A Level

Mathematics

Session: 2010 June
Type: Question paper
Code: 3890-7890; 3892-7892
Unit: 4736, 4737

OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS 4736

Decision Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book

OCR Supplied Materials:
- Printed Answer Book 4736
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Tuesday 22 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- **The questions are on the inserted Question Paper.**
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or destroyed.

1 Owen and Hari each want to sort the following list of marks into decreasing order.

31 28 75 87 42 43 70 56 61 95

(i) Owen uses bubble sort, starting from the left-hand end of the list.

(a) Show the result of the first pass through the list. Record the number of comparisons and the number of swaps used in this first pass. Which marks, if any, are guaranteed to be in their correct final positions after the first pass? **[4]**

(b) Write down the list at the end of the second pass of bubble sort. **[1]**

(c) How many more passes are needed to get the value 95 to the start of the list? **[1]**

(ii) Hari uses shuttle sort, starting from the left-hand end of the list.

Show the results of the first and the second pass through the list. Record the number of comparisons and the number of swaps used in each of these passes. **[4]**

(iii) Explain why, for this particular list, the total number of comparisons will be greater using bubble sort than using shuttle sort. **[2]**

Shuttle sort is a quadratic order algorithm.

(iv) If it takes Hari 20 seconds to sort a list of ten marks using shuttle sort, approximately how long will it take Hari to sort a list of fifty marks? **[2]**

2 A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex.

A *simply connected* graph is one that is both simple and connected.

(i) Explain why it is impossible to draw a graph with exactly three vertices in which the vertex orders are 2, 3 and 4. **[1]**

(ii) Draw a graph with exactly four vertices of orders 1, 2, 3 and 4 that is neither simple nor connected. **[2]**

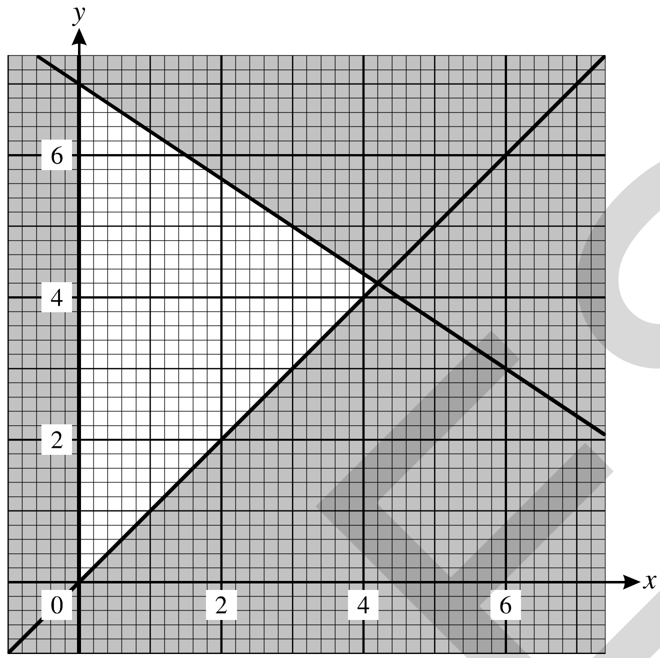
(iii) Explain why there is no simply connected graph with exactly four vertices of orders 1, 2, 3 and 4. State which of the properties 'simple' and 'connected' cannot be achieved. **[2]**

(iv) A simply connected Eulerian graph has exactly five vertices.

(a) Explain why there cannot be exactly three vertices of order 4. **[1]**

(b) By considering the vertex orders, explain why there are only four such graphs. Draw an example of each. **[3]**

3 The constraints of a linear programming problem are represented by the graph below. The feasible region is the unshaded region, including its boundaries.



(i) Write down the inequalities that define the feasible region. **[3]**

The objective is to maximise $P_1 = x + 6y$.

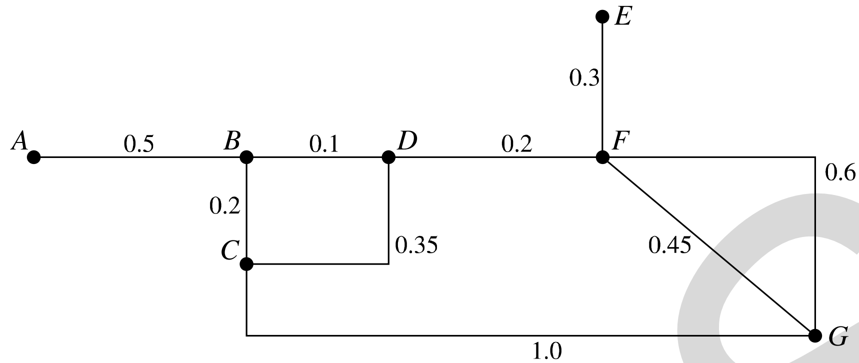
(ii) Find the values of x and y at the optimal point, and the corresponding value of P_1. **[3]**

The objective is changed to maximise $P_k = kx + 6y$, where k is positive.

(iii) Calculate the coordinates of the optimal point, and the corresponding value of P_k when the optimal point is not the same as in part **(ii)**. **[2]**

(iv) Find the range of values of k for which the point identified in part **(ii)** is still optimal. **[2]**

4 The network below represents a small village. The arcs represent the streets and the weights on the arcs represent distances in km.



(i) Use Dijkstra's algorithm to find the shortest path from A to G. You must show your working, including temporary labels, permanent labels and the order in which permanent labels are assigned. Write down the route of the shortest path from A to G. **[5]**

Hannah wants to deliver newsletters along every street; she will start and end at A.

(ii) Which standard network problem does Hannah need to solve to find the shortest route that uses every arc? **[1]**

The total weight of all the arcs is 3.7 km.

(iii) Hannah knows that she will need to travel AB and EF twice, once in each direction. With this information, use an appropriate algorithm to find the length of the shortest route that Hannah can use. Show all your working. (You may find the lengths of shortest paths between vertices by inspection.) **[5]**

There are street name signs at each vertex except for A and E. Hannah's friend Peter wants to check that the signs have not been vandalised. He will start and end at B.

The table below shows the complete set of shortest distances between vertices B, C, D, F and G.

	B	C	D	F	G
B	–	0.2	0.1	0.3	0.75
C	0.2	–	0.3	0.5	0.95
D	0.1	0.3	–	0.2	0.65
F	0.3	0.5	0.2	–	0.45
G	0.75	0.95	0.65	0.45	–

(iv) Apply the nearest neighbour method to this table, starting from B, to find an upper bound for the distance that Peter must travel. **[2]**

(v) Apply Prim's algorithm to the matrix formed by deleting the row and column for vertex G from the table. Start building your tree at vertex B.

Draw your tree. Give the order in which vertices are built into your tree and calculate the total weight of your tree. Hence find a lower bound for the distance that Peter must travel. **[4]**

5 Jenny is making three speciality smoothies for a party: *fruit salad*, *ginger zinger* and *high C*.

Each litre of *fruit salad* contains 600 calories and has 120 mg of sugar and 100 mg of vitamin C.
Each litre of *ginger zinger* contains 800 calories and has 80 mg of sugar and 40 mg of vitamin C.
Each litre of *high C* contains 500 calories and has 120 mg of sugar and 120 mg of vitamin C.

Jenny has enough milk to make 5 litres of *fruit salad* or 3 litres of *ginger zinger* or 4 litres of *high C*. This leads to the constraint

$$12x + 20y + 15z \leqslant 60$$

in which x represents the number of litres of *fruit salad*, y represents the number of litres of *ginger zinger* and z represents the number of litres of *high C*.

Jenny wants there to be no more than 5000 calories and no more than 800 mg of sugar in total in the smoothies that she makes.

(i) Use this information to write down and simplify two more constraints on the values of x, y and z, other than that they are non-negative. **[4]**

Jenny wants to maximise the total amount of vitamin C in the smoothies. This gives the following objective.

$$\text{Maximise } P = 100x + 40y + 120z$$

(ii) Represent Jenny's problem as an initial Simplex tableau. Use the Simplex algorithm, choosing the first pivot from the z column and showing all your working, to find the optimum. How much of each type of smoothie should Jenny make? **[13]**

(iii) Show that if the first pivot had been chosen from the x column then the optimum would have been achieved in one iteration instead of two. **[5]**

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ADVANCED GCE

MATHEMATICS **4737**

Decision Mathematics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- Insert for Questions 5 and 6 (inserted)
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes

* 4 7 3 7 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- There is an **insert** for use in Questions **5** and **6**.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 The famous fictional detective Agatha Parrot is investigating a murder. She has identified six suspects: Mrs Lemon (L), Prof Mulberry (M), Mr Nutmeg (N), Miss Olive (O), Capt Peach (P) and Rev Quince (Q). The table shows the weapons that could have been used by each suspect.

		Suspect					
		L	M	N	O	P	Q
Axe handle	A		✓	✓			✓
Broomstick	B		✓		✓		
Drainpipe	D	✓		✓			
Fence post	F			✓	✓		
Golf club	G	✓				✓	✓
Hammer	H	✓			✓	✓	

(i) Draw a bipartite graph to represent this information. Put the weapons on the left-hand side and the suspects on the right-hand side. **[1]**

Agatha Parrot is convinced that all six suspects were involved, and that each used a different weapon. She originally thinks that the axe handle was used by Prof Mulberry, the broomstick by Miss Olive, the drainpipe by Mrs Lemon, the fence post by Mr Nutmeg and the golf club by Capt Peach. However, this would leave the hammer for Rev Quince, which is not a possible pairing.

(ii) Draw a second bipartite graph to show this incomplete matching. **[1]**

(iii) Construct the shortest possible alternating path from H to Q and hence find a complete matching. Write down which suspect used each weapon. **[2]**

(iv) Find a different complete matching in which none of the suspects used the same weapon as in the matching from part **(iii)**. **[2]**

2 In an investigation into a burglary, Agatha has five suspects who were all known to have been near the scene of the crime, each at a different time of the day. She collects evidence from witnesses and draws up a table showing the number of witnesses claiming sight of each suspect near the scene of the crime at each possible time.

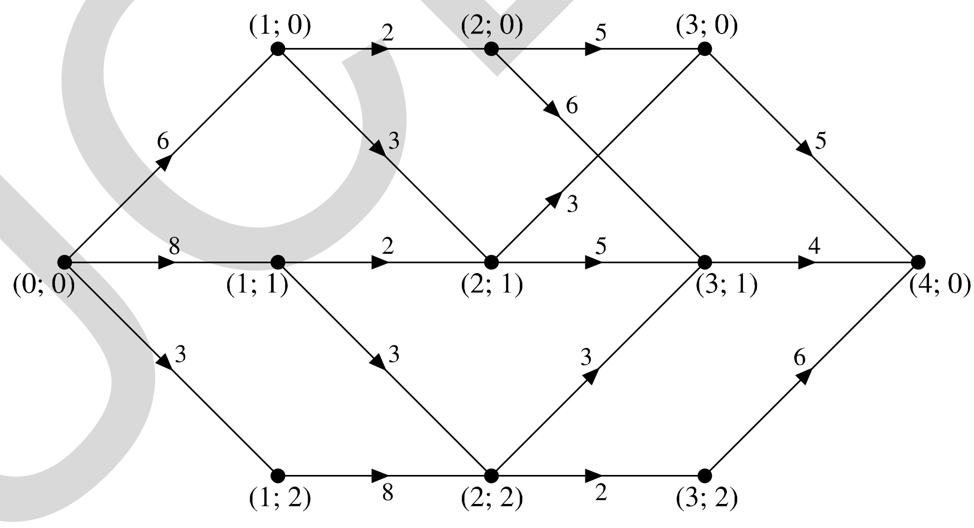
			Time				
			1 pm	2 pm	3 pm	4 pm	5 pm
Suspect	Mrs Rowan	R	3	4	2	7	1
	Dr Silverbirch	S	5	10	6	6	6
	Mr Thorn	T	4	7	3	5	3
	Ms Willow	W	6	8	4	8	3
	Sgt Yew	Y	8	8	7	4	3

(i) Use the Hungarian algorithm on a suitably modified table, reducing rows first, to find the matchings for which the total number of claimed sightings is maximised. Show your working clearly. Write down the resulting matchings between the suspects and the times. **[9]**

Further enquiries show that the burglary took place at 5 pm, and that Dr Silverbirch was not the burglar.

(ii) Who should Agatha suspect? **[1]**

3 **(i)** Set up a dynamic programming tabulation to find the minimum weight route from (0; 0) to (4; 0) on the following directed network.



Give the route and its total weight. **[9]**

(ii) Explain carefully how the route is obtained directly from the values in the table, without referring to the network. **[2]**

4 Euan and Wai Mai play a zero-sum game. Each is trying to maximise the total number of points that they score in many repeats of the game. The table shows the number of points that Euan scores for each combination of strategies.

		Wai Mai		
		X	Y	Z
Euan	A	2	−5	3
	B	−1	−3	4
	C	3	−5	2
	D	3	−2	−1

(i) Explain what the term 'zero-sum game' means. **[1]**

(ii) How many points does Wai Mai score if she chooses X and Euan chooses A? **[1]**

(iii) Why should Wai Mai never choose strategy Z? **[2]**

(iv) Delete the column for Z and find the play-safe strategy for Euan and the play-safe strategy for Wai Mai on the table that remains. Is the resulting game stable or not? State how you know. **[4]**

The value 3 in the cell corresponding to Euan choosing D and Wai Mai choosing X is changed to −5; otherwise the table is unchanged.

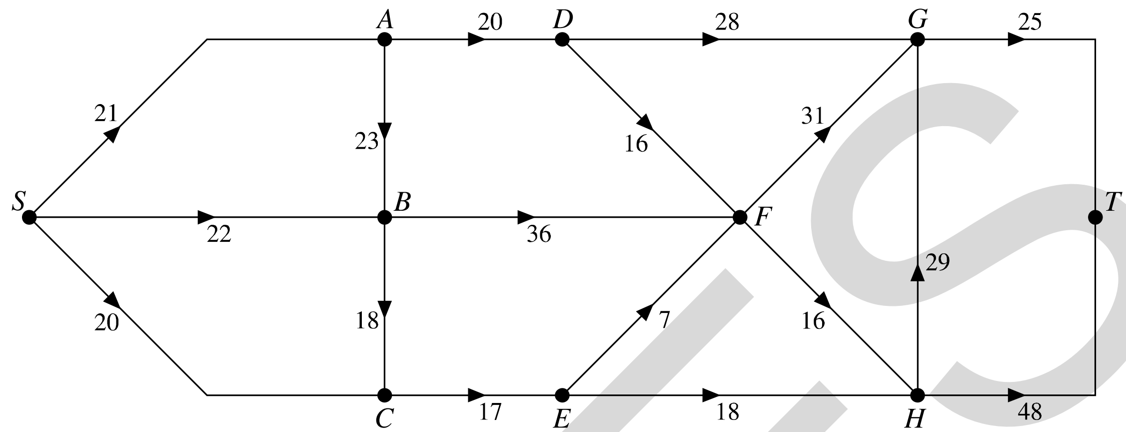
Wai Mai decides that she will choose her strategy by making a random choice between X and Y, choosing X with probability p and Y with probability $1 - p$.

(v) Write down and simplify an expression for the expected score for Wai Mai when Euan chooses each of his four strategies. **[3]**

(vi) Using graph paper, draw a graph showing Wai Mai's expected score against p for each of Euan's four strategies and hence calculate the optimum value of p. **[4]**

5 Answer this question on the insert provided.

The network represents a system of irrigation channels along which water can flow. The weights on the arcs represent the maximum flow in litres per second.



(i) Calculate the capacity of the cut that separates $\{S, B, C, E\}$ from $\{A, D, F, G, H, T\}$. **[2]**

(ii) Explain why neither arc SC nor arc BC can be full to capacity. Explain why the arcs EF and EH cannot both be full to capacity. Hence find the maximum flow along arc HT. When arc HT carries its maximum flow, what is the flow along arc HG? **[4]**

(iii) Show a flow of 58 litres per second on the diagram in the insert, and find a cut of capacity 58. **[3]**

The direction of flow in HG is reversed.

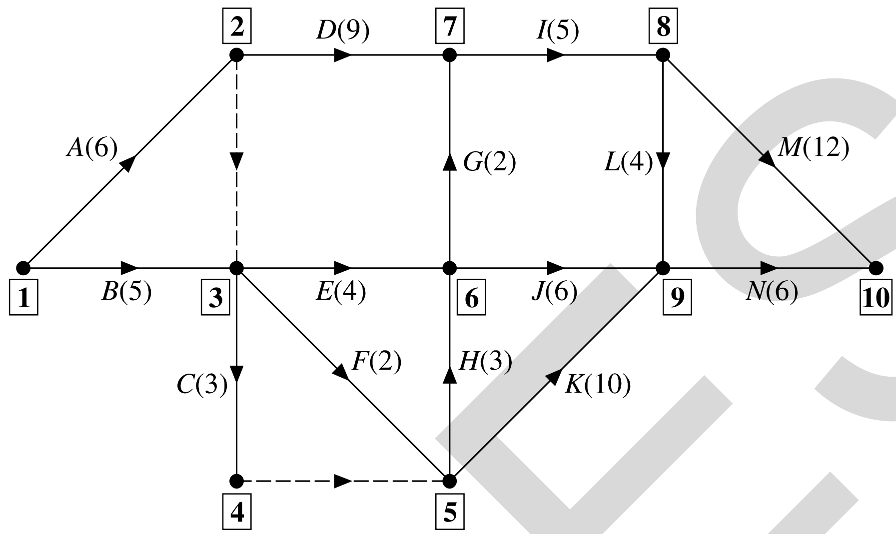
(iv) Use the diagram in the insert to show the excess capacities and potential backflows for your flow from part **(iii)** in this case. **[2]**

(v) Without augmenting the labels from part **(iv)**, write down flow augmenting routes to enable an additional 2 litres per second to flow from S to T. **[2]**

(vi) Show your augmented flow on the diagram in the insert. Explain how you know that this flow is maximal. **[2]**

6 **Answer parts (i), (ii) and (iii) of this question on the insert provided.**

The activity network for a project is shown below. The durations are in minutes. The events are numbered $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, etc. for reference.



(i) Complete the table in the insert to show the immediate predecessors for each activity. **[3]**

(ii) Explain why the dummy activity is needed between event $\boxed{2}$ and event $\boxed{3}$, and why the dummy activity is needed between event $\boxed{4}$ and event $\boxed{5}$. **[2]**

(iii) Carry out a forward pass to find the early event times and a backward pass to find the late event times. Record your early event times and late event times in the table in the insert. Write down the minimum project completion time and the critical activities. **[5]**

Suppose that the duration of activity K changes to x minutes.

(iv) Find, in terms of x, expressions for the early event time and the late event time for event $\boxed{9}$. **[4]**

(v) Find the maximum duration of activity K that will not affect the minimum project completion time found in part **(iii)**. **[1]**

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ADVANCED GCE

MATHEMATICS 4737

Decision Mathematics 2

INSERT for Questions 5 and 6

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes

Candidate Forename		Candidate Surname	

Centre Number						Candidate Number				

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- This insert should be used to answer Question **5** and Question **6** parts **(i)**, **(ii)** and **(iii)**.
- Write your answers to Question **5** and Question **6** parts **(i)**, **(ii)** and **(iii)** in the spaces provided in this insert, and attach it to your Answer Booklet.

INFORMATION FOR CANDIDATES

- This document consists of **4** pages. Any blank pages are indicated.

5 **(i)** Capacity of the cut that separates $\{S, B, C, E\}$ from $\{A, D, F, G, H, T\}$ =

.... litres per second

(ii) Neither arc SC nor arc BC can be full to capacity since ....

....

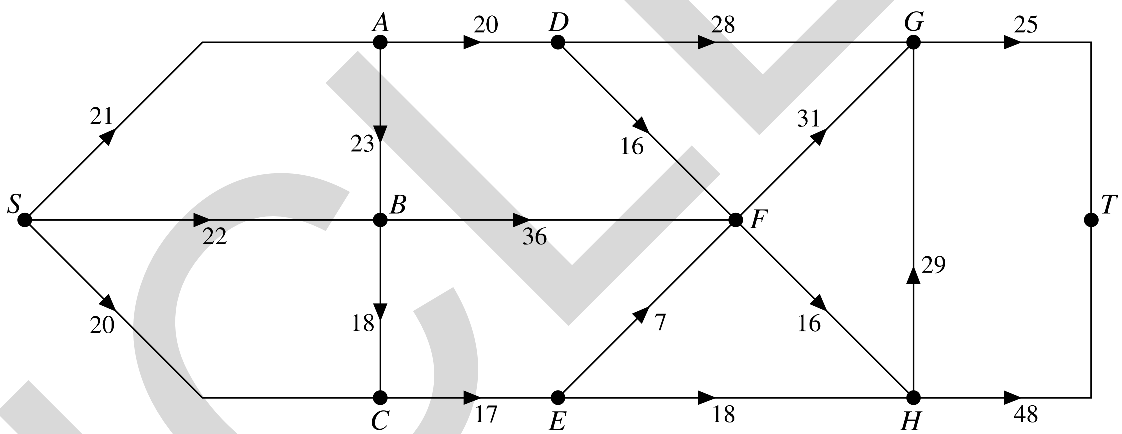
Arcs EF and EH cannot both be full to capacity since ....

....

Maximum flow along arc HT is .... litres per second

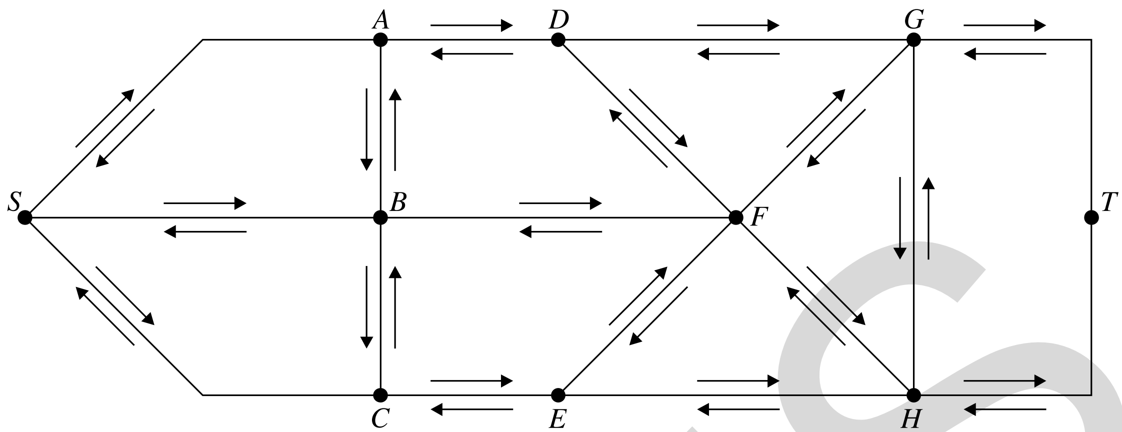
When arc HT carries its maximum flow, the flow along arc HG is .... litres per second

(iii) Flow of 58 litres per second



Cut of capacity 58 litres per second ....

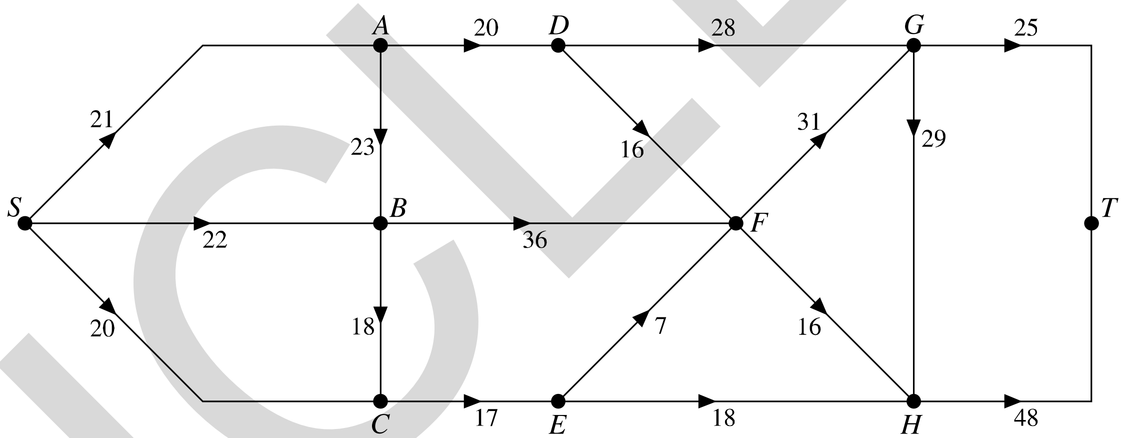
(iv)



(v) Augment flow in route .... by .... litres per second

Augment flow in route .... by .... litres per second

(vi)



Flow is maximal because ....

....

....

6 **(i)**

Activity	Duration	Immediate predecessors
A	6	
B	5	
C	3	
D	9	
E	4	
F	2	
G	2	
H	3	
I	5	
J	6	
K	10	
L	4	
M	12	
N	6	

(ii) Dummy activity is needed between event $\boxed{2}$ and event $\boxed{3}$ because ....

....

Dummy activity is needed between event $\boxed{4}$ and event $\boxed{5}$ because ....

....

(iii)

Event	$\boxed{1}$	$\boxed{2}$	$\boxed{3}$	$\boxed{4}$	$\boxed{5}$	$\boxed{6}$	$\boxed{7}$	$\boxed{8}$	$\boxed{9}$	$\boxed{10}$
Early event time										
Late event time										

Minimum project completion time = ... minutes

Critical activities: ....

Answer part (iv) and part (v) in your answer booklet.