Cambridge Assessment

A Level

Mathematics

Session:	2010 June
Type:	Question paper
Code:	3890-7890; 3892-7892
Unit:	4728, 4729, 4730, 4731

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

4728

Mechanics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book

OCR Supplied Materials:
- Printed Answer Book 4728
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Tuesday 15 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- **The questions are on the inserted Question Paper.**
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72.**
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or destroyed.

1 A block B of mass 3 kg moves with deceleration $1.2\,\text{m s}^{-2}$ in a straight line on a rough horizontal surface. The initial speed of B is $5\,\text{m s}^{-1}$. Calculate

(i) the time for which B is in motion, **[2]**

(ii) the distance travelled by B before it comes to rest, **[2]**

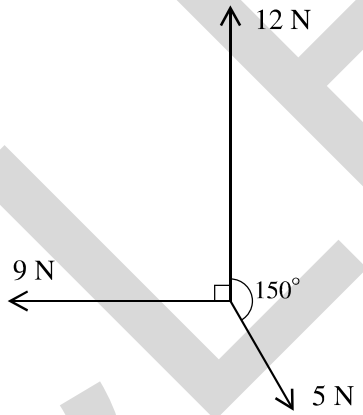
(iii) the coefficient of friction between B and the surface. **[4]**

2 Two particles P and Q are moving in opposite directions in the same straight line on a smooth horizontal surface when they collide. P has mass 0.4 kg and speed $3\,\text{m s}^{-1}$. Q has mass 0.6 kg and speed $1.5\,\text{m s}^{-1}$. Immediately after the collision, the speed of P is $0.1\,\text{m s}^{-1}$.

(i) Given that P and Q are moving in the same direction after the collision, find the speed of Q. **[4]**

(ii) Given instead that P and Q are moving in opposite directions after the collision, find the distance between them 3 s after the collision. **[5]**

3



Three horizontal forces of magnitudes 12 N, 5 N, and 9 N act along bearings 000°, 150° and 270° respectively (see diagram).

(i) Show that the component of the resultant of the three forces along bearing 270° has magnitude 6.5 N. **[2]**

(ii) Find the component of the resultant of the three forces along bearing 000°. **[2]**

(iii) Hence find the magnitude and bearing of the resultant of the three forces. **[5]**

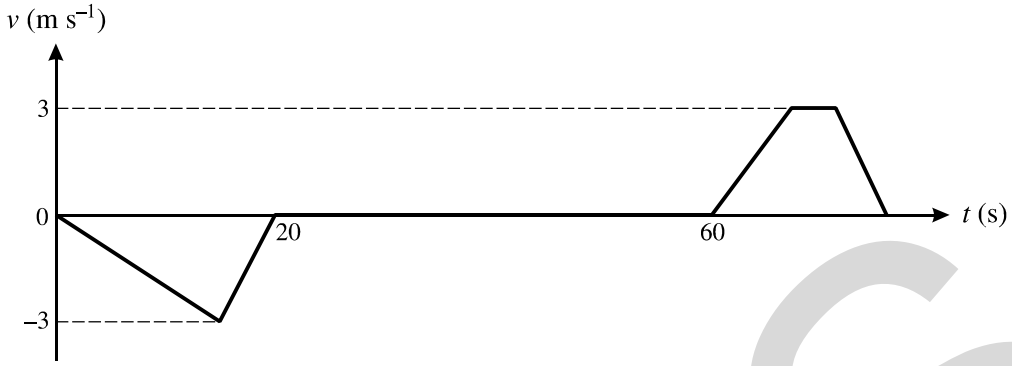
4 A particle P moving in a straight line has velocity $v\,\text{m s}^{-1}$ at time t s after passing through a fixed point O. It is given that $v = 3.2 - 0.2t^2$ for $0 \leqslant t \leqslant 5$. Calculate

(i) the value of t when P is at instantaneous rest, **[2]**

(ii) the acceleration of P when it is at instantaneous rest, **[3]**

(iii) the greatest distance of P from O. **[5]**

5



The diagram shows the (t, v) graph for a lorry delivering waste to a recycling centre. The graph consists of six straight line segments. The lorry reverses in a straight line from a stationary position on a weighbridge before coming to rest. It deposits its waste and then moves forwards in a straight line accelerating to a maximum speed of $3\,\text{m s}^{-1}$. It maintains this speed for 4 s and then decelerates, coming to rest at the weighbridge.

(i) Calculate the distance from the weighbridge to the point where the lorry deposits the waste. **[2]**

(ii) Calculate the time which elapses between the lorry leaving the weighbridge and returning to it. **[4]**

(iii) Given that the acceleration of the lorry when it is moving forwards is $0.4\,\text{m s}^{-2}$, calculate its final deceleration. **[3]**

6 A block B of mass 0.85 kg lies on a smooth slope inclined at 30° to the horizontal. B is attached to one end of a light inextensible string which is parallel to the slope. At the top of the slope, the string passes over a smooth pulley. The other end of the string hangs vertically and is attached to a particle P of mass 0.55 kg. The string is taut at the instant when P is projected vertically downwards.

(i) Calculate

(a) the acceleration of B and the tension in the string, **[5]**

(b) the magnitude of the force exerted by the string on the pulley. **[2]**

The initial speed of P is $1.3\,\text{m s}^{-1}$ and after moving 1.5 m P reaches the ground, where it remains at rest. B continues to move up the slope and does not reach the pulley.

(ii) Calculate the total distance B moves up the slope before coming instantaneously to rest. **[6]**

[Question 7 is printed overleaf.]

7

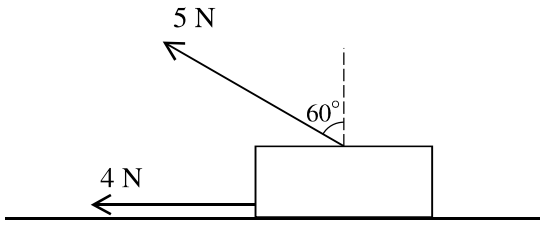


Fig. 1

A rectangular block B of weight 12 N lies in limiting equilibrium on a horizontal surface. A horizontal force of 4 N and a coplanar force of 5 N inclined at 60° to the vertical act on B (see Fig. 1).

(i) Find the coefficient of friction between B and the surface. **[6]**

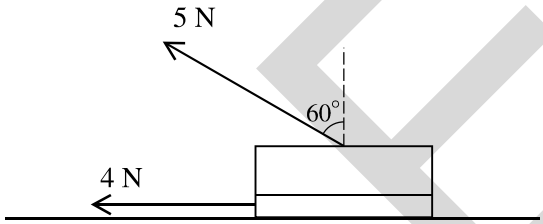


Fig. 2

B is now cut horizontally into two smaller blocks. The upper block has weight 9 N and the lower block has weight 3 N. The 5 N force now acts on the upper block and the 4 N force now acts on the lower block (see Fig. 2). The coefficient of friction between the two blocks is μ.

(ii) Given that the upper block is in limiting equilibrium, find μ. **[2]**

(iii) Given instead that $\mu = 0.1$, find the accelerations of the two blocks. **[6]**

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MATHEMATICS

Mechanics 2

4729

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Friday 18 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
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- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

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- The total number of marks for this paper is **72.**
- This document consists of **4** pages. Any blank pages are indicated.

1 A particle is projected horizontally with a speed of $7\,\text{m s}^{-1}$ from a point 10 m above horizontal ground. The particle moves freely under gravity. Calculate the speed and direction of motion of the particle at the instant it hits the ground. **[6]**

2 **(i)**

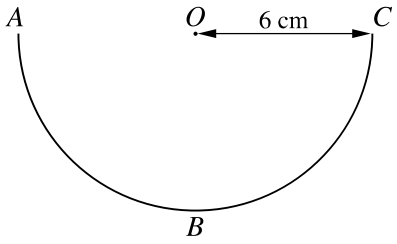


Fig. 1

A uniform piece of wire, ABC, forms a semicircular arc of radius 6 cm. O is the mid-point of AC (see Fig. 1). Show that the distance from O to the centre of mass of the wire is 3.82 cm, correct to 3 significant figures. **[2]**

(ii)

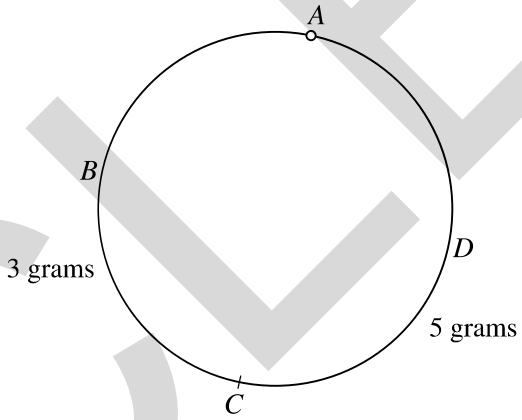


Fig. 2

Two semicircular pieces of wire, ABC and ADC, are joined together at their ends to form a circular hoop of radius 6 cm. The mass of ABC is 3 grams and the mass of ADC is 5 grams. The hoop is freely suspended from A (see Fig. 2). Calculate the angle which the diameter AC makes with the vertical, giving your answer correct to the nearest degree. **[5]**

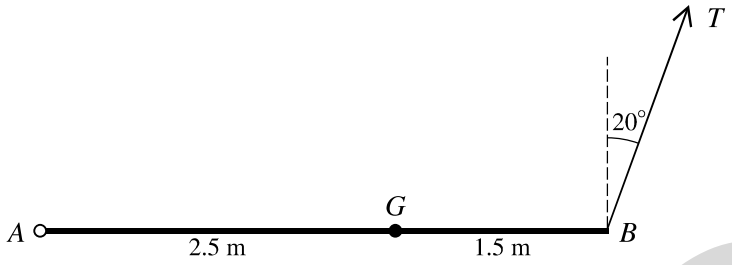
3 The maximum power produced by the engine of a small aeroplane of mass 2 tonnes is 128 kW. Air resistance opposes the motion directly and the lift force is perpendicular to the direction of motion. The magnitude of the air resistance is proportional to the square of the speed and the maximum steady speed in level flight is $80\,\text{m s}^{-1}$.

(i) Calculate the magnitude of the air resistance when the speed is $60\,\text{m s}^{-1}$. **[5]**

The aeroplane is climbing at a constant angle of 2° to the horizontal.

(ii) Find the maximum acceleration at an instant when the speed of the aeroplane is $60\,\text{m s}^{-1}$. **[4]**

4

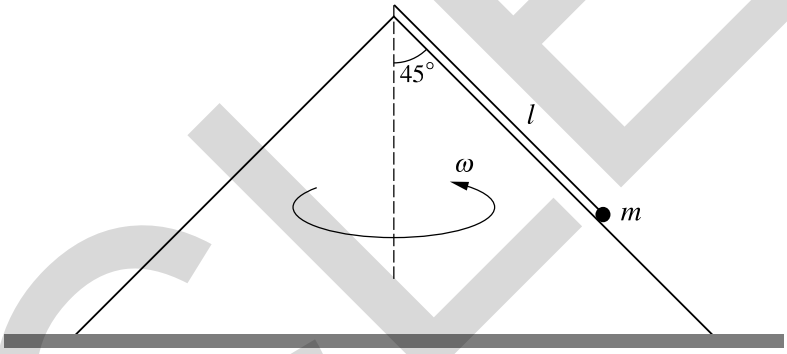


A non-uniform beam AB of length 4 m and mass 5 kg has its centre of mass at the point G of the beam where $AG = 2.5$ m. The beam is freely suspended from its end A and is held in a horizontal position by means of a wire attached to the end B. The wire makes an angle of 20° with the vertical and the tension is T N (see diagram).

(i) Calculate T. **[3]**

(ii) Calculate the magnitude and the direction of the force acting on the beam at A. **[7]**

5



One end of a light inextensible string of length l is attached to the vertex of a smooth cone of semi-vertical angle 45°. The cone is fixed to the ground with its axis vertical. The other end of the string is attached to a particle of mass m which rotates in a horizontal circle in contact with the outer surface of the cone. The angular speed of the particle is ω (see diagram). The tension in the string is T and the contact force between the cone and the particle is R.

(i) By resolving horizontally and vertically, find two equations involving T and R and hence show that $T = \frac{1}{2}m(\sqrt{2}g + l\omega^2)$. **[6]**

(ii) When the string has length 0.8 m, calculate the greatest value of ω for which the particle remains in contact with the cone. **[4]**

[Questions 6 and 7 are printed overleaf.]

6 A particle A of mass $2m$ is moving with speed u on a smooth horizontal surface when it collides with a stationary particle B of mass m. After the collision the speed of A is v, the speed of B is $3v$ and the particles move in the same direction.

(i) Find v in terms of u. **[3]**

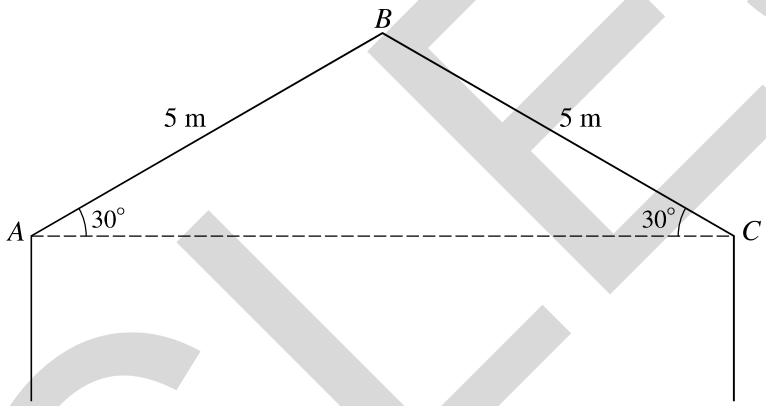
(ii) Show that the coefficient of restitution between A and B is $\frac{4}{5}$. **[2]**

B subsequently hits a vertical wall which is perpendicular to the direction of motion. As a result of the impact, B loses $\frac{3}{4}$ of its kinetic energy.

(iii) Show that the speed of B after hitting the wall is $\frac{3}{5}u$. **[4]**

(iv) B then hits A. Calculate the speeds of A and B, in terms of u, after this collision and state their directions of motion. **[8]**

7



A small ball of mass 0.2 kg is projected with speed 11 m s^{-1} up a line of greatest slope of a roof from a point A at the bottom of the roof. The ball remains in contact with the roof and moves up the line of greatest slope to the top of the roof at B. The roof is rough and the coefficient of friction is $\frac{1}{2}$. The distance AB is 5 m and AB is inclined at 30° to the horizontal (see diagram).

(i) Show that the speed of the ball when it reaches B is 5.44 m s^{-1}, correct to 2 decimal places. **[6]**

The ball leaves the roof at B and moves freely under gravity. The point C is at the lower edge of the roof. The distance BC is 5 m and BC is inclined at 30° to the horizontal.

(ii) Determine whether or not the ball hits the roof between B and C. **[7]**

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MATHEMATICS

Mechanics 3

4730

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Tuesday 15 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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INFORMATION FOR CANDIDATES

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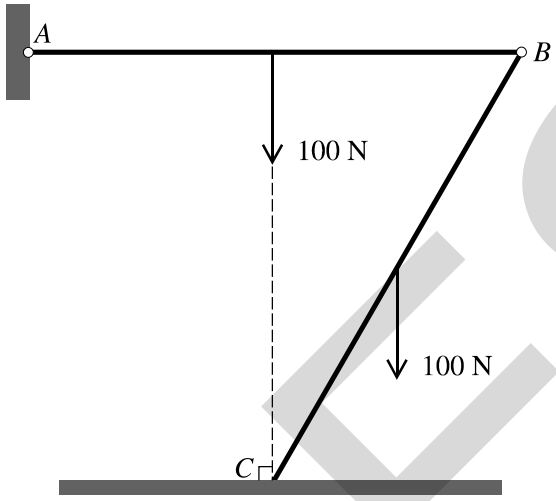
- The total number of marks for this paper is **72.**
- This document consists of **8** pages. Any blank pages are indicated.

1 A small ball of mass 0.8 kg is moving with speed $10.5\,\text{m s}^{-1}$ when it receives an impulse of magnitude 4 N s. The speed of the ball immediately afterwards is $8.5\,\text{m s}^{-1}$. The angle between the directions of motion before and after the impulse acts is α. Using an impulse-momentum triangle, or otherwise, find α. **[6]**

2



Two uniform rods AB and BC are of equal length and each has weight 100 N. The rods are freely jointed to each other at B, and A is freely jointed to a fixed point. The rods are in equilibrium in a vertical plane with AB horizontal and C resting on a rough horizontal surface. C is vertically below the mid-point of AB (see diagram).

(i) By taking moments about A for AB, find the vertical component of the force on AB at B. Hence find the vertical component of the contact force on BC at C. **[3]**

(ii) Calculate the magnitude of the frictional force on BC at C and state its direction. **[4]**

3

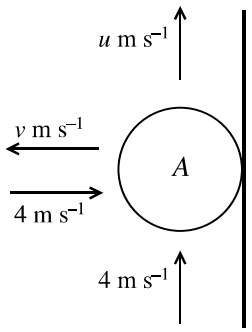


Fig. 1

A uniform smooth sphere A moves on a smooth horizontal surface towards a smooth vertical wall. Immediately before the sphere hits the wall it has components of velocity parallel and perpendicular to the wall each of magnitude $4\,\text{m s}^{-1}$. Immediately after hitting the wall the components have magnitudes $u\,\text{m s}^{-1}$ and $v\,\text{m s}^{-1}$, respectively (see Fig. 1).

(i) Given that the coefficient of restitution between the sphere and the wall is $\frac{1}{2}$, state the values of u and v. **[2]**

Shortly after hitting the wall the sphere A comes into contact with another uniform smooth sphere B, which has the same mass and radius as A. The sphere B is stationary and at the instant of contact the line of centres of the spheres is parallel to the wall (see Fig. 2). The contact between the spheres is perfectly elastic.

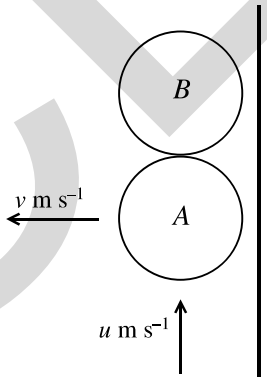


Fig. 2

(ii) Find, for each sphere, its speed and its direction of motion immediately after the contact. **[6]**

4 O is a fixed point on a horizontal plane. A particle P of mass 0.25 kg is released from rest at O and moves in a straight line on the plane. At time t s after release the only horizontal force acting on P has magnitude

$$\tfrac{1}{2400}\left(144 - t^2\right) \text{N} \quad \text{for } 0 \leqslant t \leqslant 12$$

and

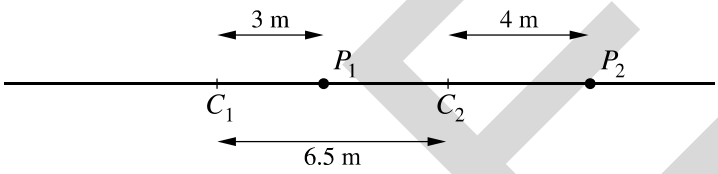
$$\tfrac{1}{2400}\left(t^2 - 144\right) \text{N} \quad \text{for } t \geqslant 12.$$

The force acts in the direction of P's motion. P's velocity at time t s is v m s^{-1}.

(i) Find an expression for v in terms of t, valid for $t \geqslant 12$, and hence show that v is three times greater when $t = 24$ than it is when $t = 12$. **[8]**

(ii) Sketch the (t, v) graph for $0 \leqslant t \leqslant 24$. **[3]**

5



Particles P_1 and P_2 are each moving with simple harmonic motion along the same straight line. P_1's motion has centre C_1, period 2π s and amplitude 3 m; P_2's motion has centre C_2, period $\frac{4}{3}\pi$ s and amplitude 4 m. The points C_1 and C_2 are 6.5 m apart. The displacements of P_1 and P_2 from their centres of oscillation at time t s are denoted by x_1 m and x_2 m respectively. The diagram shows the positions of the particles at time $t = 0$, when $x_1 = 3$ and $x_2 = 4$.

(i) State expressions for x_1 and x_2 in terms of t, which are valid until the particles collide. **[3]**

The particles collide when $t = 5.99$, correct to 3 significant figures.

(ii) Find the distance travelled by P_2 before the collision takes place. **[4]**

(iii) Find the velocities of P_1 and P_2 immediately before the collision, and state whether the particles are travelling in the same direction or in opposite directions. **[4]**

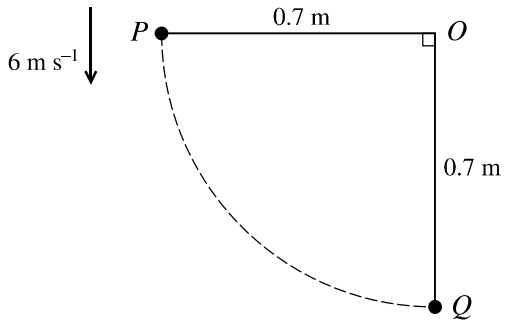
6 A bungee jumper of weight W N is joined to a fixed point O by a light elastic rope of natural length 20 m and modulus of elasticity 32 000 N. The jumper starts from rest at O and falls vertically. The jumper is modelled as a particle and air resistance is ignored.

(i) Given that the jumper just reaches a point 25 m below O, find the value of W. **[5]**

(ii) Find the maximum speed reached by the jumper. **[4]**

(iii) Find the maximum value of the deceleration of the jumper during the downward motion. **[3]**

7



A particle P is attached to a fixed point O by a light inextensible string of length 0.7 m. A particle Q is in equilibrium suspended from O by an identical string. With the string OP taut and horizontal, P is projected vertically downwards with speed 6 m s^{-1} so that it strikes Q directly (see diagram). P is brought to rest by the collision and Q starts to move with speed 4.9 m s^{-1}.

(i) Find the speed of P immediately before the collision. Hence find the coefficient of restitution between P and Q. **[3]**

(ii) Given that the speed of Q is v m s^{-1} when OQ makes an angle θ with the downward vertical, find an expression for v^2 in terms of θ, and show that the tension in the string OQ is $14.7m(1 + 2\cos\theta)$ N, where m kg is the mass of Q. **[6]**

(iii) Find the radial and transverse components of the acceleration of Q at the instant that the string OQ becomes slack. **[4]**

(iv) Show that $V^2 = 0.8575$, where V m s^{-1} is the speed of Q when it reaches its greatest height (after the string OQ becomes slack). Hence find the greatest height reached by Q above its initial position. **[4]**

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MATHEMATICS

Mechanics 4

4731

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Thursday 24 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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INFORMATION FOR CANDIDATES

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1 A wheel is rotating and is slowing down with constant angular deceleration. The initial angular speed is $80\,\text{rad}\,\text{s}^{-1}$, and after 15 s the wheel has turned through 1020 radians.

(i) Find the angular deceleration of the wheel. **[2]**

(ii) Find the angle through which the wheel turns in the last 5 s before it comes to rest. **[2]**

(iii) Find the total number of revolutions made by the wheel from the start until it comes to rest. **[3]**

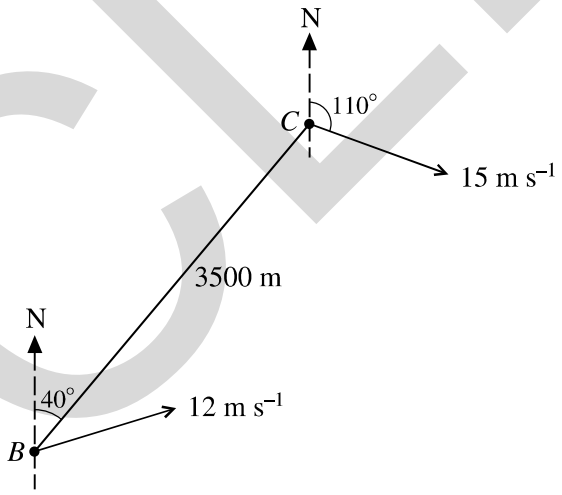
2 The region bounded by the x-axis, the y-axis, the line $x = \ln 3$, and the curve $y = \text{e}^{-x}$ for $0 \leqslant x \leqslant \ln 3$, is occupied by a uniform lamina. Find, in an exact form, the coordinates of the centre of mass of this lamina. **[9]**

3 A circular disc is rotating in a horizontal plane with angular speed $16\,\text{rad}\,\text{s}^{-1}$ about a fixed vertical axis passing through its centre O. The moment of inertia of the disc about the axis is $0.9\,\text{kg}\,\text{m}^2$. A particle, initially at rest just above the surface of the disc, drops onto the disc and sticks to it at a point 0.4 m from O. Afterwards, the angular speed of the disc with the particle attached is $15\,\text{rad}\,\text{s}^{-1}$.

(i) Find the mass of the particle. **[4]**

(ii) Find the loss of kinetic energy. **[3]**

4



From a boat B, a cruiser C is observed 3500 m away on a bearing of 040°. The cruiser C is travelling with constant speed $15\,\text{m}\,\text{s}^{-1}$ along a straight line course with bearing 110° (see diagram). The boat B travels with constant speed $12\,\text{m}\,\text{s}^{-1}$ on a straight line course which takes it as close as possible to the cruiser C.

(i) Show that the bearing of the course of B is 073°, correct to the nearest degree. **[4]**

(ii) Find the magnitude and the bearing of the velocity of C relative to B. **[3]**

(iii) Find the shortest distance between B and C in the subsequent motion. **[3]**

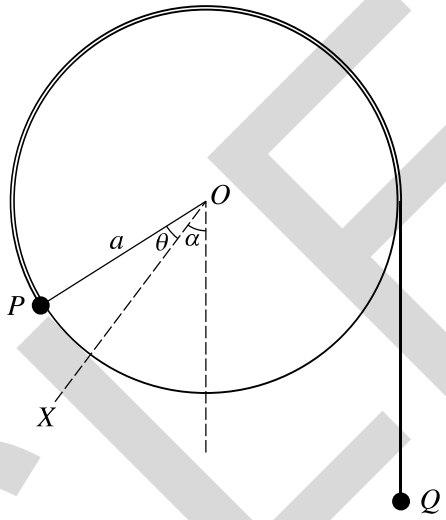
5 A uniform rod AB has mass m and length $6a$. The point C on the rod is such that $AC = a$. The rod can rotate freely in a vertical plane about a fixed horizontal axis passing through C and perpendicular to the rod.

(i) Show by integration that the moment of inertia of the rod about this axis is $7ma^2$. **[5]**

The rod starts at rest with B vertically below C. A couple of constant moment $\frac{6mga}{\pi}$ is then applied to the rod.

(ii) Find, in terms of a and g, the angular speed of the rod when it has turned through one and a half revolutions. **[6]**

6



A light pulley of radius a is free to rotate in a vertical plane about a fixed horizontal axis passing through its centre O. Two particles, P of mass $5m$ and Q of mass $3m$, are connected by a light inextensible string. The particle P is attached to the circumference of the pulley, the string passes over the top of the pulley, and Q hangs below the pulley on the opposite side to P. The section of string not in contact with the pulley is vertical. The fixed line OX makes an angle α with the downward vertical, where $\cos\alpha = \frac{4}{5}$, and OP makes an angle θ with OX (see diagram).

You are given that the total potential energy of the system (using a suitable reference level) is V, where

$$V = mga(3\sin\theta - 4\cos\theta - 3\theta).$$

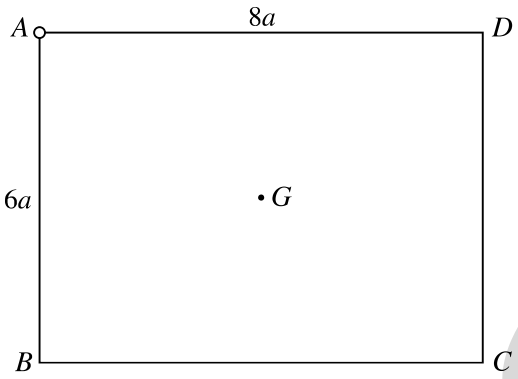
(i) Show that $\theta = 0$ is a position of stable equilibrium. **[5]**

(ii) Show that the kinetic energy of the system is $4ma^2\dot{\theta}^2$. **[2]**

(iii) By differentiating the energy equation, then making suitable approximations for $\sin\theta$ and $\cos\theta$, find the approximate period of small oscillations about the equilibrium position $\theta = 0$. **[5]**

[Question 7 is printed overleaf.]

7



The diagram shows a uniform rectangular lamina $ABCD$ with $AB = 6a$, $AD = 8a$ and centre G. The mass of the lamina is m. The lamina rotates freely in a vertical plane about a fixed horizontal axis passing through A and perpendicular to the lamina.

(i) Find the moment of inertia of the lamina about this axis. **[3]**

The lamina is released from rest with AD horizontal and BC below AD.

(ii) For an instant during the subsequent motion when AD is vertical, show that the angular speed of the lamina is $\sqrt{\frac{3g}{50a}}$ and find its angular acceleration. **[5]**

At an instant when AD is vertical, the force acting on the lamina at A has magnitude F.

(iii) By finding components parallel and perpendicular to GA, or otherwise, show that $F = \frac{\sqrt{493}}{20}mg$. **[8]**