



O Level

Mathematics

Session: 1967 June
Type: Question paper
Code: 411-412

411/1

MATHEMATICS

ORDINARY LEVEL, ALTERNATIVE A

ARITHMETIC

(One hour and a half)

Mathematical tables and squared paper are provided.

Candidates are warned that misreading a question may lead to the loss of a number of marks.

All working must be clearly shown; it should be done on the same sheet as the rest of the answer.

SECTION I. [70 marks]

Answer all the questions in this section.

You must not use Mathematical tables in working questions 1-4.

- 1 (i) Express as a fraction in its lowest terms

$$\frac{\frac{5}{9} - \frac{7}{15}}{1 - (\frac{5}{9} \times \frac{7}{15})}$$

- (ii) Express 3375 as the product of its prime factors. Hence write down the length of the edge of a cube whose volume is 3·375 cu. cm.

- 2 (i) A tape measure is marked in inches on one side and in centimetres on the other. Given that 1 in. equals 2·54 cm. find, correct to the nearest mm., the length on the centimetre side which corresponds to 17·3 in.

- (ii) Taking £1 to be equivalent to 2·79 dollars, express 2 dollars

(a) as a decimal of £1, giving your answer correct to three places of decimals;

(b) in shillings and pence, correct to the nearest penny.

- 3 (i) A motorist travelling at a steady speed of 96 m.p.h. covers a section of a motorway in 25 minutes. After a speed-limit is imposed he finds that, when travelling at the maximum speed allowed, he takes 7 minutes longer than before to cover the same section. Calculate the speed limit.

(ii) A farm of 221 acres includes 60 acres of marshland, the rest being divided between grassland and arable land in the ratio of 5:2. If 7 acres of marsh are turned into grassland, while 10 acres of grassland are ploughed for use as arable land, find, in its lowest terms, the new ratio of grassland to arable land.

- 4 (i) A boy who was asked to find $3\frac{1}{2}\%$ of a sum of money misread the question and found $5\frac{1}{2}\%$ of it. His answer was £247. 10s. Find what the answer should have been.

(ii) A man invested £8,000 and, after paying income tax at 8s. 3d. in the £ on the first year's interest, he had £282 of

the interest left. Calculate the rate per cent at which interest was paid.

5 (i) Use tables to calculate, correct to three significant figures, the value of

$$\sqrt[3]{2.092 \times (0.2176)^2}.$$

(ii) The inner and outer diameters of a length of cylindrical plastic pipe are 1.6 in. and 2 in. Given that 1 cu. in. of this plastic weighs 0.525 oz. calculate, correct to the nearest lb., the weight of 100 ft. of the pipe.

[Take π to be 3.142.]

SECTION II [30 marks]

Answer two questions in this section.

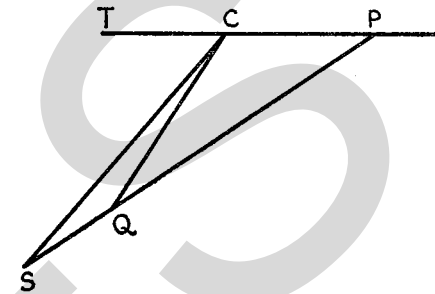
6 The container of a crop-spraying machine is a rectangular tank 4 ft. long, 4 ft. wide and 2 ft. high. As the machine moves forward liquid is pumped from the tank and sprayed on to a strip of land 22 ft. wide. If the pump empties the tank in an hour, and the machine is moving at 3 m.p.h., find the number of gallons sprayed per acre. [Take 1 cu. ft. to be $6\frac{1}{4}$ gallons.]

Find also the speed at which the machine must move in order to reduce by 5 gallons the amount sprayed per acre.

7 A householder pays 6d. per unit for the first 100 units of electricity used, and $1\frac{1}{2}$ d. per unit for any further amount. Taking 1 in. to represent 100 units of electricity on one axis, and 1 in. to represent £1 on the other axis, draw a graph to show the cost of electricity up to 900 units. Find from your graph the number of units used by a householder whose bill is £6. 15s.

Using the same axes, draw a graph to show what the cost would be if, instead, a uniform rate of 2.4d. per unit were charged for all units used. Use your graphs to find the saving on a bill for 340 units.

8

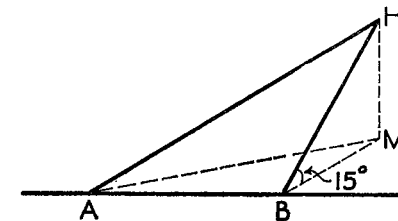


In the diagram P is a point on the coast 1.5 miles east of a coastguard station C . S is a point on the opposite coast 3.2 miles from C . A ship sailing from P to S is intercepted at Q by a boat from the coastguard station.

Given that $\hat{TCS} = 63^\circ$ and $QS = 0.5$ miles, calculate

- (i) PQ ;
- (ii) the bearing of Q from P .

9



In the diagram A and B are points on a straight level road crossing a hillside which slopes at 15° to the horizontal. AH is a path from the road to a house on the hill. Given that $AB = 55$ yd., $\hat{HBA} = 90^\circ$ and $\hat{HAB} = 52^\circ$, calculate

- (i) HB ;
- (ii) the vertical height, HM , of H above the road;
- (iii) the angle HAM , which AH makes with the horizontal.

MATHEMATICS

ORDINARY LEVEL, ALTERNATIVE A

GEOMETRY

(Two hours)

Candidates are warned that misreading a question may lead to the loss of a number of marks.

In questions involving calculations, no proofs are required but essential steps of the working must be shown.

SECTION I. [70 marks]

Answer all the questions in this section.

- 1 Without using set square or protractor, construct
 (i) the rectangle $ABCD$ in which $AB = 2.9$ in. and the diagonal $AC = 3.7$ in.;
 (ii) the trapezium $AXBC$ in which BX is parallel to CA and $\hat{CAX} = 60^\circ$.
 Measure AX .
- 2 (i) $PQRST$ is a pentagon in which PT is parallel to RS . The sides QR and TS meet, when produced, at U . Given that $\hat{P} = \hat{Q} = \hat{QRS}$, and $\hat{RUS} = 35^\circ$, calculate \hat{P} and \hat{T} .
 (ii) The area of a rhombus is 16 sq. in. and the length of one of its diagonals is 6 in. Calculate the length of the other diagonal.
- 3 (i) A chord of a circle is 48 cm. in length and is 7 cm. from the centre of the circle. Calculate the radius of the circle, and hence calculate the length of a second chord which is 15 cm. from the centre of the circle.

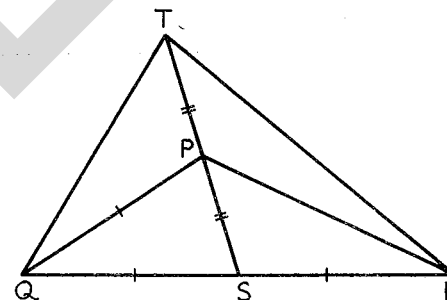
(ii) The triangle ABC in which angle A is obtuse is inscribed in a circle. The angle $ABC = 22^\circ$ and the angle $ACB = 40^\circ$. M is the mid-point of the major arc BC . Calculate \hat{BMC} and \hat{BAM} .

4 $WXYZ$ is a parallelogram. A line through W meets ZY at T and XY produced at U . Prove that the triangles WZT , UYT are similar.

Given that $\frac{ZT}{TY} = \frac{3}{2}$ and the area of the parallelogram is 20 sq. in., calculate

- (i) the area of the trapezium $WXYZ$;
 (ii) the area of $\triangle UYT$.

5



In the given figure $SP = PT$ and $PQ = QS = SR$.

Prove that

- (i) $\hat{TPQ} = \hat{PSR}$;
 (ii) $TQ = PR$;
 (iii) the triangles TPQ and TPR are equal in area.

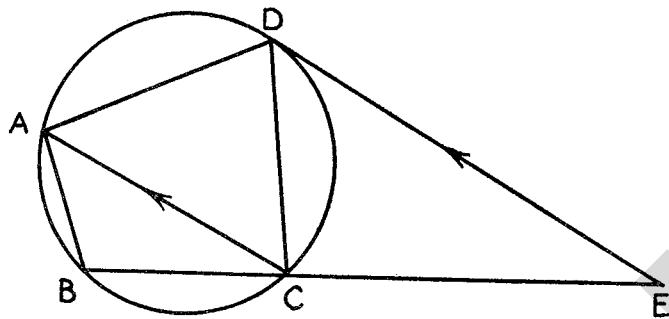
6 ABC is a triangle in which \hat{A} is a right-angle. The perpendicular from A to BC meets BC at D , and the bisector of \hat{B} meets AD at E and AC at F . Prove that $AE = AF$.

Given also that $\hat{ABC} = 62^\circ$, prove that AE is greater than BE .

SECTION II. [30 marks]

Answer two questions in this section.

- 7 Prove that if a straight line touch a circle, and from the point of contact a chord be drawn, the acute angle which this chord makes with the tangent is equal to the angle in the alternate segment.



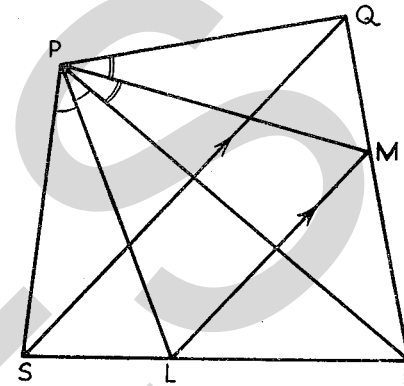
In the given figure AC is parallel to the tangent DE .

Prove that

- (i) $\triangle ADC$ is isosceles;
 - (ii) $\hat{ABC} = 2\hat{DAC}$.
- 8 (i) AB is a fixed line of length 4 in., and R is a point such that the area of $\triangle ABR$ is 5 sq. in. S is the mid-point of AR . State the locus of R and the locus of S , giving reasons and a sketch.

(ii) T is a fixed point outside a fixed circle, centre O . A variable line through T meets the circle at X and Y . Prove that the locus of the mid-point of XY is an arc of the circle on OT as diameter.

- 9 Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.



In the given figure PL and PM bisect \hat{SPR} and \hat{RPQ} respectively, and LM is parallel to SQ .

- (i) Complete the statement

$$\frac{SL}{LR} = \frac{MR}{MR}$$

- (ii) Prove that $PS = PQ$.

- 10 Without using set square or protractor, construct

(i) the triangle ABC in which $BC = 1.8$ in., $CA = 1.9$ in., $AB = 2.3$ in.;

(ii) the circle which touches BC and also touches AB produced and AC produced (i.e. the escribed circle of $\triangle ABC$ which is opposite A);

(iii) the point D on this circle such that \hat{DBC} is obtuse and the area of $\triangle DBC$ is equal to the area of $\triangle ABC$.

[No written explanation of the constructions is required.]

MATHEMATICS

411/3

ORDINARY LEVEL, ALTERNATIVE A

ALGEBRA

(One hour and a half)

Mathematical tables and squared paper are provided.

Candidates are warned that misreading a question may lead to the loss of a number of marks.

All working must be clearly shown; it should be done on the same sheet as the rest of the answer.

SECTION I. [70 marks]

Answer all the questions in this section.

- 1 (i) Simplify

$$(3x-1)(2x+1) - (2x-1)^2.$$

- (ii) Solve the equation

$$\frac{4p-1}{3} - \frac{3p-1}{2} = \frac{5-2p}{4}.$$

- 2 (i) Express as a single fraction in its lowest terms

$$\frac{3}{2x-1} - \frac{2x}{4x^2-1}.$$

(ii) When $x = 0$ the value of the expression $ax^2 + bx + c$ is -3 . When $x = 3$ the value is 6 , and when $x = -2$, the value is 11 . Find a , b , c .

- 3 Factorise

(i) $2pq + pr - sr - 2sq$;

(ii) $6x^2 + 17x - 14$;

(iii) $a^2 - b^2 - (a+b)$.

- 4 Solve the equations

(i) $(2x+1)^2 = 4$;

(ii) $2x^2 + 7x + 1 = 0$, giving your answers correct to two decimal places.

5 One side of a rectangle is d in. long. The other side is 2 in. shorter. The side of a square is 2 in. shorter still. The sum of

the areas of the square and the rectangle is 148 sq. in. Find an equation for d and solve it.

6 (i) A shopkeeper pays $\pounds c$ for 1 cwt. of apples. He sells them at b shillings per lb., thus making $p\%$ profit on his outlay. Find an expression for p in terms of b and c and reduce it to its simplest form.

- (ii) Given that

$$\frac{2x-3y}{2x+y} = \frac{3}{5},$$

express x in terms of y .

- 7 (i) Make x the subject of the formula

$$z = \sqrt{\frac{x}{x+y}}.$$

(ii) Given that $A = PV^n$ where $P = 0.9314$, $V = 0.6815$, $n = 0.5$, use logarithms to calculate the value of A correct to three significant figures.

SECTION II. [30 marks]

Answer two questions in this section.

- 8 (i) Solve the simultaneous equations

$$x+y = 12, \quad 2x^2 + 3y^2 = 7xy.$$

(ii) The distance, s ft., that a body moves from a fixed point is the sum of two terms, one of which varies as the time, t seconds, and the other as the square of the time. If the body moves 82 ft. in the first 2 seconds and 171 ft. in the first 3 seconds, find how far it will move in the fourth second.

- 9 (i) (a) Without using tables, evaluate

$$64^{\frac{2}{3}} \div 125^{-\frac{1}{3}}.$$

- (b) Solve the equation

$$\log_{10}(3x-5) = 2\log_{10}5 - 1.$$

(ii) The n th term of an A.P. is $4n-1$. Write down the first three terms and hence find the common difference. Find

an expression for the sum of the first n terms, giving your answer in its simplest form.

10 At the beginning of term a student bought a number of text books at a total cost of £22. A few days later he bought three more books for a further expenditure of £2. 14s. He then found that this purchase had reduced the average cost per book by 1s. 6d. Find the number of books he first bought.

11 Write down the values of $x(5-x)$ which are omitted from the table below:

x	0	0.5	1	2	2.5	3	4	4.5	5
$x(5-x)$		2.25	4				4	2.25	0

Draw the graph of $y = x(5-x)$ from $x = 0$ to $x = 5$, using scales of 1 in. to one unit on each axis.

With the same axes, draw the graph of $y = \frac{6}{x+1}$ from $x = 0$ to $x = 5$.

Use your graphs to find the range of values of x for which $x(5-x)$ is greater than $\frac{6}{x+1}$.

MATHEMATICS

412/1

ORDINARY LEVEL, ALTERNATIVE B

PAPER 1

(Two hours and a half)

Answer all the questions in Section I and any four in Section II.

All working must be clearly shown; it should be done on the same sheet as the rest of the answer.

Mathematical tables, squared paper and plain paper are provided.

SECTION I. [52 marks]

1 Evaluate, using tables where necessary,

(i) $17 \times 2,418$ correct to four significant figures;

(ii) $\frac{1}{5} \div (\frac{1}{3} \div \frac{1}{2})$;

(iii) $(-1)^3 \times (-6) \div (-2)$;

(iv) $5 \times \sqrt{0.9}$;

(v) $16 - 3 \cos 53^\circ$.

2 (i) A product may be bought at 1s. 4d. per lb. or £6. 10s. 8d. per cwt. Express, in its simplest form, the ratio of the costs.

(ii) Find the percentage reduction when the price of eggs goes down from 5d. each to 4s. 4½d. per dozen.

(iii) The average of three numbers is 58. The average of two of them is 49. Find the third number.

3 (i) Solve the equation $x^2 + 5x = 0$.

(ii) Solve the equation $y^2 + 5y = 6$.

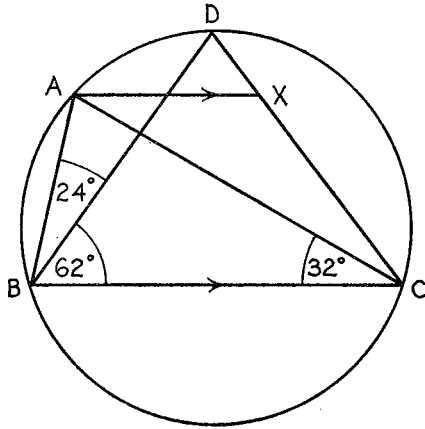
(iii) A man goes on holiday for x days and takes £ f with him to spend. He spends £ g each day for the first y days.

Write down expressions for (a) the number of pounds he has left after y days, (b) the number of pounds he may spend each day for the remainder of his holiday.

4 In the triangle ABC , D is the foot of the perpendicular from the vertex A to the side BC . Given that $AD = 5$ cm., $BD = 8$ cm., and $\hat{C} = 33^\circ$, calculate \hat{B} and the length of AC .

The arc of the circle, centre D and radius DB , cuts DA produced at X . Taking π to be 3.142, calculate the area of the region bounded by BA , AX and the arc XB .

5



In the figure, AX is parallel to BC , $\hat{A}BD = 24^\circ$, $\hat{D}BC = 62^\circ$ and $\hat{A}CB = 32^\circ$. Calculate $\hat{A}XC$, giving reasons.

If the tangent to the circle at C meets AX produced at Y , prove that the triangle XYC is isosceles.

SECTION II. [48 marks]

Answer any four questions in this section.

6 A length of string is wound into a ball, which may be taken to be a sphere of volume 35 cu. in. Calculate the radius of the ball. [Volume of a sphere = $\frac{4}{3}\pi r^3$. Take π to be 3.142.]

The cross-section of the string is a circle of diameter 0.08 in. Assuming that only 60% of the volume of the ball is occupied by the material of the string, calculate the length of string in the ball, to the nearest foot.

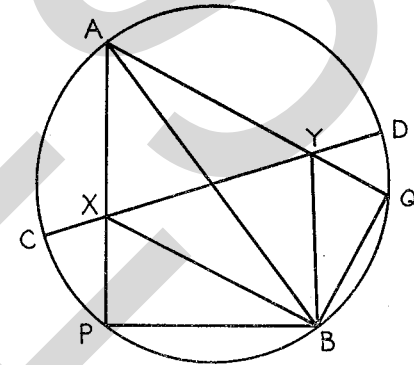
7 (i) Without using tables, evaluate $\sqrt{\frac{p}{q}}$ when $p = 64^{\frac{3}{2}}$ and $q = 3^{-2}$.

(ii) Simplify

$$\frac{(x^2y^3)^2 + (x^3y^2 \times x^2y^3)}{xy + y^2}$$

(iii) It is given that y is inversely proportional to the square of $(x+3)$. If $y = 9$ when $x = 1$, find the possible values of x when $y = 1$.

8



In the figure, AB and CD are diameters of the circle and $CX = DY$. When AX and AY are produced they meet the circle again at P and Q . By considering the diagonals of the quadrilateral $AXBY$, show that it is a parallelogram.

Prove that the triangles XBP and YBQ are similar and deduce that $AX \cdot XP = AY \cdot YQ$.

9 A , B and C are three positions on a map whose scale is 6 in. to 1 mile. $AB = 3$ in., $AC = 2$ in. and $\hat{B}AC = 50^\circ$. Calculate

(i) the area of the triangle ABC on the map;

(ii) the length of BC on the ground (in yards);

(iii) the angle of elevation of A from B , given that A is on the 500 ft. contour and B on the 250 ft. contour.

10 In a certain building, the lift ascends from ground level to the 60 ft.-level in 10 seconds, waits for 10 seconds, and descends to the ground in a further 10 seconds. The table shows the height of the lift (H ft.) on the upward and downward journeys t seconds after leaving ground level.

t (sec.)	0	2	4	6	8	10	20	22	24	26	28	30
H (ft.)	0	3	16	44	57	60	60	57	44	16	3	0

(i) Draw a graph showing the height of the lift for values of t from 0 to 30. (Take 1 in. to represent 5 seconds on the horizontal scale.)

(ii) By drawing a tangent estimate the rate at which the lift is rising when $t = 8$.

(iii) A man waiting at the 40 ft.-level starts to go down the stairs when $t = 15$. If he descends at a steady rate of 2 ft. per second find, by using the graph, the height at which the lift passes him.

11 *The plain paper provided must be used in this question.*

A parallelogram $ABCD$, whose diagonals intersect at X , is the horizontal base of a pyramid. $AX = 2$ in., $BX = 2\frac{1}{2}$ in. and $\hat{A}XB = 45^\circ$. The vertex V is vertically above the point X and the edges VB and VD are 4 in. long.

Draw the plan of the pyramid and its elevation on a plane parallel to VBD . Use your drawings to find, by further construction, the length of the edge VA .

412/2

MATHEMATICS

ORDINARY LEVEL, ALTERNATIVE B

PAPER 2

(Two hours and a half)

Answer all the questions in Section I and any four in Section II.

All working must be clearly shown; it should be done on the same sheet as the rest of the answer.

Mathematical tables are provided.

SECTION I. [52 marks]

1 (i) Simplify $(3\frac{3}{4} \times 1\frac{1}{5}) - 2\frac{5}{6}$.

(ii) Given that 8 kilometres are equivalent to 5 miles and that there are 640 acres in a square mile, calculate the number of acres in a square kilometre.

(iii) A rectangular block is f in. long, g in. wide and g in. high. Write down an expression for the total surface area of the block.

2 (i) Write down the square root of each of the following expressions

(a) $4x^2 - 12x + 9$; (b) $25x^{16}$.

(ii) Given that $a = \frac{b}{b+1}$, express b in terms of a .

(iii) The expression $cx^2 + dx$ has the value 33 when $x = 3$ and the value 7 when $x = 1$. Calculate the values of c and d .

3 (i) Calculate an interior angle of a regular polygon of 12 sides.

(ii) In the triangles ABC and PQR , $AB = PQ$ and $\hat{A} = \hat{Q}$. State **one** other fact which would ensure that the triangles are congruent.

(iii) Two circles of radius 3 in. and 5 in. touch each other. State the possible distances between the centres of the circles.

(iv) The angles of a quadrilateral, taken in order, are in the ratios 5:4:2:1. Prove that two sides of the quadrilateral are parallel.

4 (i) In the triangle ABC , the sides AB and AC are equal, $BC = 4$ in. and $\hat{B} = 70^\circ$. Calculate AB .

(ii) Calculate the volume of a cylinder of radius 2.39 in. and height 5.62 in. Give your answer correct to the nearest cubic inch. [Take π to be 3.142.]

5 Using ruler and compasses only, construct

(i) the triangle LMN in which $LM = 3$ in., $LN = 5$ in. and $\angle M = 120^\circ$;

(ii) the perpendicular from N on to LM produced, meeting LM produced at K ;

(iii) the position of the point P on NK which is equidistant from LN and LM produced.

Measure NK and hence calculate the area of the triangle LMN .

SECTION II. [48 marks]

Answer any four questions in this section.

6 A manufacturer X makes an article for £300. He sells it to Y at a profit of 20% and Y resells it for £342. Find the percentage profit or loss which Y makes on the transaction.

Labour normally accounts for 65% of the manufacturer's costs. Calculate the labour charges.

If, during the coming year, these labour charges are to increase by 5% and the remaining charges are to increase by $6\frac{2}{3}\%$, calculate the percentage increase in the total cost of manufacturing the article.

7 Two rectangular rooms each have an area of 240 sq. ft. If the length of one of the rooms is x ft. and the other room is 4 ft. longer, write down the width of each room in terms of x .

If the widths of the rooms differ by 3 ft., form an equation in x and show that it reduces to $x^2 + 4x - 320 = 0$. Solve this equation and hence find the difference between the perimeters of the rooms.

8 The point P is 5 miles due north of the point Q . A man walks from Q in a direction $N. 26^\circ E.$ (026°). Calculate how far he walks before he is

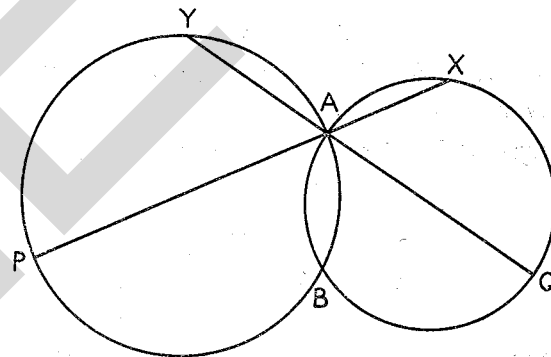
- equidistant from P and Q ;
- as close as possible to P ;
- north-east of P .

9 The sides AB, BC of a rhombus $ABCD$ are inclined to each other at an angle of 60° and the shorter diagonal AC is 6 cm. long. Calculate

- the length CD ;
- the length BD ;
- the area of the rhombus.

The perpendicular from A on to BC cuts BD at X and meets BC at Y . Calculate the lengths of the sides of the triangle BXY .

10 Prove that the angle in a semicircle is a right-angle.



In the figure, AP and AQ are diameters. Prove that P, B and Q lie on a straight line.

PA, QA produced meet the circles again at X, Y . Prove that $PYXQ$ is a cyclic quadrilateral and that PX bisects BXY .

11 A ship leaves a port P which lies in latitude $20^\circ N$. It sails due east through 30° of longitude and then due south to Q , which lies on the equator. Calculate the distance it has travelled. [Take the radius of the earth to be 3,960 statute miles and π to be 3.142.]

On the return journey it sails due west through 30° of longitude and then due north back to P . Show that the difference in length between the outward and inward journeys is approximately 125 statute miles.

Using this value of 125 miles and taking 1 knot to be 6080 ft. per hr., calculate the difference in times between the two journeys, assuming that, on each journey, the ship sails at an average speed of 25 knots.

UCLES