Mendick, H. (2008) Subtracting difference: troubling transitions from GCSE to AS level mathematics, *British Educational Research Journal, 34*(6), 711–732.

Noyes, A. & Sealey, P. (2011) Managing learning trajectories: the case of 14–19 mathematics, *Educational Review*, *63*(2), 179–193.

Noyes, A. & Sealey P. (2012) Investigating participation in Advanced level mathematics: a study of student drop-out, *Research Papers in Education*, *27*(1), 123–138.

Office for National Statistics (2015) MYE2: Population Estimates by single year of age and sex for local authorities in the UK, mid-2013 http://www.ons.gov.uk/ons/rel/pop-estimate/population-estimates-for-uk--england-and-wales--scotland-and-northern-ireland/2013/rft---mid-2013-uk-population-estimates.zip

UKCES (2012) *UK Commission's Employer Skills Survey 2011: UK results. Evidence Report 45*. London: UKCES. Retrieved from https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/303374/ukces-employer-skills-survey-11.pdf

Wiliam, D., Brown, M., Kerslake, D., Martin, S. & Neill, H. (1999) The transition from GCSE to A-level in mathematics: a preliminary study, *Advances in Mathematics Education*, *1*(1), 41–56.

Statistics and Mechanics: Comparing the Applied Mathematics of international Mathematics qualifications

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Introduction

In this article, we report on data collated as part of a large-scale study investigating how A level Mathematics and Further Mathematics prepare students for the mathematical demands of university study in a range of subjects. We investigate and compare the applied mathematical content (Mechanics and Statistics) in a range of international Mathematics qualifications and conclude that the A level has notable differences to similar qualifications in other jurisdictions. In particular, the existing modular structure at A level introduces significant variability into the mathematical backgrounds of students studying what is theoretically the same qualification. Although this problem will be rectified by the introduction of prescribed content from 2016, two other differences emerged during this investigation. First, whilst Mechanics content at A level is primarily studied in Mathematics and/or Further Mathematics, in nearly every other jurisdiction this content is studied within the Physics course. Secondly, there appears to be no international consensus about what statistical content is taught at this level. These findings may have implications for ongoing reform at A level, particularly with respect to the applied content in Further Mathematics, and may also prove interesting for employers and universities with a global reach who currently use Mathematics qualifications for admissions or recruitment purposes.

Background

As part of ongoing qualification reform in England and Wales, A level Mathematics and Further Mathematics are being reformed for first teaching in 2016. The reforms have significant implications for the structure and content of post-compulsory Mathematics in the UK. All A levels are moving from a modular to a linear system, meaning that students will be required to take all of their examinations at the end of the two-year course, rather than throughout as is currently the case. Additionally, the AS level and the A level are being 'decoupled'. The A level is currently a two-year course; students sit examinations during the first year which contribute to their final A level grades which also counts as a qualification in its own right (the Advanced Subsidiary or 'AS' level). However, in the reformed A levels, the AS level will become a stand-alone qualification and will no longer count towards a student's overall A level grade.

Students are able to study two Mathematics A levels: Mathematics and Further Mathematics[1]. The four main awarding bodies (AQA, Oxford, Cambridge and RSA (OCR) Examinations, Pearson Edexcel, and WJEC) all offer their own versions of both A levels, and students and schools are able to select which awarding body's specification they would like to study. Currently, there is a great deal of flexibility in the structure of both subjects, particularly in relation to the applied content. Further Mathematics must be studied alongside or after A level Mathematics, and its content builds on material covered in Mathematics.

In A level Mathematics, students sit four 'Core' Pure Mathematics modules, and two 'Applied' modules. The Applied modules can be chosen from Mechanics, Statistics and Decision Mathematics, and students can either take one module from two different strands, or multiple modules from the same area (e.g., Mechanics 1 and Statistics 1, or Mechanics 1 and 2). For example, a student interested in studying Engineering or Physics at university may be encouraged to specialise in Mechanics, whilst a prospective Biologist or Social Scientist may choose only Statistics modules (Lee, Harrison, & Robinson, 2007; A Level Content Advisory Board, [ALCAB] 2014). However, students are rarely able to choose their own modules as these decisions are predominantly made by the school/college. Schools/colleges often lack the resources to offer different modules for individual students and instead tailor their module

1. Two awarding bodies also offer an AS/A level in Statistics, but these are taken by very few students.

selection to the needs of the overall cohort. Consequently, students are often restricted to studying a mixture of modules, usually Statistics 1 and Mechanics 1 or Statistics 1 and Decision 1, rather than specialising in a particular area of Applied Mathematics.

In A level Further Mathematics, students must study two Further Pure Mathematics modules, and an additional four modules which can be a mixture of supplementary Further Pure modules and Applied modules. As the Applied modules have prerequisite modules in the same strand, the more advanced Applied modules, Statistics 3-4 and Mechanics 3-5, can only be studied in Further Mathematics. Consequently, students and teachers currently have considerable freedom regarding which modules they study at A level, which causes a degree of variability in the mathematical background of students when they reach university. This is exacerbated by the fact that awarding bodies often include different content in different modules, meaning that students taking the same modules in different specifications may not necessarily have covered the same content (see the National HE STEM Programme Wales, 2012, for a closer examination of differences in content division between awarding bodies).

However, in the reformed A level Mathematics, 100 per cent of the content will be prescribed by The Office of Qualifications and Examinations Regulation (Ofqual) (see Department for Education, 2014, for details). Additionally, this prescribed content includes both Statistics and Mechanics content, meaning that all students will have some grounding in both of these areas regardless of their school or specification. However, in Further Mathematics, only half of the content will be prescribed. The awarding bodies are able to decide what additional content they will include, which is likely to be applied as all of the prescribed content in Further Mathematics is Pure Mathematics material. This content does not necessarily need to follow the Statistics/Mechanics/Decision Mathematics framework, as awarding bodies may decide to introduce more specialised content such as 'Mathematics for Economists' or 'Mathematics for Biologists'. Nevertheless, it is likely that uptake of such specialised modules would be very low as most schools/colleges would be unable to afford to offer modules to individual students, and thus it is likely that awarding bodies will instead choose to offer optional Mechanics, Statistics or Decision content in Further Mathematics. Although the introduction of prescribed content will ameliorate the situation, there will therefore still be some variability in the mathematical backgrounds of students who have studied Further Mathematics, particularly with reference to Applied Mathematics.

Consequently, the changes to post-compulsory Mathematics qualifications in the UK and the development of the new specifications provide an opportunity to consider how the applied content of the existing A levels compare to their international counterparts.

Method

Mathematics qualifications from a number of international jurisdictions were analysed in order to ascertain the Applied Mathematics content of those qualifications most aligned with A level Mathematics and Further Mathematics. Mathematics qualifications from Alberta (Canada), Hong Kong, New South Wales and Victoria (Australia), Singapore, Scotland and the United States were investigated.

These jurisdictions were chosen for their similarity to A level, in terms of being used as an entrance qualification for undergraduate study and the age at which students sit their final examinations. Additional factors were the presence of an extension Mathematics course or additional content for the most able students, and the availability of curriculum materials in the English language. Specifications and content outlines were used to make judgements, obtained from awarding body and Ministry for Education websites. However, it should be noted that not all relevant documents may have been publicly available, or in the English language, and therefore there may have been information relating to these qualifications that was not used in this study (see Elliott, 2013, for more information about the limitations of comparability studies).

Analysis of applied content was separated into Mechanics and Statistics. Decision Mathematics was not investigated in the current study as it will not be included in the reformed A levels due to its perception as a 'soft' option (ALCAB, 2014). Although awarding bodies may well incorporate some Decision Mathematics into Further Mathematics, this investigation was restricted to the areas of Applied Mathematics that will become compulsory in the reformed A levels. Nevertheless, Decision Mathematics remains an interesting area for future comparison.

It was noted where particular topics occurred and in which module or course they appeared. Mathematics qualifications considered to be equivalent to A level Mathematics or Further Mathematics were used. However, during the course of the study it became apparent that Mathematics qualifications in the majority of the jurisdictions investigated did not include any Mechanics content. Upon closer examination, this content was found to be incorporated into Physics courses. Consequently, instances where a Mechanics topic occurred in a comparable Physics qualification are depicted in the results.

Qualifications

GCE A level – United Kingdom

In the UK, students take A levels at age 18. Their grades are typically used to gain entry to university and/or employment. Students usually take three or four subjects for the full two-year course and two Mathematics qualifications are available: A level Mathematics and A level Further Mathematics[2]. As described above, A levels are currently modular, and students must sit a mixture of core and applied units in both Mathematics qualifications.

For the purposes of this study A level Mathematics and Further Mathematics specifications offered by the four main awarding bodies were investigated: those offered by AQA, OCR, Pearson Edexcel and WJEC. It should be noted that OCR has two A level Mathematics/Further Mathematics specifications: the standard specification and the course developed in collaboration with the Mathematics in Education and Industry (MEI) organisation, which offers different content and unique optional units such as 'Further Pure with Technology' and 'Numerical Methods'. Both of these courses were investigated. All modules from all five specifications were analysed.

A level specifications analysed: AQA (2013), OCR (2013b), OCR MEI (2013a), Pearson Edexcel (2013), WJEC (2013).

2. Students can also gain an additional AS/A level in Additional Further Mathematics if they take enough units, but very few students choose to do so.

Alberta High School Diploma – Alberta, Canada

Students in Alberta sit the Alberta High School Diploma (HSD) in Grades 10–12 and graduate when they are 17–18 years old. Students must earn 100 credits across all modules, with compulsory modules in English Language Arts, Social Studies, Mathematics and Science.

As Mathematics is compulsory, there are three streams for students to choose from depending on ability. For the purposes of this study we focused on the most advanced stream: the '-1 course' sequence. This course consists of three units: 10C, 20-1, and 30-1. This sequence is also compulsory for students wishing to study Mathematics 31, an additional unit which extends knowledge of calculus and introduces some practical applications.

There is very little applied content in the main three modules, but Mathematics 31 includes three electives in Applied Mathematics: (1) Applications of Calculus to Physical Sciences and Engineering; (2) Applications of Calculus to Biological Sciences; and (3) Applications of Calculus to Business and Economics. There is also some more basic Statistics in the less-advanced Mathematics 'sequences'.

Alberta HSD courses analysed: 10C, 20-1, 30-1, Mathematics 31, Physics 20-30 (Alberta Education, 1995; 2008; 2014).

Hong Kong Diploma of Secondary Education – Hong Kong

The Hong Kong Diploma of Secondary Education (HKDSE) is taken by students in their final year of secondary school, when aged 16–17. Students take four core subjects in Chinese Language, English Language, Mathematics and Liberal Studies, as well as two or three elective subjects of their choice. Mathematics is unique amongst HKDSE subjects in that it contains two parts. In addition to the Compulsory part, it also contains an 'Extended part' for the most able students. The Extended part is designed to cater for students who intend to "...pursue further studies which require more mathematics; or follow a career in fields such as natural sciences, computer sciences, technology or engineering." (Curriculum Development Council and The Hong Kong Examinations and Assessment Authority [HKEAA], 2014a).

There is no Mechanics in either the Compulsory or Extended part, although there is some Statistics in both parts. Consequently, the Physics course was also investigated for the Mechanics comparisons.

HKSDE courses analysed: Mathematics (Compulsory, Extended), Physics (Curriculum Development Council and The HKEAA, 2014a; 2014b).

Higher School Certificate – New South Wales, Australia

Students in New South Wales (NSW) take the Higher School Certificate (HSC) at the end of Year 12, aged 17–18, making the HSC broadly equivalent to A level. The HSC consists of Preliminary courses, taken in Year 11, and HSC courses, taken in Year 12. Mathematics is not a compulsory subject in the HSC but there are a wide range of Mathematics courses available to choose from, dependent on ability and planned future progression into Higher Education (HE). For the purposes of this study only Mathematics Extension 1 (ME1) and Mathematics Extension 2 (ME2) are investigated.

There is very little statistical content in either ME1 or ME2, although there is some statistical content in the less advanced courses. However, there are Mechanics topics in both units.

HSC modules analysed: Mathematics Extension 1, Mathematics Extension 2. (Board of Studies New South Wales 1997; 2011a; 2011b).

Advanced Highers – Scotland

Advanced Highers (AHs) are the highest qualification offered by the Scottish Qualifications Authority (SQA), and are classified as a SQA Level 7 qualification. AHs contain more content and require more advanced skills than their counterparts, Highers (SQA Level 6), and are thus considered to be closer to the A level in terms of difficulty (see Johnson and Haywood, 2008, for a closer comparison of the demands of Advanced Highers and A level). Consequently, they are taken by the most able students, with 13,316 students entered for AHs in 2013 (SQA, 2013).

There are two AH courses in Mathematics available to students: Mathematics and Applied Mathematics. Mathematics is the considerably more popular subject – 3,314 students entered in 2013 compared to 361 for Applied Mathematics (SQA, 2013). Students taking Applied Mathematics are required to also take AH Mathematics, making up approximately 10.9 per cent of the total entries for this course. Whilst AH Mathematics has the larger number of candidates, it does not cover any Mechanics or Statistics and therefore Applied Mathematics is the qualification of focus for the comparisons in this study. Nonetheless, the low entry size for this course should be noted as the majority of AH students will thus not have studied any Applied Mathematics.

In Applied Mathematics, students take one core module and then choose between Mechanics and Statistics. It is important to note that, from 2015, the SQA will be splitting the Applied Mathematics AH into new separate Statistics and Mechanics courses. However, as these new qualifications are yet to be assessed it was felt to be more appropriate to compare the current qualifications, in line with the *Cambridge approach to comparative evidence from other jurisdictions* (Elliott, 2013).

AH modules analysed: Statistics 1-2, Mechanics 1-2, Mathematics 1-3 (SQA 2004; 2007; 2010).

Singapore-Cambridge GCE A level – Singapore

Students in Singapore sit Singapore-Cambridge A levels, designed and administered in collaboration with the Ministry of Education and Cambridge International Examinations. Singapore A levels are offered at three levels of study: H1, H2 and H3, with H3 being the most advanced (Ministry of Education Singapore, 2013).

For the purposes of this study only H2 and H3 Mathematics are investigated, due to their closer comparability with UK A level Mathematics and Further Mathematics than H1. H3 covers Pure Mathematics content in more depth than H2 whilst also introducing some more Mechanics content through the 'Differential Equations as Mathematical Methods' strand. Statistics is only included in the H2 course.

Singapore GCE A level courses analysed: Mathematics H2, Mathematics H3, Physics H2 (Singapore Examinations and Assessment Board [SEAB], 2013a, 2013b; 2013c).

Advanced Placement exams – United States and Canada

Advanced Placement (AP) exams are administered by the College Board and are taken primarily by students in the United States and Canada, although they may be taken by students around the world. They are not part of compulsory education but are instead taken by students intending to progress to HE. Although they are unlikely to feature in admissions requirements for US universities, which focus more on the Scholastic Aptitude Test (SAT), they may positively affect applications for scholarships. They can also be used to earn college credit and to enter Higher level college courses (College Board, 2014a).

There are four AP courses Mathematics or Mathematics-related courses available: Calculus AB, Calculus BC, Statistics and Physics C (Mechanics). For the purposes of this study, only Physics C (Mechanics) will be considered. This is because whilst it is a Physics course, it is the only AP course which contains any Mechanics content. Additionally, AP Statistics is classified as a Group B subject by the University and Colleges Admissions Service (UCAS), meaning that it is not considered to be comparable to A level in terms of demand (UCAS, 2006). Consequently, AP courses are only used in the Mechanics comparisons.

AP courses analysed: Physics C (College Board, 2014b).

Victorian Certificate of Education – Victoria, Australia

In Victoria, students sit the Victorian Certificate of Education (VCE) as the culmination of their secondary school education. The VCE is different in structure and in content to the HSC in New South Wales. In order to attain the VCE students must take a minimum of 16 units over Years 11 and 12. There are 12 Mathematics units in total available to students taking the VCE which are designed to satisfy a range of different abilities and needs.

For the purposes of this study we concentrated on the most advanced courses: Mathematical Methods Computer Algebra Systems (CAS) 1–4, Further Mathematics (FM) 3+4, and Specialist Mathematics (SM) 3+4. However, it is important to recognise that the Further Mathematics units should not be directly equated with A level Further Mathematics; this is because whilst there is some overlap in content between the two, a VCE student would need to study the additional Specialist Mathematics and Mathematical Methods units to cover the same range of content.

Further Mathematics in the VCE has a statistical focus, whilst Mechanics content is concentrated in Specialist Mathematics. There is also some more basic statistical content in the less advanced units.

VCE courses analysed: Mathematical Methods (CAS) 1–4, Further Mathematics 3+4, Specialist Mathematics 3+4, Physics (Victorian Curriculum and Assessment Authority [VCAA], 2010; 2012).

Results and Discussion

Mechanics

Mechanics is one of the two predominant areas of Applied Mathematics studied at this level (the other being Statistics) and forms the basis for further study in Engineering and the Physical Sciences, as well as Mathematics. Consequently, the Mechanics content of international qualifications has implications for the preparedness of students for a range of undergraduate courses, not just Mathematics.

In order to compare the Mechanics content of Mathematics qualifications, the specifications and assessment materials of the qualifications already outlined were utilised. The topics which recurred most often were listed, as well as which specific modules or units they were included in (see Table 1).

Table 1: Mechanics content

Topic	*AQA*	*OCR*	*OCR MEI*	*Pearson Edexcel*	*WJEC*	*AP*	*AH Applied Maths*	*Alberta*	*Hong Kong*	*NSW*	*Singapore*	*Victoria*
Vectors	C4|FP3|M1	C4|FP3	C4|M1	C4|FP3|M1	M2	Physics C	M1	Physics 20	Extension	ME2	Maths H2	SM 3+4
Kinematics	M1|M2	M1	M1	M1		Physics C	M1	31	Physics	ME2	Physics H2	GM 1+2
Newton's laws of motions applied to problems involving force	M1	M1	M1	M1	M1	Physics C	M1	Physics 20	Physics	ME2	Maths H3	Physics
Force as a vector	M1	M1	M1	M1	M2	Physics C	M1	Physics 20	Physics	Physics	Physics H2	SM 3+4
Work, energy and power	M2	M2	M2	M2	M2	Physics C	M2	31	Physics		Physics H2	Physics
Impulse and momentum in two dimensions	M1|M3	M3	M2	M1	M1(1D)	Physics C	M2	Physics 30	Physics	Physics	Physics H2	Physics
Centre of mass	M2	M4	M2	M3	M1	Physics C		31			Physics H2	Physics
Simple harmonic motion	M5	M3	M3	M3	M3	Physics C	M2	31		ME2	Physics H2	
Elastic springs and strings (inc. Hooke's law)	M2	M3	M3	M3	M2|M3		M2	31			Maths H3	Physics
Equilibrium of rigid bodies	M2	M2|M3	M2	M3	M1|M3	Physics C	M1		Physics			SM 3+4
Rotation of a rigid body	M4	M4	M4	M5		Physics C	Physics					
Stability and oscillations	M5	M4	M4	M3|M4		Physics C	M2	Physics 20		ME1	Physics H2	
Moment of inertia	M4	M4	M4	M5		Physics C	Physics	31	Physics	Physics		
Relative motion	M3	M4	M1	M4	M2	Physics C	M1				Physics H2	Physics
Angular motion	M2	M4	M4	M5	M2	Physics C	M2			ME2		
Linear motion under a variable force	M2	M3	M4	M3			M2	31				GM 1+2
Problems involving variable mass	M5		M4	M5				31			Maths H3	
Coefficient of restitution (elastic/inelastic collision)	M3	M2	M2	M2		Physics C*		Physics 30*	Physics*			
Motion of a projectile	M1	M2	M1	M2	M2	Physics C	M1	Physics 20	Physics Ext.	ME1	Physics H2	Physics
Moments	M2	M2	M2	M1	M1		M2		Physics	Physics	Physics H2	
Circular motion:												
Uniform motion in a circle	M2	M2	M3	M3	M2	Physics C	M2	Physics 20	Physics Ext.	ME2	Physics H2	Physics
Motion in a vertical circle	M2	M3	M3	M3	M2	Physics C			Physics Ext.			Physics

* denotes that elastic and inelastic collisions are covered but knowledge of the coefficient of restitution is not required

However, it should be noted that there are limitations to this approach: noting if and where a topic occurs does not address how much depth is given to these topics or the required level of understanding. Additionally, the occurrence of certain Mechanics topics in Physics courses means that the level of underlying Mathematics required when studying these topics is uncertain. For example, whether students are expected to have knowledge of calculus when studying Physics, and if so, is this taught within the Physics course or must students also study the Mathematics course? A deeper analysis of these factors is beyond the remit of the current study, which intends to highlight which Applied Mathematics topics students study in these jurisdictions, but would be an interesting avenue for future research. It should also be borne in mind when considering the results.

Results

The following Mechanics topics are covered by all qualifications under consideration and occur in the less advanced A level Mechanics modules (M1 and M2):

- Vectors
- Kinematics
- Newton's laws of motion applied to problems involving force
- Force as a vector
- Work, energy and power
- Impulse and momentum in two dimensions[3]
- Motion of a projectile
- Uniform motion in a circle[4].

There are also several topics that are covered in M1 and M2 of all the A level specifications, but not in all of the other qualifications, even when Physics courses are included:

- Moments
- Equilibrium of rigid bodies[5].

There are no topics that are included in the majority of international qualifications but not in A level.

Excluding the above topics, there is a reasonable amount of variation between specifications. However, it should be noted that in the international qualifications investigated, all topics are compulsory. Conversely, there is no guarantee that an A level Mathematics or Further Mathematics student will have studied *any* of the Mechanics topics, especially those occurring in the more advanced modules (M3–5).

Discussion

Through examination of Table 1, it initially appears that existing A levels cover considerably more Mechanics content than their international counterparts. However, the optionality of the modular structure has significant implications for the content an A level student will actually study. If a student studies all four Mechanics modules (or all five if they are taking the AQA or Pearson Edexcel specifications), they will certainly have covered Mechanics in greater depth than students taking other qualifications.

However, whilst it is technically possible for a student to take four or five Mechanics modules if they are taking the full A levels in Mathematics and Further Mathematics, entry numbers are very low for the more advanced modules. In June 2013 for the OCR specification, there were only 696 entries for M3 and 126 for M4, compared to the 6,216 candidates certificating for either A level or AS Further Mathematics. For the MEI specification, there were 717 entries for M3 and just 86 for M4, in comparison to the 6,302 candidates certificating for this specification in either A level or AS Further Mathematics. Additionally, M4 is often self-taught by students either out of personal interest or as advanced preparation for university study. Consequently, the number of schools/colleges routinely offering these modules is likely to be even lower than these figures suggest.

This suggests that only a very small minority of students studying Further Mathematics study the more advanced Mechanics modules. By contrast, M1 is the second most popular Applied module in both specifications, and M2 is the fifth most popular. It is therefore assumed that the majority of A level Further Mathematics candidates will have studied M1 and M2 only[6]. When this is taken into account, the Mechanics content of A levels is more in line with the international qualifications considered, but only when comparable Physics courses are also included.

These findings are corroborated by other research in this area. Leppinen (2008) found that 65% of first year undergraduate Mathematics students had studied at most two Mechanics modules at A level; 28% had studied three, with just 7% studying four or more. Ward-Penny, Johnston-Wilder, and Johnston-Wilder (2013) have found further evidence to suggest that uptake of the higher Mechanics modules is low, with less than half of all schools in their sample offering M3 for Further Mathematics students. Only 11% offered M4, and 61% of these were independent or grammar schools. Whilst the optionality inherent in A levels introduces flexibility for both teachers and students and allows them to study modules of particular interest, it introduces a degree of uncertainty for universities and consequently students' preparedness for tertiary study. Research indicates that this uncertainty has implications for undergraduate study: in a study of Engineering departments at 19 universities, Lee et al. (2007) found that only 17% of the lecturers they surveyed knew which Mechanics modules their students had studied within A level Mathematics. As a result, 58% of the lecturers assumed a level of Mechanics knowledge that their students did not in fact have.

Nevertheless, for all of the other qualifications under consideration, vectors were the only Mechanics-related content always covered within even the most advanced Mathematics courses. The rest (excluding AHs) incorporate a significant proportion of their Mechanics topics into their Physics qualifications. This suggests that there is a difference between what the UK and other jurisdictions perceive to be an appropriate place for this content.

It is difficult to know the rationale behind different jurisdictions' decisions about where to include specific content. However, it may be partially due to the non-modularity of these international qualifications. Whilst there are often a range of Mathematics courses to choose from, as in Alberta and Australia, all topics within a course or sequence are usually compulsory, unlike in A levels. Furthermore, where there is some optional content, as in the HKDSE, the content is usually compulsory for the most able students (i.e., those who would be most likely to take A level Further Mathematics if they were resident in the UK). This therefore necessarily

3. Only covered in one dimension by WJEC.
4. Covered in M3 for the MEI and Pearson Edexcel courses.
5. Covered in M3 for Pearson Edexcel.
6. This is true of all A level specifications apart from WJEC, which only offers three Mechanics units. Because of this, an A level Further Mathematics student will have had to study M1–3, unless they opted to study an additional Further Pure unit.

entails a reduction in content when compared to A level as all topics must be taught and studied. This may lead awarding bodies or Ministries for Education to incorporate any additional Mechanics content into their Physics courses in order to allow adequate time for study.

However, the difference in content may be due to a difference in belief about just how mathematical Physics qualifications should be. There is some evidence to suggest that A level Physics does not contain an appropriate amount of Mathematics to fully prepare students for Physics or related degrees, unless they also study at least A level Mathematics. The Institute of Physics (2011) found that this caused two problems: prospective undergraduates did not expect there to be so much Mathematics in their Physics course, and students struggled to apply Mathematics in physical contexts or understand where mathematical concepts might be useful to solve Physics problems. These problems were attributed to the lack of Mathematics in A level Physics by both lecturers and undergraduates. The fact that UK universities usually require both A level Physics and Mathematics for admission to undergraduate Physics and Engineering courses is perhaps indicative of this.

Nevertheless, the comparisons made in this study indicate that this appears to be a problem particular to A level, with other jurisdictions ensuring that their Physics courses are suitably mathematical. Whilst this means that students taking these qualifications must study both Mathematics and Physics, students in the UK also need to take both subjects to be similarly prepared. It could be argued that if students take qualifications in both Mathematics and Physics, they are likely to encounter the same content regardless of which subject it occurs in. However, the reforms to A level Mathematics and Further Mathematics offer an interesting opportunity to consider why these differences arise and why the A level differs so much to its international counterparts. Although the changes mean all A level Mathematics students will study some Mechanics, the perceived disconnect between Mathematics and Physics may well continue.

Statistics

Statistics is the second branch of Applied Mathematics investigated in this study. As well as further study in Mathematics, knowledge of Statistics is required in undergraduate courses that entail quantitative analysis, such as Psychology, Economics and other Social Sciences. The statistical content of international qualifications thus has implications for students in a variety of university courses, including some that may be traditionally considered non-mathematical.

All of the qualifications outlined above, excluding AP exams, are included for analysis. The most common topics were listed and compared, as well as which specific modules or units they occur in (see Table 2).

As well as Statistics content, probability is also included for comparison here. Whilst probability and Statistics are definitively not the same area of Mathematics, they are often included in the same areas or units of Mathematics qualifications and both are therefore included in this study. It should also be noted that some statistical content is often included in Social and Biological Science qualifications at this level (e.g., in A level Psychology) but investigation was restricted to Mathematics courses only.

Results

Initial data collation indicated that there was very little statistical content in either the Alberta '-1 course' sequence or the NSW HSC's Maths Extension 1 and 2. Consequently, comparisons were expanded to include the less advanced Mathematics courses for these qualifications. Where a topic occurs in one of these courses, it is clearly marked in Table 2.

There appears to be very little consensus as to either which or how many topics are included in the Statistics components of the international qualifications under consideration. Excluding the first three topics (Measures of central tendency, Measures of dispersion, and Visual presentation of data) as they are usually included as a review of earlier qualifications, the most common topics are:

- Binomial distribution
- Normal distribution
- Poisson distribution
- Product moment correlation coefficient
- Central Limit Theorem
- Conditional and independent events (probability)
- Expectation algebra.

The following topics are covered in the majority of the international qualifications but the minority of A level specifications:

- z-score[7]
- Permutation and combination[8].

Discussion

Once again, it initially appears that A levels cover the most Statistics content. However, as with Mechanics, the modular structure has implications for the content an A level Further Mathematics student will actually study. Whilst S1 is the most popular Applied module and S2 is the fourth (in both the OCR and MEI specifications), entries for S3 and S4 in 2013 were even lower than for the corresponding Mechanics modules. For the OCR specification, there were 399 entries for S3 and 82 for S4, whilst in the MEI specification there were 500 entries for S3 but just 28 for S4. Consequently, it is highly likely that an A level Further Mathematics student will have studied S1 and S2 only[9]. Although this data is for the OCR and MEI courses only, the small entry numbers for these modules are corroborated by Ward-Penny et al. (2012), who found that only 18% of sampled schools/colleges offered Further Mathematics students S3, with just 4% offering S4.

There are two key findings from these Statistics comparisons. First, which may not be immediately obvious from Table 2, is that the way in which Statistics is taught in other jurisdictions is notably different to A level Mathematics. In other jurisdictions it is predominantly taught in a practical way and involves the handling and collection of real data. This is particularly the case in Victoria and Hong Kong, where students have to undertake statistical coursework. Conversely, the existing A level specifications currently teach Statistics in a largely theoretical manner, with no requirement for students to handle real data. However, the statistical content in the reformed A levels will focus on practical applications of Statistics and real, large data sets, with compulsory content on hypothesis testing and sampling methods (ALCAB, 2014). This has been developed in consultation with the Royal Statistical Society with the belief that handling real data sets will prove to be more engaging and useful for students. Developing skills in hypothesis testing and

7. AQA only.
8. OCR, MEI and WJEC only.
9. As with Mechanics, the WJEC specification only has S1-S3 so a Further Mathematics student taking the course to A2 will have had to study all three Statistics modules unless they opt to study an additional Further Pure module.

Table 2: Statistics content

Topic	*AQA*	*OCR*	*OCR MEI*	*Pearson Edexcel*	*WJEC*	*AH Applied Maths*	*Alberta*	*Hong Kong*	*NSW*	*Singapore*	*Victoria*
Statistics:											
Measures of central tendency†	S1	S1	S1	S1	S1	S1	30 - 3		GM		FM 3+4
Measures of dispersion†	S1	S1	S1	S1	S1	S1	20 - 2	Compulsory	GM		FM 3+4
Visual presentation of data†		S1	S1	S1	S1		20 - 3		GM		FM 3+4
Binomial distribution	S1	S1	S1	S2	S1	S1		Extended	ME1	H2	CAS 3+4
Normal distribution	S1	S2	S2	S1	S2	S1	20 - 2	Extended	GM	H2	CAS 3+4
Poisson distribution	S2	S2	S2	S2	S1	S1		Extended		H2	CAS 3+4
Sampling methods		S2	S3	S3		S1		Compulsory	GM	H2	
Hypothesis testing	S2|S3	S2	S1	S2	S2|S3	S1				H2	
t-test (one- and two-tailed)	S2	S3	S1|S3	S2	S2	S1				H2	
Product moment correlation coefficient	S1	S1	S2	S1	S2	S2				H2	FM 3+4
Non-parametric tests (Wilcoxon/Mann-Whitney)		S4	S3			S2					
Least-squares regression line	S1	S1	S2	S1	S3					H2	FM 3+4
Chi-squared test for goodness of fit	S4	S3	S3	S3		S2					
Chi-squared test for use in contingency table	S2		S2	S3		S2					
Type I and II error	S2	S2	S4	S4							
Central Limit Theorem	S1	S2	S3	S3	S3	S1		Extended		H2	
"Uses and abuses"/applications							20 - 2	Compulsory			FM 3+4
Geometric distribution	S4	S1						Extended			
t distribution	S4		S3	S4	S3	S2					
Bernouilli trials and Markov chains			FP3		S1			Extended			CAS 3+4
Confidence intervals	S1|S2|S3	S3	S3	S3	S2	S1 | S2	20 - 2	Extended			
z-score	S3					S1	20 - 2	Extended	GM		FM 3+4
Probability:											
Calculation of expectation and variance	S2	S1	S1	S1	S1	S1		Extended		H2	
Conditional and independent events	S1	S1	S1	S1	S1	S1	30 - 2	Compulsory	GM	H2	CAS 1
Expectation algebra	S1	S3	S3	S1	S1	S1		Compulsory		H2	CAS 2
Probability generating functions		S4	S4								FM 3+4
Moment generating functions		S4	S4								
Estimators	S4	S4	S4	S3|S4	S3			Extended			
Probability density function	S2	S2	S3	S2	S1						CAS 3+4
Cumulative distribution function	S2		S3	S1	S1						CAS 3+4
Bayes' Theorem	S3	S4	D2		S1	S1		Extended			
Permutation and combination		S1	S3		S1		30 - 1	Compulsory		H2	CAS 2

† These topics are usually covered before A level or equivalent qualifications. Their inclusion here denotes a review for the teaching of more advanced Statistics.

Topics displayed in yellow occur in less advanced Mathematics courses.

sampling may also prove useful for students who intend to progress to undergraduate courses that require statistical but not necessarily mathematical knowledge, such as Psychology and the Social Sciences.

The second key finding is the number of international qualifications that contain very little Statistics. The Singapore GCE A level and HKDSE are the only other qualifications to have compulsory Statistics content, with further Statistics being available in Module 2 of the Extended part in the HKDSE. Conversely, even when the less advanced courses are included, NSW and Alberta have very little statistical content at all.

It is not clear why there is such a disparity between the statistical content of the different qualifications. A possible explanation may lie in the intended audience of these qualifications. The advanced Mathematics qualifications investigated in this study are primarily intended for students progressing to Mathematics or Physical Sciences courses at university. Consequently, this leads to a strong emphasis on areas of Pure Mathematics such as calculus rather than Statistics in order to better prepare students for the demands of their university courses.

This explanation however would still leave the majority of students progressing to Mathematics and Physical Science degrees with very little experience of Statistics. An alternative explanation may be that Statistics is taught to a higher level earlier in the school curriculum in these jurisdictions, and therefore there is a reduced demand for Statistics at this level. A whole-scale analysis of General Certificate of Secondary Education (GCSE) level Mathematics qualifications is beyond the scope of this article but may prove to be an interesting area for future research.

Conclusion

This article has shown that there are substantial differences between the applied content of A level Mathematics and Further Mathematics and their international counterparts. Significant differences in both Mechanics and Statistics content were identified, which raises the question of how the content of Mathematics qualifications at this level is decided. There appears to be very little international consensus in how much Statistics or Mechanics a student should be exposed to before beginning university or entering the workplace. Additionally, the UK is unique amongst the jurisdictions investigated for incorporating its Mechanics content into Mathematics courses, rather than Physics.

However, it is difficult to fully appreciate the differences in international Mathematics qualifications without investigating their preceding qualifications (e.g., GCSE level courses) as well as analysing the Pure Mathematics content. This study also does not investigate the cultural contexts in which these qualifications are taken and how they are used by students and other stakeholders. Rather, this article acts as an exploratory basis from which to consider how pre-university Mathematics is taught and assessed across the world.

Acknowledgements

I would like to thank Ellie Darlington, Research Division, for her helpful advice with this article.

References

A Level Content Advisory Board. (2014). *Report of the ALCAB Panel on Mathematics and Further Mathematics*.

Alberta Education. (1995). *Mathematics 31 Program of Study*. Alberta, Canada.

Alberta Education. (2008). *Mathematics Grades 10–12 Program of Study*. Alberta, Canada.

Alberta Education. (2014). *Physics 20–30 Program of Studies*. Alberta, Canada.

AQA. (2013). *General Certificate of Education Mathematics 6360 2014 Specification*. Manchester, UK.

Board of Studies New South Wales. (1997). *Mathematics 4 Unit Year 12 Syllabus*. North Sydney, Australia.

Board of Studies New South Wales. (2011a). *General Mathematics Stage 6 Syllabus*. North Sydney, Australia.

Board of Studies New South Wales. (2011b). *Mathematics 2/3 Unit Years 11–12 Syllabus*. North Sydney, Australia.

College Board. (2014a). *10 Years of Advanced Placement Exam Data Shows Significant Gains in Access and Success; Areas for Improvement*. Retrieved from https://www.collegeboard.org/releases/2014/class-2013-advanced-placement-results-announced

College Board. (2014b). *AP Physics Course Description*.

Curriculum Development Council and The HKEAA. (2014a). *Mathematics Curriculum and Assessment Guide (Secondary 4-6)*.

Curriculum Development Council and The HKEAA. (2014b). *Physics Curriculum and Assessment Guide (Secondary 4–6)*.

Department for Education. (2014). *Further Mathematics: GCE AS and A level subject content*. London, UK.

Elliott, G. (2013). *The Cambridge approach to comparative evidence from other jurisdictions*. Cambridge: Cambridge Assessment.

Institute of Physics. (2011). *Mind the Gap: Mathematics and the transition from A-levels to physics and engineering degrees*. Retrieved from http://www.iop.org/publications/iop/2011/file_51933.pdf

Johnson, J., & Haywood, G. (2008). *Expert Group Admission for Award Seeking Admission to the UCAS Tariff: Scottish Highers and Advanced Highers*. Cheltenham, UK.

Lee, S., Harrison, M. C., & Robinson, C. L. (2007). Recent changes in A-level Mathematics: is the availability and uptake of mechanics declining yet more? *Teaching Mathematics and its Applications*, *26*(3), 111-118.

Leppinen, D. (2008). Reflections upon the teaching of Mechanics to first year university students. *MSOR Connections*, *8*(1), 3–6. doi: 10.11120/msor.2008.08010003

Ministry of Education Singapore. (2013). *The 'A' Level Experience*. Retrieved from http://www.moe.gov.sg/cpdd/alevel2006/experience/levels.htm

National HE STEM Programme Wales. (2012). *Studying Mathematics at University: Enhancing the Transition from School or College*. Retrieved from http://www.wimcs.ac.uk/document_repository/Outreach%20and%20Transition%20case%20studies/WIMCS%20Teacher%20Fellows/WIMCS%20Teacher%20Fellows%20Mathematics%20Final%20Report%20%5BEnglish%5D.pdf

OCR. (2013a). *MEI Structured Mathematics*. Cambridge, UK.

OCR. (2013b). *OCR Advanced GCE in Mathematics 67746 Specification*.

Pearson Edexcel. (2013). Specification GCE Mathematics.

SQA. (2004). *Advanced Higher Physics Arrangements Documents C)69 13 Edition 4. Glasgow, Scotland.*

SQA. (2007). *Advanced Higher Applied Mathematics Arrangement Documents*. *Glasgow, Scotland.*

SQA. (2010). *Advanced Higher Mathematics Arrangement Documents C100 13 Edition 6*. *Glasgow, Scotland.*

SQA. (2013). *Annual Statistical Report*. Glasgow, Scotland.

SEAB. (2013a). *Physics Higher 2 (Syllabus 9646)*. Retrieved from https://www.seab.gov.sg/content/syllabus/alevel/2015Syllabus/9646_2015.pdf. SEAB, Singapore.

SEAB. (2013b). *Mathematics Higher 2 (Syllabus 9740)*. Retrieved from https://www.seab.gov.sg/content/syllabus/alevel/2015Syllabus/9740_2015.pdf. SEAB, Singapore.

SEAB. (2013c). *Mathematics Higher 3 (Syllabus 9824)*. SEAB, Singapore.

UCAS. (2006). *Expert Group Report for Awards Seeking Admission to the UCAS Tariff: The Advanced Placement Programme*. Cheltenham, UK.

Victorian Curriculum and Assessment Authority. (2010). *Mathematics: Victorian Certificate of Education Study Design*. Victoria, Australia.

Victorian Curriculum and Assessment Authority. (2012). *Physics: Victorian Certificate of Education Study Design*. Victoria, Australia.

Ward-Penny, R., Johnston-Wilder, S., & Johnston-Wilder, P. (2013). Discussing perception, determining provision: teachers' perspectives on the applied options of A-level mathematics. *Teaching Mathematics and its Applications*, *32*(1), 1–18. doi: 10.1093/teamat/hrs011

WJEC. (2013). *GCE Mathematics Specification (2009 onwards) Version 2*. Retrieved from http://www.wjec.co.uk/qualifications/qualification-resources.html?subject=Mathematics&level=GCEASA