Cambridge Assessment

A Level

Mathematics

Session:	2010 June
Type:	Question paper
Code:	3890-7890; 3892-7892
Unit:	4725, 4726, 4727

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE
MATHEMATICS **4725**
Further Pure Mathematics 1

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Friday 11 June 2010
Morning

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Prove by induction that, for $n \geqslant 1$, $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$. **[5]**

2 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 1 & -4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 0 \\ -2 & 2 \end{pmatrix}$. Find

(i) **AB**, **[2]**

(ii) **BA** – 4**C**. **[4]**

3 Find $\sum_{r=1}^{n}(2r-1)^2$, expressing your answer in a fully factorised form. **[6]**

4 The complex numbers a and b are given by $a = 7 + 6\text{i}$ and $b = 1 - 3\text{i}$. Showing clearly how you obtain your answers, find

(i) $|a - 2b|$ and $\arg(a - 2b)$, **[4]**

(ii) $\frac{b}{a}$, giving your answer in the form $x + \text{i}y$. **[3]**

5 **(a)** Write down the matrix that represents a reflection in the line $y = x$. **[2]**

(b) Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$, **[2]**

(ii) $\begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$. **[2]**

6 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z - 3 + 4\text{i}| = 5$, **[2]**

(b) $|z| = |z - 6|$. **[2]**

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 + 4\text{i}| \leqslant 5 \quad \text{and} \quad |z| \geqslant |z - 6|.$$ **[2]**

7 The quadratic equation $x^2 + 2kx + k = 0$, where k is a non-zero constant, has roots α and β. Find a quadratic equation with roots $\frac{\alpha + \beta}{\alpha}$ and $\frac{\alpha + \beta}{\beta}$. **[7]**

8 **(i)** Show that $\dfrac{1}{\sqrt{r+2}+\sqrt{r}} \equiv \dfrac{\sqrt{r+2}-\sqrt{r}}{2}$. **[2]**

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r+2}+\sqrt{r}}.$$ **[6]**

(iii) State, giving a brief reason, whether the series $\displaystyle\sum_{r=1}^{\infty} \frac{1}{\sqrt{r+2}+\sqrt{r}}$ converges. **[1]**

9 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & a & -1 \\ 0 & a & 2 \\ 1 & 2 & 1 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of **A**. **[3]**

(ii) Three simultaneous equations are shown below.

$$\begin{aligned} ax + ay - z &= -1 \\ ay + 2z &= 2a \\ x + 2y + z &= 1 \end{aligned}$$

For each of the following values of a, determine whether the equations are consistent or inconsistent. If the equations are consistent, determine whether or not there is a unique solution.

(a) $a = 0$

(b) $a = 1$

(c) $a = 2$

[6]

10 The complex number z, where $0 < \arg z < \frac{1}{2}\pi$, is such that $z^2 = 3 + 4\mathrm{i}$.

(i) Use an algebraic method to find z. **[5]**

(ii) Show that $z^3 = 2 + 11\mathrm{i}$. **[1]**

The complex number w is the root of the equation

$$w^6 - 4w^3 + 125 = 0$$

for which $-\frac{1}{2}\pi < \arg w < 0$.

(iii) Find w. **[5]**

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MATHEMATICS 4726

Further Pure Mathematics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Thursday 27 May 2010
Morning

Duration: 1 hour 30 minutes

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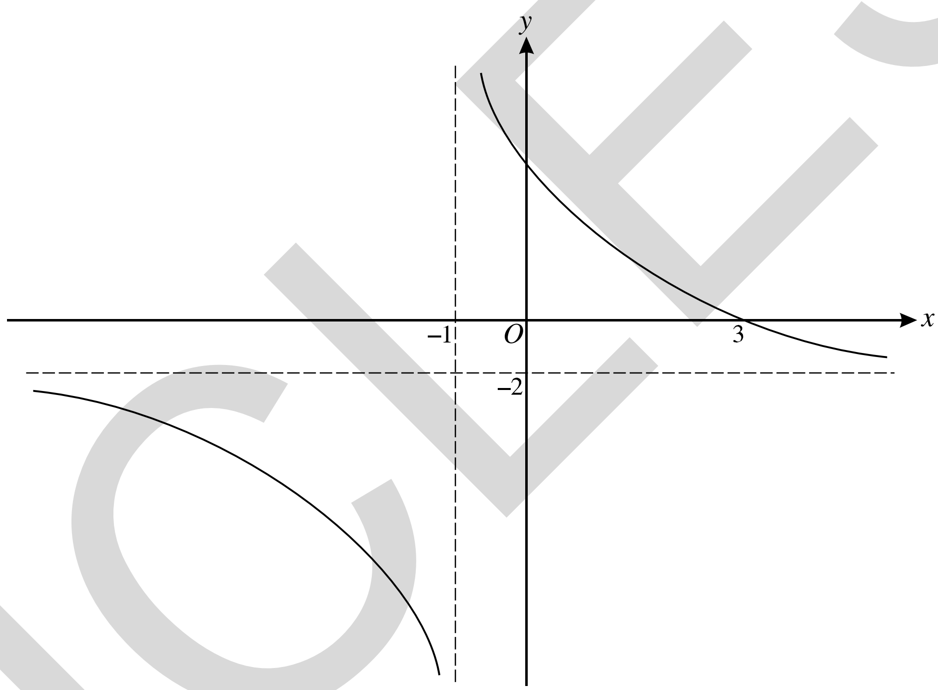
1 It is given that $f(x) = \tan^{-1} 2x$ and $g(x) = p \tan^{-1} x$, where p is a constant. Find the value of p for which $f'\left(\frac{1}{2}\right) = g'\left(\frac{1}{2}\right)$. **[4]**

2 Given that the first three terms of the Maclaurin series for $(1 + \sin x)e^{2x}$ are identical to the first three terms of the binomial series for $(1 + ax)^n$, find the values of the constants a and n. (You may use appropriate results given in the List of Formulae (MF1).) **[6]**

3 Use the substitution $t = \tan \frac{1}{2}x$ to show that

$$\int_0^{\frac{1}{3}\pi} \frac{1}{1 - \sin x}\, dx = 1 + \sqrt{3}.$$ **[6]**

4



The diagram shows the curve with equation

$$y = \frac{ax + b}{x + c},$$

where a, b and c are constants.

(i) Given that the asymptotes of the curve are $x = -1$ and $y = -2$ and that the curve passes through (3, 0), find the values of a, b and c. **[3]**

(ii) Sketch the curve with equation

$$y^2 = \frac{ax + b}{x + c},$$

for the values of a, b and c found in part **(i)**. State the coordinates of any points where the curve crosses the axes, and give the equations of any asymptotes. **[4]**

5 It is given that, for $n \geqslant 0$,

$$I_n = \int_0^{\frac{1}{2}} (1-2x)^n e^x \, dx.$$

(i) Prove that, for $n \geqslant 1$,

$$I_n = 2nI_{n-1} - 1.$$ **[4]**

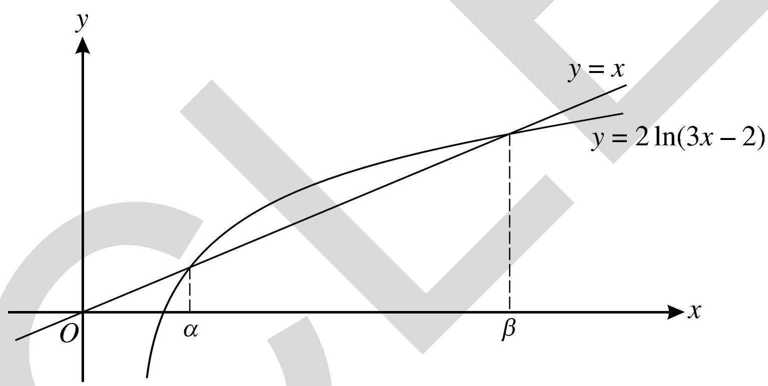
(ii) Find the exact value of I_3. **[4]**

6 **(i)** Show that $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$. **[2]**

(ii) Given that $y = \cosh(a \sinh^{-1} x)$, where a is a constant, show that

$$(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - a^2 y = 0.$$ **[5]**

7



The line $y = x$ and the curve $y = 2\ln(3x-2)$ meet where $x = \alpha$ and $x = \beta$, as shown in the diagram.

(i) Use the iteration $x_{n+1} = 2\ln(3x_n - 2)$, with initial value $x_1 = 5.25$, to find the value of β correct to 2 decimal places. Show all your working. **[2]**

(ii) With the help of a 'staircase' diagram, explain why this iteration will not converge to α, whatever value of x_1 (other than α) is used. **[3]**

(iii) Show that the equation $x = 2\ln(3x-2)$ can be rewritten as $x = \frac{1}{3}\left(e^{\frac{1}{2}x} + 2\right)$. Use the Newton-Raphson method, with $f(x) = \frac{1}{3}\left(e^{\frac{1}{2}x} + 2\right) - x$ and $x_1 = 1.2$, to find α correct to 2 decimal places. Show all your working. **[4]**

(iv) Given that $x_1 = \ln 36$, explain why the Newton-Raphson method would not converge to a root of $f(x) = 0$. **[2]**

[Questions 8 and 9 are printed overleaf.]

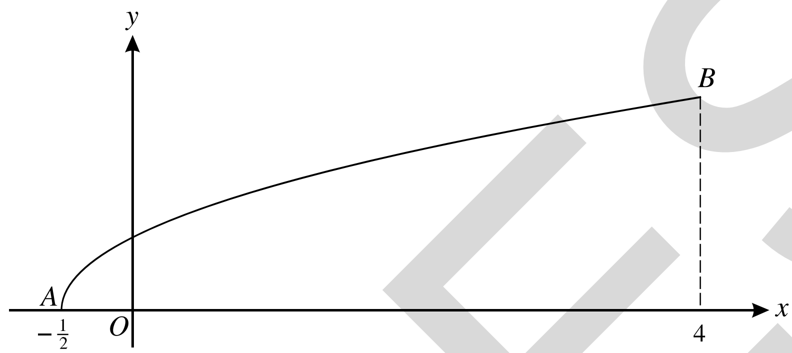
8 **(i)** Using the definition of $\cosh x$ in terms of e^x and e^{-x}, show that

$$4\cosh^3 x - 3\cosh x \equiv \cosh 3x.$$ **[4]**

(ii) Use the substitution $u = \cosh x$ to find, in terms of $5^{\frac{1}{3}}$, the real root of the equation

$$20u^3 - 15u - 13 = 0.$$ **[6]**

9



The diagram shows the curve with equation $y = \sqrt{2x+1}$ between the points $A\,(-\frac{1}{2}, 0)$ and $B\,(4, 3)$.

(i) Find the area of the region bounded by the curve, the x-axis and the line $x = 4$. Hence find the area of the region bounded by the curve and the lines OA and OB, where O is the origin. **[4]**

(ii) Show that the curve between B and A can be expressed in polar coordinates as

$$r = \frac{1}{1-\cos\theta}, \quad \text{where } \tan^{-1}\left(\tfrac{3}{4}\right) \leqslant \theta \leqslant \pi.$$ **[5]**

(iii) Deduce from parts **(i)** and **(ii)** that $\displaystyle\int_{\tan^{-1}\left(\frac{3}{4}\right)}^{\pi} \operatorname{cosec}^4\left(\tfrac{1}{2}\theta\right)\,d\theta = 24.$ **[4]**

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MATHEMATICS 4727

Further Pure Mathematics 3

Candidates answer on the Answer Booklet

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- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Monday 24 May 2010
Afternoon

Duration: 1 hour 30 minutes

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3R–9K03

1 The line l_1 passes through the points (0, 0, 10) and (7, 0, 0) and the line l_2 passes through the points (4, 6, 0) and (3, 3, 1). Find the shortest distance between l_1 and l_2. **[7]**

2 A multiplicative group with identity e contains distinct elements a and r, with the properties $r^6 = e$ and $ar = r^5a$.

(i) Prove that $rar = a$. **[2]**

(ii) Prove, by induction or otherwise, that $r^n a r^n = a$ for all positive integers n. **[4]**

3 In this question, w denotes the complex number $\cos\frac{2}{5}\pi + \mathrm{i}\sin\frac{2}{5}\pi$.

(i) Express w^2, w^3 and w^* in polar form, with arguments in the interval $0 \leqslant \theta < 2\pi$. **[4]**

(ii) The points in an Argand diagram which represent the numbers

$$1, \quad 1+w, \quad 1+w+w^2, \quad 1+w+w^2+w^3, \quad 1+w+w^2+w^3+w^4$$

are denoted by A, B, C, D, E respectively. Sketch the Argand diagram to show these points and join them in the order stated. (Your diagram need not be exactly to scale, but it should show the important features.) **[4]**

(iii) Write down a polynomial equation of degree 5 which is satisfied by w. **[1]**

4 **(i)** Use the substitution $y = xz$ to find the general solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x\cos\left(\frac{y}{x}\right),$$

giving your answer in a form without logarithms. (You may quote an appropriate result given in the List of Formulae (MF1).) **[6]**

(ii) Find the solution of the differential equation for which $y = \pi$ when $x = 4$. **[2]**

5 Convergent infinite series C and S are defined by

$$C = 1 + \tfrac{1}{2}\cos\theta + \tfrac{1}{4}\cos 2\theta + \tfrac{1}{8}\cos 3\theta + \ldots\ ,$$
$$S = \tfrac{1}{2}\sin\theta + \tfrac{1}{4}\sin 2\theta + \tfrac{1}{8}\sin 3\theta + \ldots\ .$$

(i) Show that $C + \mathrm{i}S = \dfrac{2}{2 - \mathrm{e}^{\mathrm{i}\theta}}$. **[4]**

(ii) Hence show that $C = \dfrac{4 - 2\cos\theta}{5 - 4\cos\theta}$, and find a similar expression for S. **[4]**

6 **(i)** Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 17x + 36.$$ **[7]**

(ii) Show that, when x is large and positive, the solution approximates to a linear function, and state its equation. **[2]**

7 A line l has equation $\mathbf{r} = \begin{pmatrix} -7 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$. A plane Π passes through the points (1, 3, 5) and (5, 2, 5), and is parallel to l.

(i) Find an equation of Π, giving your answer in the form $\mathbf{r.n} = p$. **[4]**

(ii) Find the distance between l and Π. **[4]**

(iii) Find an equation of the line which is the reflection of l in Π, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. **[4]**

8 A set of matrices M is defined by

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \quad C = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad E = \begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}, \quad F = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix},$$

where ω and ω^2 are the complex cube roots of 1. It is given that M is a group under matrix multiplication.

(i) Write down the elements of a subgroup of order 2. **[1]**

(ii) Explain why there is no element X of the group, other than A, which satisfies the equation $X^5 = A$. **[2]**

(iii) By finding BE and EB, verify the closure property for the pair of elements B and E. **[4]**

(iv) Find the inverses of B and E. **[3]**

(v) Determine whether the group M is isomorphic to the group N which is defined as the set of numbers {1, 2, 4, 8, 7, 5} under multiplication modulo 9. Justify your answer clearly. **[3]**