

A possible formula to determine the percentage of candidates who should receive the new GCSE grade 9 in each subject

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Motivation and disclaimer

Reformed GCSEs will be awarded from summer 2017 onwards beginning with Mathematics and English with other subjects being awarded in later years. Part of the reform is a change to the grading scale from the symbols A*-G to the numbers 1-9 with 9 representing the highest grade. The research in this report was motivated in response to the publication of a recommendation submitted to the Ofqual board in August 2014. The recommendation was, that in every examination, the 8/9 boundary would be set so that 20 per cent of those candidates awarded at least a grade 7 will be awarded a grade 9¹. This led to concerns that this would make it too difficult to achieve the top grade in some subjects whilst making it too easy in others.

Whilst this report promotes a particular approach, which has been discussed with colleagues as a potential solution, it is acknowledged that setting a new grading standard that will be relatively comparable across different subjects is not an easy task and is likely to be controversial. This report summarises my own thoughts on approaching the issue for the first awarding of reformed GCSEs in each subject and does not necessarily reflect the view of Cambridge Assessment as a whole.

¹ Notes of the relevant board meeting are available from

https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/377771/2014-09-12-board-paper-for-new-gcses-in.pdf.

Introduction

In September 2014 Ofqual announced their intended approach towards setting standards in the new GCSEs². This included a proposal that, for the reformed GCSEs, grade 9 should be awarded to the top 20 per cent of candidates achieving at least grade 7. In addition, in terms of expected pass rates, grade 7 will be equivalent to grade A in current GCSEs. This report examines a weakness of this proposal and suggests a simple formula that should improve things.

Given the way in which GCSE results will be used, it is important that grade 9 represents a fairly similar standard of attainment across awarding organisations (AOs) and across subjects. For example the Department for Education's guidance on the new progress 8 measures³ for school accountability states that "Points will be allocated to the new GCSEs on a 1 - 9 point scale corresponding to the new 1 to 9 grades, e.g. a grade 9 will get 9 points in the performance measures." Given that for this purpose (and inevitably for others) point scores will be used interchangeably across subjects, it is important that a grade 9 in one subject indicates roughly the same level of achievement as grade 9 in another. Although precisely defining inter-subject comparability can be tricky, clearly we do not want the new rule to create any obvious disparities between subjects.

Comparability of grades across subjects is not a new research issue. However, the currently suggested rule may exacerbate this problem in some cases, and create new problem in others.

For example,

- Research published by Ofqual on inter-subject comparability⁴ suggested that, at present, the A* grade for Design and Technology is easier to achieve than the A* grade for German. However, based on JCQ data⁵, under the originally proposed rule the percentage of Design and Technology candidates awarded grade 9 would only be slightly lower than the percentage currently awarded grade A* (3.5% compared to 5.1%). In contrast, the proportion of German candidates awarded grade 9 would be substantially less than the percentage awarded A* at present (4.7% compared to 8.5%).
- The same research indicated that, at present, the A* grade is of roughly equivalent difficulty between Media, Film and TV (MFT) studies and Biology. However, under the originally suggested rule, this comparability would not be maintained. The proportion of grade 9s in MFT would be only slightly lower than the proportion currently awarded grade A* (3.1% compared to 3.3%) indicating that grade 9 would scarcely be more difficult than the current A*. However, the proportion awarded grade 9 in Biology would be substantially lower than the proportion currently awarded to 13.2%) suggesting a major increase in difficulty.

Although they are only taken by small numbers of candidates, perhaps the greatest impact of the new rules would be on students studying Classical Greek and Latin. Although various statistical techniques have consistently revealed these GCSE subjects as being amongst the most difficult, the currently suggested rules would make this situation much worse. In Latin, the proportion of pupils awarded grade 9 would be substantially lower than the current A* percentage (14.7%)

² See <u>https://www.gov.uk/government/news/setting-standards-for-new-gcses-in-2017</u>.

³ Available from

https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/456438/Progress_8_school_performance_measure_2 015_updated_August_2015.pdf.

⁴ See Figure 8 in <u>https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/486936/3-inter-subject-comparability-pf-exam-standards-in-gcse-and-a-level.pdf.</u>

⁵ From <u>http://www.jcq.org.uk/Download/examination-results/gcses/2015/gcse-and-entry-level-certificate-results-summer-2015</u>. Tables for all UK candidates. The same source of data is used several times throughout this report. The predicted percentage to achieve grade 9 is just calculated as one fifth of those currently achieving grade A or above.

compared to 48.6%⁶). For Classical Greek, the proportion of pupils achieving the top grade would plummet from 66.9% to 17.0%. Each of these scenarios would represent a significant increase in the difficulty of achieving the top grade within subjects where achieving the top grade is possibly already harder than for other GCSE subjects.

To address such issues, this report presents a simple, alternative formula for calculating the percentage of students who should be awarded grade 9 in the reformed GCSEs. This approach may also help with the issue of achieving comparability at grade 9 between different AOs. The thrust of the argument in this report is as follows:

- 1. The greater the proportion of a cohort that achieves at least grade A, the higher the ability of these candidates is likely to be. This can be seen:
 - a. Between subjects, by noting that in subjects with a greater proportion of grade As a greater proportion of these tend to get grade A*
 - b. Between AOs within the same subjects, by noting that, within any subject, the AO predicted (from concurrent attainment) to have the highest cumulative percentage of grade As also tends to be predicted to have a higher percentage of these achieving grade A*.
- 2. For these reasons, in order to maintain a reasonable level of comparability between subjects and between AOs, it is inadequate to award grade 9 to a flat percentage of those achieving at least grade 7 across all subjects.
- 3. Instead we suggest adopting the following simple formula:

Proportion of those achieving at least grade 7 who should be awarded grade 9 = $7\%+0.5^*$ (Percentage of candidates awarded grade 7 or above).

For example, if 20% of people get grade 7 or above then the percentage of these awarded grade 9 should be 17%. If (as for Biology grade A) 42% get grade 7 or above, then 28% of these should be awarded grade 9. If (as for grade A in Latin) around 74% of candidates get grade 7 or above, then 44% of these should be awarded grade 9.

- 4. This rule fits with the stated aims for grade 9 in that:
 - a. Overall, across all subjects, the percentage of those achieving at least grade 7 who will be awarded grade 9 is very close to the 20% suggested by the currently intended rule.
 - b. Across almost every subject, the percentage of candidates achieving grade 9 will be substantially lower than the percentage who achieved grade A*.

The remaining sections of this report expound this argument in more detail.

⁶ Based on published OCR results for June 2015 (<u>http://www.ocr.org.uk/Images/247533-provisional-exam-statistics-june-2015-now-includes-gcse.pdf</u>).

The greater the proportion of a cohort achieving at least grade A, the higher the ability of these candidates is likely to be.

This empirical fact is firstly illustrated between subjects by Figure 1. Based upon JCQ's publication of GCSE results for summer 2015, the points on this chart show the relationship between the cumulative percentage of candidates achieving grade A in each of 49 subjects⁷ and the percentage of these candidates (that is, of those achieving at least grade A) who achieve grade A*. As can be seen, there is a strong relationship between the two (correlation=0.79). This implies that, compared to current approaches to A*, the flat 20% rule will be overly generous to subjects where a small proportion of candidates achieve grade A, and extremely harsh for subjects where many candidates achieve at least grade A.

This is illustrated further by the two lines on the graph. The horizontal line represents the flat 20% rule, whereas the sloped line represents the newly proposed rule for determining the percentage of candidates to be awarded grade 9. As can be seen, the flat rule would result in many GCSEs being awarded a greater number of grade 9s than have historically been awarded A*. In addition, some subjects towards the right hand side of the figure would see a much greater reduction in the number of top grades awarded than others. The sloped line on figure 1 represents the results of the new formula suggested in this report. This formula addresses both problems. Under the suggested formula, only three subjects (Construction, "Other Technology" and Manufacturing - all with very low entry numbers) would see the percentage of candidates awarded grade 9 exceed the percentage who had been awarded A*. Furthermore, the reduction in the proportion of candidates awarded the top grade is more consistent across subjects.



Figure 1: The relationship between the cumulative percentage achieving grade A in a subject and the percentage of these candidates awarded the top grade

One (incorrect) criticism of the above analysis might be that the pattern seen in Figure 1 is the result of a lack of inter-subject comparability – that is, that subjects with a greater percentage of grade As tend to award grade A*s to a greater percentage of these because the A* is too easy in these subjects. However, using previous studies of inter-subject comparability it is possible to

⁷ Note that the "subjects" include some broader grouping such as "Other Modern Languages", "Other Sciences", "Other Technology" and "All other subjects". However, the essential pattern revealed by this chart remains even once these subjects are removed.

demonstrate that this is not the case. For example, Figure 2 uses data from Coe (2008)⁸ to plot the percentage of pupils getting grade A* of those that achieve at least a grade A against the estimated Rasch-Thurstone difficulty of the A* grade. As can be seen, there is no pattern for subjects where the greatest proportion of grade A candidates are awarded grade A* to be easier – if anything, they appear to be more difficult. Despite this, the flat 20% rule suggests an increased difficulty for grade 9 compared to A* for subjects on the right hand side of the figure whilst maintaining a similar level of difficulty for those on the left. There can be no justification in terms of inter-subject comparability for this approach.



Figure 2: The relationship between the percentage of pupils getting grade A* of those that achieve at least a grade A and the Rasch difficulty of the A* grade in different subjects.

Next, we demonstrate that the relationship between awarding more grade As and awarding A*s to a greater percentage of these is not only true across subjects but is also predicted *within* subjects by inter-board screening. Figure 3 is based on aggregated predictions for June 2013 collated as part of the analysis for Benton and Sutch (2014)⁹. Specifically, Figure 3 shows, for each subject within each AO, how the predicted percentage of candidates to achieve grade A (using concurrent attainment) relates to the predicted percentage of these who will achieve grade A^{*10}. Predictions from concurrent attainment, rather than actual awarded percentages, are used to remove the impact of any slight lack of comparability between AOs. This chart confirms the patterns seen earlier.

⁸ Coe, R. (2008) Comparability of GCSE examinations in different subjects: an application of the Rasch model, *Oxford Review of Education*, 34(5), 609-636. Data for Figure 2 is derived from tables 2 and 3 in this paper with the figures for vocational GCSEs and short courses removed. This paper uses data from examinations taken in summer 2004.

⁹ Benton, T., and Sutch, T. (2014) Analysis of the use of Key Stage 2 data in GCSE predictions. Ofqual, Ofqual/14/5471.

¹⁰ That is, the 100*(predicted % A*)/(predicted cum % A).



Figure 3: The relationship between the prediction from concurrent attainment of the cumulative percentage achieving grade A in a subject for each AO and the predicted percentage of these candidates awarded the top grade

However, the results in Figure 3 are partially caused by the between-subject pattern we have seen earlier. Figure 4, removes this influence by centring all of the results within each subject. In order to do this, the predictions on both the x and y axes are adjusted by subtracting the mean prediction within that subject. For example, for one AO within the data used for analysis, 20% of Mathematics candidates were predicted to achieve at least grade A, and 35% of these were predicted to achieve grade A*. This compares to averages of 18.5% and 34% across all four relevant AOs¹¹. Thus the centred figures for Mathematics for the AO in question would be 1.5% and 1% on the x and y axes respectively.

Figure 4 shows that, even when the effect of subjects have been removed, AOs that are predicted to have a greater proportion of grade A (and above) candidates are also predicted to have a greater proportion of these that should achieve grade A*. Note that Figure 4 has removed both the effect of inter-subject comparability (as results have been centred within each subject) and inter-board comparability (as it is based upon predictions from concurrent attainment rather than final awards). This implies that the pattern seen in Figure 4 can only have one reasonable explanation – that where we see a greater proportion of grade A (and above) candidates, the ability of the candidates within this group also tends to be higher. With this in mind, and given the requirement for comparability for the use of GCSEs within school accountability, it is inappropriate to treat all such groups of candidates as if they shared the same level of ability.

Figure 4 also suggests that the newly proposed formula may help to address the inter-board comparability at grade 9. Specifically it shows that once inter-board differences at grade 7 (or equivalently grade A) have been addressed (via Key Stage 2 predictions or any other method), these can help to inform comparability at higher grades such as grade 9. Of course, in future years, once the initial grade 9 standard has been set, there may be more effective ways to address this issue.

¹¹ For the purposes of this chart simple, unweighted, arithmetic averages were used. That is, just the mean of the four relevant figures for the four AOs.



Figure 4: The relationship between the *centred* predictions from concurrent attainment of the cumulative percentage achieving grade A in a subject for each AO and the *centred* predicted percentage of these candidates awarded the top grade

The development of the new formula

The above analyses reveal some weaknesses with the flat 20% formula. The next stage is to develop an alternative. Given that the empirical analysis shows that higher pass rates at grade A tend to be associated with higher achievement amongst those reaching at least this level, it seemed natural to develop a rule based upon a consistent shape for the ability¹² distribution with only the central tendency shifting between different subjects and AOs. This is illustrated in Figure 5. This figure shows (notional) fixed boundaries for grade 7 (which is equivalent to grade A) and grade 9. Two normal distributions are shown that differ only in terms of their mean. As can be seen, the distribution with the higher mean has a greater proportion of candidates above the grade 7 threshold, and also a greater proportion of these who are above the grade 9 threshold.



Figure 5: The effect of a shift in the mean of a distribution

To turn this idea into a working rule we first choose a value to represent the pass mark for grade A. Actually any value can be chosen but this was fixed at 0.8 as this corresponds to the 78.8th percentile of the standard normal distribution – that is, the level only exceeded by 21.2% of the distribution which corresponds to the current percentage of GCSE entries that achieve grade A or above. Next we note that, given this threshold for grade A, if we assume a standard normal distribution for achievement within any individual GCSE, then the mean of this distribution can be derived from the grade A pass rate via the following formula:

$$\mu = 0.8 - \Phi^{-1}(1 - Grade A pass rate)$$

Where Φ^{-1} is the inverse of the standard normal cumulative distribution function. Now we simply pick a value for the threshold for grade 9 (T₉) on the standard normal distribution and can then infer that the percentage of candidates achieving grade 9 should be determined by the following formula:

Proportion achieving grade 9 = 1 -
$$\Phi(T_9 - \mu) = 1 - \Phi(T_9 - 0.8 + \Phi^{-1}(1 - Grade A pass rate))$$

The formula above was applied to each subject listed in the JCQ GCSE results data for 2015 with the value of T_9 chosen to ensure that overall, as intended by the current rule, the proportion of GCSEs awarded grade 9 would be one fifth of the proportion awarded grade A or above. A value for T_9 of 1.78 achieved this goal.

¹² The term "ability" is used very loosely here. I am not assuming that all subjects measure the same construct. What I mean is that I want to look at the relationship between the percentage of candidates above one threshold, and the percentage above another as the average of a fixed distribution changes.

Using this value for T₉, applying this approach to each subject listed in the JCQ GCSE results data for 2015 leads to 4.22% of GCSEs overall being awarded grade 9. This is set against the fact that 21.2% of GCSEs were awarded grade A or above in 2015 so that overall 19.9% of GCSEs awarded A or above would be awarded grade 9¹³. The proposed proportion of grade A and above candidates who would be awarded grade 9 in each subject using the above formula is plotted against the cumulative percentage achieving grade A in Figure 6. As can be seen, the relationship is an almost perfect straight line meaning that use of complicated formulae based on the cumulative normal distribution can be avoided with almost no loss of accuracy and replaced with a rule of the type displayed at the beginning of this report. Using regression to find a line of best fit through these points yields coefficients of 6.3% for the intercept, and 0.53 for the slope (see Table 1). These coefficients could have been used directly for our proposed rule. However, in order to simplify the application of the rule (so that the necessary values can be easily calculated mentally) the slope coefficient was rounded down to one significant figure (0.5). In order to maintain the overall percentage of candidates at grade A and above who would be awarded grade 9 at 20%¹⁴ it was then necessary to adjust the intercept upwards slightly to 7%. This provided the coefficients that have been used in the proposed rule.

|--|

		Proposed value		
Coefficient	Original Estimated Value	after rounding adjustments		
Intercept	6.3%	7%		
Slope	0.53	0.5		



Figure 6: Relationship between percentage grade A and above in GCSE subjects and proposed percentage to achieve grade 9

¹³ Calculated as 4.22/21.2.

¹⁴ Actual final figure is 19.9%.

Aims, assumptions and limits on alternative formulae

Although, as described above, the initial thinking used to derive the suggested formula was based upon normal distributions, this should not be seen as a key assumption of applying the rule in practice. Rather, the newly suggested rule was derived with the following three aims:

- 1. Grade 9 will be defined such that roughly 20% of all GCSEs awarded grade A or above would be awarded grade 9.
- 2. Across most subjects, a smaller percentage would be awarded grade 9 than are currently awarded A*.
- 3. The new grade 9 should not create new and unjustifiable issues with inter-subject comparability. This is particularly true for subjects such as Latin and Classical Greek but also (to some extent) for the Single Sciences.

The use of normal distributions was purely a means to an end to (successfully) meet the three criteria above. Clearly there also exist other possible rules that could fulfil the above criteria. However, it has already been shown that, in order to meet the third criterion above, any rule must allow the percentage of grade 7¹⁵ (and above) candidates that will be awarded grade 9 to increase along with the grade 7 pass rate; that is, subjects with higher proportions of candidates at grade 7 or above must be allowed to award grade 9 to a higher proportion of these¹⁶. A linear relationship between these two factors represents the simplest possible way in which such a relationship could be captured. As such, there exists no simpler rule that will meet all of the criteria above. It would be possible to choose different coefficients in the formula (as opposed to 7% and 0.5) to meet criterion 3 but any such rule must also satisfy criteria 1 and 2. This places significant bounds on the range of possible coefficients.

To illustrate this, it is possible to calculate a similar rule to the one above but for use at grade A*. This is done by fitting a regression line to the relationship between the proportion of pupils getting grade A or above in a subject and the proportion of these awarded grade A*. This was done based on published JCQ data from 2015 and illustrated in Figure 7 below. The regression line indicates that the expected proportion of A grade (or above) students to be awarded A* equals 11% plus 0.7 times the percentage of all candidates who achieved grade A or above.

¹⁵ Note that it has already been decided that grade 7 will be linked to grade A. This makes the language confusing as when talking about future awarding we need to say grade 7 whereas when talking about historical data analysis we need to say grade A. For the purposes of this paper the terms "grade 7" and "grade A" are used essentially interchangeably.

¹⁶ Note that a rule based on A* does not avoid this. We just hit the same issue in different form; namely that where a greater proportion of candidates have achieved at least A* it is reasonable that a greater proportion of these would be deserving of grade 9.



Figure 7: The relationship between the percentage of candidates awarded A or above and the percentage of these awarded A* (regression line included)

The coefficients in this A*-based regression (namely 11% and 0.7) provide some bounds for the possible coefficients in any modified grade 9 rule of the type proposed¹⁷. For example, we can infer that the intercept in the formula for grade 9 should certainly be smaller than that used in the A*-based regression formula (11%). If this is not done it would imply that, for the subjects with the lowest percentages of candidates achieving grade A or above, grade 9 would be no more demanding than A* thus violating the second of our criteria. Next, supposing that the intercept was set at this maximum level (11%), it would be necessary to set the slope coefficient to 0.35 in order to fulfil criterion 1 (based on the JCQ 2015 data).

Similarly, we can also note that the intercept for the formula could not reasonably be set below 0%, and that, supposing it was set to equal 0%, it would be necessary to set the slope coefficient to 0.78 in order to fulfil criterion 1.

Using similar logic to this (and assuming that the intercept will be set to equal a whole number) there are precisely 12 possible sets of coefficients for a grade 9 rule of this type that would fulfil both criteria 1 and 2, as well as providing the simplest possible solution to criterion 3. These are listed in Table 2 below. Note that, although all of these would meet the letter of the criteria above, some of them fit with criterion 2 more fully than others. For example, setting the intercept at 11% (and the slope to 0.35) would (by definition) make grade 9 hardly any harder than A* for subjects that have historically awarded few A grades. By implication, this would mean that most of the increased difficulty of grade 9 relative to A* would be amongst subjects that have historically any grade 9 formula with the intercept coefficient close to zero would appear to lead to an inequitable change in the difficulty of the top grade relative to A*; placing a lot of the increased difficulty amongst subjects with historically lower grade A pass rates. For this reason, the solutions away from either the top or bottom of Table 1 would appear ideal. The originally suggested set of coefficients (7% and 0.5) falls into this category. The suggested use of 0.5 of the slope coefficient is aesthetically pleasing and the fact it is rounded to

¹⁷ That is, where the percentage of candidates awarded grade 9 of those awarded grade 7 and above is allowed to increase linearly along with the proportion awarded grade 7 and above.

a single digit helps avoid the appearance of spurious precision. As stated earlier, it also leads to a rule that can be easily applied using mental arithmetic.

Table 2: Possible coefficients for a modified grade 9 rule that would fulfil the necessary criteria (proposed coefficients highlighted)

Intercept	Required Slope
0%	0.78
1%	0.74
2%	0.70
3%	0.66
4%	0.62
5%	0.58
6%	0.54
7%	0.50
8%	0.47
9%	0.43
10%	0.39
11%	0.35

Finally, ignoring the possible coefficients close to the top and bottom of Table 2, we can compare the impact of the different possible rules. This is done in Table 3 below. Table 3 shows the expected percentage of candidates to achieve grade 9 in the two subjects with the highest and lowest percentage of candidates who achieved grade A. These subjects are chosen as they are the ones where the different possible rules would have the largest effects. The percentage of candidates who would achieve grade 9 is expressed both as a percentage of candidates achieving grade A and above, and as a percentage of all candidates. As can be seen, although there are clearly visible differences when expressed as a percentage of candidates achieving grade A or above, the differences in the overall percentage of grade 9s awarded is much smaller. In particular, compared to the proposed coefficients, the choice of a higher intercept (9%) makes a difference of less than 2 percentage points to the overall percentage of candidates awarded grade 9 in any subject. This implies there is no compelling reason to use a different set of coefficients to those that have been proposed.

Warded grade 5 in different Subjects								
	Cum % A (2015)	Cum % A* (2015)	% of A and above who are grade 9			Overall % grade 9		
Subject			Coefs= (3%,0.66)	Coefs= (9%,0.43)	Coefs= (7%,0.50)	Coefs= (3%,0.66)	Coefs= (9%,0.43)	Coefs= (7%,0.50)
Engineering	6.4%	1.2%	7.22%	11.75%	10.20%	0.46%	0.75%	0.65%
Other Modern Languages	63.3%	36.8%	44.78%	36.22%	38.65%	28.34%	22.93%	24.47%

Table 3: The effect of different choices of coefficients on the percentage of candidates awarded grade 9 in different subjects

Final thoughts

One issue that has not been addressed in this report is the effect of each rule on the positioning of boundaries on the mark scale and ensuring that these are reasonably spaced out. There is no reason to suspect that the newly proposed rule will cause more problems than the flat 20% rule. Moreover, the fact that the newly proposed rule results in pass rates closer to those defined by grade A*, which are themselves defined arithmetically *purely* to ensure adequate spacing of the boundaries, would suggest it may also lead to fewer problems in this aspect of awarding. In future, it may be sensible to consider making grade 9 a key boundary and then setting other grade boundaries in the light of this in order to ensure sensible spacing. In addition, although this report suggests a statistical approach to determining the location of grade 9, it would be helpful if, after the first awarding of reformed GCSEs in each subject, a qualitative description could be written of the types of skills displayed by candidates who achieved this grade. This may help users to understand the meaning of these achievements as well as providing descriptive information to assist with expert scrutiny of scripts around this boundary in future years.