

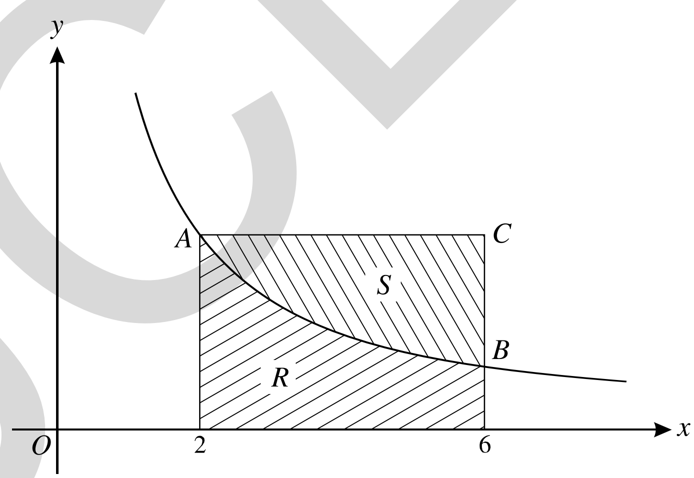
(i) The curve $y = \ln x$ is transformed by R followed by T. Find the equation of the resulting curve. **[2]**

(ii) Find, in terms of S and T, a sequence of transformations that transforms the curve $y = x^3$ to the curve $y = \left(\frac{1}{9}x - 4\right)^3$. You should make clear the order of the transformations. **[2]**

3 **(i)** Express the equation $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$ in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are constants. **[3]**

(ii) Hence solve, for $-180° < \theta < 180°$, the equation $\operatorname{cosec}\theta(3\cos 2\theta + 7) + 11 = 0$. **[3]**

4



The diagram shows part of the curve $y = \frac{k}{x}$, where k is a positive constant. The points A and B on the curve have x-coordinates 2 and 6 respectively. Lines through A and B parallel to the axes as shown meet at the point C. The region R is bounded by the curve and the lines $x = 2$, $x = 6$ and $y = 0$. The region S is bounded by the curve and the lines AC and BC. It is given that the area of the region R is $\ln 81$.

(i) Show that $k = 4$. **[3]**

(ii) Find the exact volume of the solid produced when the region S is rotated completely about the x-axis. **[4]**

5 **(i)** Solve the inequality $|2x+1| \leqslant |x-3|$. **[5]**

(ii) Given that x satisfies the inequality $|2x+1| \leqslant |x-3|$, find the greatest possible value of $|x+2|$. **[2]**

6 **(i)** Show by calculation that the equation

$$\tan^2 x - x - 2 = 0,$$

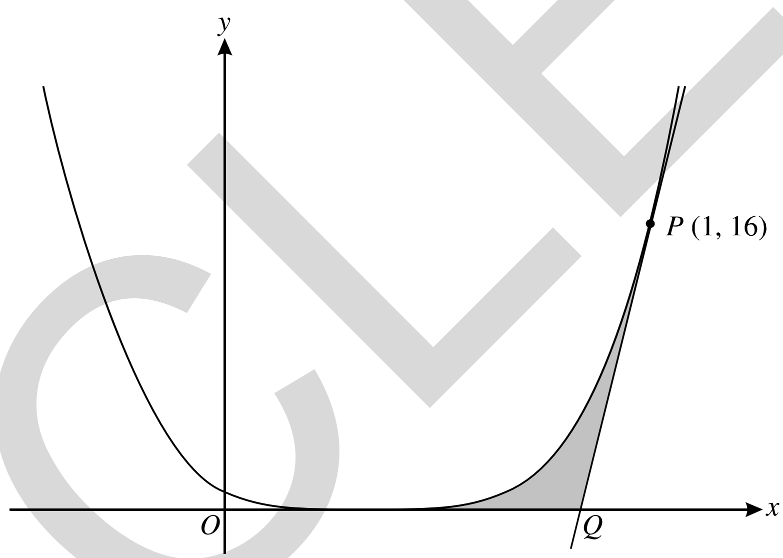
where x is measured in radians, has a root between 1.0 and 1.1. **[3]**

(ii) Use the iteration formula $x_{n+1} = \tan^{-1}\sqrt{2+x_n}$ with a suitable starting value to find this root correct to 5 decimal places. You should show the outcome of each step of the process. **[4]**

(iii) Deduce a root of the equation

$$\sec^2 2x - 2x - 3 = 0.$$ **[3]**

7



The diagram shows the curve with equation $y = (3x-1)^4$. The point P on the curve has coordinates $(1, 16)$ and the tangent to the curve at P meets the x-axis at the point Q. The shaded region is bounded by PQ, the x-axis and that part of the curve for which $\frac{1}{3} \leqslant x \leqslant 1$. Find the exact area of this shaded region. **[10]**

8 **(i)** Express $3\cos x + 3\sin x$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. **[3]**

(ii) The expression $\mathrm{T}(x)$ is defined by $\mathrm{T}(x) = \dfrac{8}{3\cos x + 3\sin x}$.

(a) Determine a value of x for which $\mathrm{T}(x)$ is not defined. **[2]**

(b) Find the smallest positive value of x satisfying $\mathrm{T}(3x) = \frac{8}{9}\sqrt{6}$, giving your answer in an exact form. **[4]**

[Question 9 is printed overleaf.]

UCLES

9 The functions f and g are defined for all real values of x by

$$f(x) = 4x^2 - 12x \qquad \text{and} \qquad g(x) = ax + b,$$

where a and b are non-zero constants.

(i) Find the range of f. **[3]**

(ii) Explain why the function f has no inverse. **[2]**

(iii) Given that $g^{-1}(x) = g(x)$ for all values of x, show that $a = -1$. **[4]**

(iv) Given further that $gf(x) < 5$ for all values of x, find the set of possible values of b. **[4]**

UCLES

OCR
RECOGNISING ACHIEVEMENT

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

© OCR 2010

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE
MATHEMATICS 4724
Core Mathematics 4

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Friday 11 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

UCLES

© OCR 2010 [R/102/2711] OCR is an exempt Charity

1 Expand $(1 + 3x)^{-\frac{5}{3}}$ in ascending powers of x, up to and including the term in x^3. **[5]**

2 Given that $y = \dfrac{\cos x}{1 - \sin x}$, find $\dfrac{dy}{dx}$, simplifying your answer. **[4]**

3 Express $\dfrac{x^2}{(x-1)^2(x-2)}$ in partial fractions. **[5]**

4 Use the substitution $u = \sqrt{x+2}$ to find the exact value of

$$\int_{-1}^{7} \frac{x^2}{\sqrt{x+2}}\, dx.$$ **[7]**

5 Find the coordinates of the two stationary points on the curve with equation

$$x^2 + 4xy + 2y^2 + 18 = 0.$$ **[7]**

6 Lines l_1 and l_2 have vector equations

$$\mathbf{r} = \mathbf{j} + \mathbf{k} + t(2\mathbf{i} + a\mathbf{j} + \mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = 3\mathbf{i} - \mathbf{k} + s(2\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})$$

respectively, where t and s are parameters and a is a constant.

(i) Given that l_1 and l_2 are perpendicular, find the value of a. **[3]**

(ii) Given instead that l_1 and l_2 intersect, find

(a) the value of a, **[4]**

(b) the angle between the lines. **[3]**

7 The parametric equations of a curve are $x = \dfrac{t+2}{t+1}$, $y = \dfrac{2}{t+3}$.

(i) Show that $\dfrac{dy}{dx} > 0$. **[6]**

(ii) Find the cartesian equation of the curve, giving your answer in a form not involving fractions. **[5]**

8 **(i)** Find the quotient and the remainder when $x^2 - 5x + 6$ is divided by $x - 1$. **[3]**

(ii) **(a)** Find the general solution of the differential equation

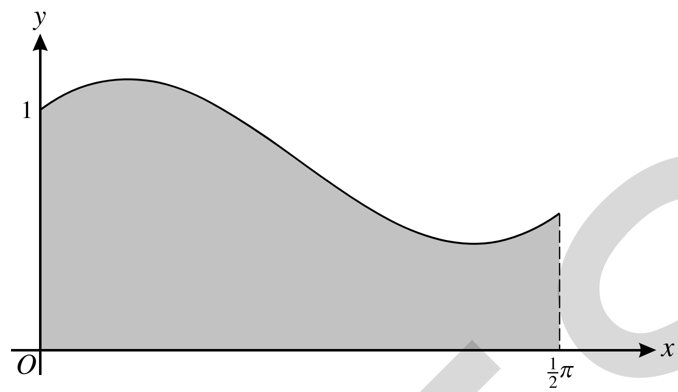
$$\left(\frac{x-1}{x^2 - 5x + 6}\right)\frac{dy}{dx} = y - 5.$$ **[3]**

(b) Given that $y = 7$ when $x = 8$, find y when $x = 6$. **[4]**

UCLES

9 **(i)** Find $\int (x + \cos 2x)^2 \, dx$. **[9]**

(ii)



The diagram shows the part of the curve $y = x + \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The shaded region bounded by the curve, the axes and the line $x = \frac{1}{2}\pi$ is rotated completely about the x-axis to form a solid of revolution of volume V. Find V, giving your answer in an exact form. **[4]**

UCLES

UCLES

OCR
RECOGNISING ACHIEVEMENT

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.