A Study of the Effects of Context in the Assessment of the Mathematical Learning of 10/11 year olds

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Abstract

This paper reports the third stage of a study of the effects of context in assessing the mathematical learning of 10/11 year olds. The aim of the study is to identify (a) the cognitive level/s at which context aids or obstructs pupil performance , (b) which of five identified element/s of the context contribute to this effect and (c) how they do so. The effects of the presence (or absence) of five elements of context are identified. 140 pupils from 4 schools completed parallel contextualised and context-free questions in three areas of mathematics. Ten pupils were followed up in one-to-one interviews to study their performance on questions against five stages of problem-solving. 'Operatives' are identified in the form of one or two key ideas which a pupils identifies and carries through each phase of solving a problem. The results suggest that mathematics in a presented context provides pupils with something with which to reason as well as a goal towards which to work.

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INTRODUCTION

In 1985 the Assessment Performance Unit (APU) in England and Wales identified the importance of context in assessing mathematical learning by stating that:

> "Mathematics performance cannot be assessed on its own; the mathematics must be communicated in some way and presentation influences performance." (APU 1985 p838)

This paper presents some of the findings of part of a longitudinal study to explore the question of how the context in which mathematics is presented influences pupil performance and to identify the ways in which it does so. Implicit in this exploration of the effects of context lies a concern with the processes within problem solving.

Krutetskii (1976) has identified the importance of going beyond the results of a test, i.e. the quantitative aspect of assessment, and looking at the qualitative aspects in order to reveal the mental processes behind children's thinking. He refers to this as providing "the psychological essence of results" in assessing mathematical learning. (Krutetskii 1976 p13) By attending to these qualitative aspects of assessment (as this study attempts to do) it may be possible to understand more fully how pupils approach a mathematical problem in a *presented context*(see below). A case study approach to how children solve the kinds of problems they meet in current mathematical assessment procedures should provide some insight into the effects of different elements of context and building on this, how to devise more effective assessment of mathematical learning in context.

This project arose initially from an interest in the results of moving from norm-referenced testing to criterion-referenced testing in the United Kingdom when such testing was adopted with the introduction of the new General Certificate in Secondary Education. The stated aim of such testing is that it

> "provides an evaluative description of the qualities which are to be assessed (e.g. an account of what pupils know and can do) without reference to the performance of others." (Brown 1988 p4)

This approach to testing is often described in terms of rewarding pupils for *positive achievement* and Butterfield(1995) describes how it later became linked with progression within the UK National Curriculum as a whole.

The introduction of criterion-referenced testing coincided with attempts to link the teaching of mathematics more with the 'real world'of pupils and assessment in mathematics, in turn, tried to reflect such teaching and to assess more and more in context. A cursory study of test items performed by pupils at any of the Key Stages in the mathematics curriculum indicates that often pupils move through such problems with success at various stages of a problem but do not get 'the right answer' at the end of the problem. The question that arises in relation to criterion referenced testing is whether or not pupils **are** being rewarded for what they **can do** when they are assessed in this way, as is the stated intention**.** If they can proceed through important early stages of a problem successfully, are they being rewarded for doing so? If not, it seems even more crucial that we understand the effects of context so that pupils are not penalised when they are assessed in this way. This means understanding more fully the role of the different elements of context and possibly controlling them so that something like optimum contextualisation may be achieved in assessing mathematical learning.

METHODOLOGY

This stage of the project has involved 10 pupils who initially took part in the trialling of a set of parallel contextualised questions (CQs) and context-free questions (CFQs) with 139 11 year-olds (see Appendix 1). The 10 pupils were selected on the basis of their performance on these questions where particular situations presented themselves, e.g. a pupil gained a correct answer on the CQ but incorrect on the parallel CFQ (or vice versa), they may have answered the wrong question or they may have used an interesting strategy. The six questions came from three different areas of mathematics: Number, Shape and Space and Data Handling. Taped interviews were conducted with the pupils (five girls and five boys) in which they were given their completed question papers and asked to talk through what they had done. Interviews were carried out using questions within a semi-structured approach which reflected the framework adopted for problem solving (see below). The disadvantages of interviewing some time after an event has occurred have been identified by Ericsson and Simon (1993) who note the pitfalls of asking children to 'rehearse' their thought processes at a distance in time, e.g. the possibility that they will 'make up' something because they are expected to answer or that they will say what they think the interviewer wants to hear. Whatever the drawbacks, however, this stage of the study was considered important in exploring the feasibility of identifying stages of problem solving in the recorded thoughts of the children against a structure to be used later in the study.

The framework adopted for the interviews and ultimately, the analysis, reflects to an extent the kinds of knowledge used in problem solving identified by Mayer et al (1984). These are Recognition, Understanding, Strategy and Execution, and we have added a fifth, Verification. However, they are viewed here as stages in the process of solving problems as opposed to kinds of knowledge and as such, they provide a framework for the analysis of the interviews.

The analysis of interviews with pupils about each of the questions was approached with two concerns in mind. Firstly we were interested to see when and how context may have intervened in the problem solving process. The aim was to identify any commonalities that existed at each stage amongst the group as a whole in relation to the effect context may have had on progression through a problem and the point at which this may have occurred. Secondly, we were interested in the performance of pupils on the paired CQ/CFQ questions and to probe for differences in their problem solving procedures in the parallel questions.

The focus of the study is with the *presented context* of a question, i.e. not just the everyday theme in which a question is set but all the elements that are contained within its presentation. The elements of context have been identified as pictures, words, numbers, symbols and graphics. At this stage of the study they are being used only as an indication of the richness of the context in which a question is set but will figure more strongly in later parts of our investigation.

Performance on individual questions

In the course of analysing the individual pupils' commentaries about how they proceeded through a problem, it emerged that in dealing with CQs, pupils appeared to focus on one or two aspects within a question and to carry these through the remaining stages in solving the problem. We have called these *operatives* and they took the form of a specific concept, a mathematical operation, a number or a process. The criterion for identifying an operative was that it appeared in at least three of the RUSE stages (not the V stage, see below). The notion of an *operativ*e has provided a useful tool in trying to make sense of how the pupils went about solving these problems.

Although we consider the Verification stage an important aspect of problem solving, the results reported below indicated that few pupils carried out a check of their results and if they did, it was most often done by repeating what they had already done.
As a result, data relating to this part of the analysis is very limited.

ANALYSIS

<u>Number</u>

CQ1

In CQ1 (see Appendix 2) where the question was to find how many 6-packs of cola would be needed at a party for 42 people, the most frequent operatives were 6, 42 and 'times table' and a less frequent alternative was 'price'. Since the question asked how many packs were needed, focussing on price could lead a pupil to answer the wrong question. Simon's response to this question is an example of the effects of the latter.

At the Understanding stage Simon interprets the question in this way:

> "How many packs must he buy....what they're trying to do is....
> say he as 42 cans to buy and they're £1.30 each."

and later, at the Strategy stage, he says:

> "You need 42 so first you add up in 6.....You can keep $1.30
> in your head."

He focussed on the price sign in the picture accompanying the question and carried this through to reach a solution to what he thought was the problem set.

In the one other case (Kim) where there was an incorrect solution to this question, there appeared to be no operative in play. At the Understanding stage, Kim appeared totally distracted by the context when she said:

> "If I'm having a party.....you go to the cash and carry. I'm too
> young - have to be 12 to go there."

At the Strategy stage she said

> "Not sure at first. Put a division sign........I was going to do
> times."

She seemed to have an intuitive sense of what was required and attempted to carry out the correct operation. In the event however, she got the answer wrong (10r2). Had she understood what the question was asking, she would have realised that her answer would not have made sense. When asked whether she had checked her answer (the Verification stage) she replies:

> "Not sure....we weren't allowed to use calculators."

CFQ4

Clearly the fact that a question is context free immediately means that there is a much more limited choice of operatives than in a contextualised question. The most frequent operative for the parallel Number question, CFQ4 (54÷6), was '6', with the number 54 recorded only four times. 'Times table' appeared less frequently as did 'divide'. Whereas 8 pupils were successful on the CQ, only 4 got the correct answer in this parallel CFQ. The effect of lack of context here appeared to be for pupils to concentrate on number and carrying out a mathematical operation (division) which they appeared not fully to understand, but relied on parts of the algorithm they could recall. Sarah provides an example when, at the Strategy stage, she says:

> "Can't get it into 5. Add it together."

and Kelly says

> "Put the 5 and 4 at the top and the 6 underneath."

However, Sarah succeeds because her Strategy was to revert to multiplication and she says

> "Find 6 into 54. Times table."

and comes up with the right answer, whereas Kelly could not link it with any Strategy and got the wrong answer.

Summary

The presence of the context in CQ1 appears to have provided pupils with a way into selecting a strategy to solve this problem which was not the case with CFQ4. This led to fewer calculation errors in the contextualised problem compared with doing the straight 'division sum' in CFQ4 and also provided pupils with a way of checking their answer at the Verification stage.

Shape and Space

CQ2

CQ2 (see Appendix 3) was a 'Shape and Space' question in which there appeared a rectangle (10cm x 6cm) representing a piece of card of given dimensions (50cm x 30cm) and pupils were asked to divide it into 10 equal pieces in order to make invitations for a party. Here the main operatives were 10 and 'divide by'. Only three of the pupils focussed on the fact that the pieces had to be of equal size. Only two of the ten pupils got all parts of this question right (i.e. drew the lines dividing the card into equal pieces and gave both dimensions correctly). Most of the 7 who did not succeed in all aspects of the question failed at the Strategy level because they could not formulate a specific strategy and relied on trial and error methods. The actual act of measuring presented a problem although they knew that this was what had to be done. Kelly, for example, had identified the fact that the pieces had to be "exactly the same size"

and goes on to say:

"First of all I drew my little one and did 10 of them and they all turned out to be equal and so I just carried on."

Drawing the "little one" consisted of marking off 1 cm along the width, repeating the process and finding that there were 10 of them. What she ended up with in her drawing were 10 equal pieces (1cm x 6cm) for which she gave the wrong dimensions (6cm x 10 cm, the dimensions of the original rectangle).

Sarah, on the other hand, had a strategy (at least at the intuitive level). Firstly, she, like Kelly, had identified the fact that the pieces "need to be the same size" and says:

"10 friends, 10 invitations. Have to be the same size."

Although at the Strategy stage she refers to the operative 10 when she says

"Think of 50.....like in 5 cms for each, because if 10.... 5 would go into 50."

she is beginning to think of 10 as a divisor and the operative is taking on a different role. At the Execution stage, she simply says

"Used a ruler....no problems"

and divided the rectangle into 50 equal pieces (but gave the correct dimensions, 5cm x 3 cm). She had lost sight of the operative '10' in terms of '10 equal pieces' at the Execution stage and when asked if she had checked her answer (Verification stage) she said

"Yes. They're all the same size."

CFQ5

The parallel Shape and Space CFQ5 (see Appendix 4) simply asked pupils to divide a rectangle of the same dimensions as that in CQ2 into 10 equal pieces. Although only 2 pupils were successful on all aspects of this question, 4 managed to complete the drawing satisfactorily. The pupils were, as it were, obliged to approach this question numerically. The most frequent operative was 'equal' but in several cases the pupil simply said "Same as the other one" (meaning the parallel CQ2). Some pupils recognised that the concept of area could be helpful but did not know what its relevance was and how they could use it to solve the problem.

Summary

There was evidence of difficulty in solving both these problems because of the plethora of demands imposed by both the contextualised and context-free versions question. This CQ was heavily contextualised involving all five possible elements of context (picture, words, number, symbol and graphics). Problems seemed to be greater in the CQ where some pupils were unable to carry an operative through the problem to solve it in its entirety and the concern that was uppermost was to divide the rectangle and not to attend to the dimensions.

Data Handling

CQ3

This question (see Appendix 5)presented pupils with a tally chart of birthdays in each month for the children in a class (see Appendix). They were asked how many children there were in a class and how many more birthdays there were in one month than another. The chief operatives used were 'counting', 'adding' and 'how many'. Pupils appeared to understand quite readily what was required. Strategies used, however, varied. Some used the tallies as they appeared such as Sarah, who said:

> "Easily done because these are 5s."

Others, however, such as David, made a tally of the tallies! His operative was adding and he began at the Understanding stage with the identification of a 1:1 relationship by saying

> "For every line there is one person in the month...."

and at the Strategy stage

> "I've got to add them all up to get the answer."

and then at the Execution stage

> "Four lines down, added them....then another 1 went across....
> added them up."

The other most frequent Strategy was to add the tally for each month and then to sum those. All 10 pupils completed this problem successfully.

CFQ6 Data Handling

Any problem to do with Data Handling can be said to be *pre-contextualised* since it is dealing with information that has been gathered with respect to some situation or other. However, an attempt was made in this question (see Appendix 6) to de-contextualise to the extent where the problem was as completely related to mathematical concepts as possible as opposed to being embedded in an 'real life' activity.

The question contained a tally chart of the frequency of numbers appearing in a list, with the numbers having no other point of reference (see Appendix). Success on this item was achieved by 6 out of the 10 pupils, as opposed to the 100% success rate in the parallel CQ3. 'Counting' and 'add' were the most frequent operatives and while all 9 pupils who attempted the question got the total number of children right, 3 did not find the difference between the frequency of two numbers in the list. Kelly was one of these. She had used the Strategy of counting in threes:

> "Started counting individually and thought :This isn't going to work 'CQs I'll be here all day!' so then I counted in 3's, started to do 'times table'....."

This contrasted with the parallel CQ3 where she counted in ones all the way through and completed the whole question correctly. However, she appeared not to be able to understand the idea of finding the difference of the frequencies of two numbers appearing in a list (as opposed to the frequency of the occurrence of birthdays) and simply put '0' as the answer for the second part of the question. The performance of other pupils also suggested that, like Kelly, the presence of numbers as data in themselves caused some confusion at the Strategy stage probably arising from the fact that the problems presented were all concerned with numbers, or numbers of numbers.

Summary

In CQ3 the familiar context appeared to allow pupils immediate access to the question although there was some variety at the Strategy stage. In the parallel CFQ6, however, the fact that there was a context of a kind that was numerical caused difficulties and appeared to undermine the confidence of pupils and to confuse them when it came to finding something apparently as simple as the difference between the frequencies of two numbers.

DISCUSSION

Table 1 (see below) presents the numbers of correct responses to each pair of parallel CQs and CFQs.

TABLE I HERE

The results show that of the three areas of mathematics represented by the problems in the study, context seemed to intervene most strongly in the contextualised Shape and Space question (CQ 2).

CQ2 was the most richly contextualised problem of the three and contained all of the possible elements of context identified - words, picture, graphics, symbols, and numbers. There was much more information from which a pupil could choose when it came to selecting an operative and even when a useful operative was chosen, it occasionally could be 'lost' as the pupil progressed through the stages of solving the problem. Case(1985) refer to the "load on working memory" when problems are being solved (i.e. how much has to be retained throughout the process of solution) and the fact that "level of working memory" (the tying together of structures from different domains) determines a limit to the ability of an individual to deal with all aspects of a problem successfully. (Case 1985 p561) A 'rich' context such as that in the Shape and Space question clearly places heavy demands on the pupil by supplying several *different bits* of *different kinds* of information which they have to understand before they begin to solve the problem. This leads to the increased likelihood either that a correct operative may be chosen but 'lost' as they progress through the problem, that wrong operatives will be selected, or that none at all will be selected. The latter results in a situation where different ideas operate at different stages of the problem solving process and there is no single point of reference to provide continuity as the pupil progresses through the problem which can affect their success or failure. In this case, they can also be led astray by confusing the mathematical cues given. They *saw* their drawing divided into equal pieces and assumed they had the correct solution without counting *how many* they had drawn.

In the parallel context-free question (CFQ4), with the 'story-line' dropped and with four rather than five elements of context, pupils still struggled. Although 'context-free' in the 'real life' sense, there are still several elements from which to choose operatives (a diagram, words and numbers) and again, pupils seemed unable to sustain a continuity of operatives that led them to answer the whole of the question successfully.

There was some evidence of an intuitive sense of what could help solve the problem (e.g. calculating the area of the rectangle) but appearing either not to have, or not to be able to select, the appropriate skills or concepts to carry it through. They appeared no to have the "flexibility of thought" that Krutetskii(1978) describes as a component of mathematical abilities which requires switching from one mental operation to another. (Krutetskii 1978 p87) As a result, several pupils got one part of the question (either the drawing or the dimensions) right, but not both with only two achieving the latter.

In CQ1, the presented context allowed the possibility for the pupil to choose the wrong operative as a result of a visual cue (the price sign) that was superfluous to solving the problem. This again was a richly contextualised question insofar as it contained four of the five possible elements of context (words, numbers, a picture and symbols). This led one of the pupils to select the wrong operative (although within their solution, they had answered the question asked). Wood(1988) refers to the need for pupils to "regulate" their thinking in their approach to a problem and it is clear that in this situation, the pupil was diverted from doing just this by an element in the presented context. (Wood 1988 p 192) They have to ensure that they relate *what is asked* in a problem to *what they do.* In the parallel CFQ3 (essentially a straightforward 'short division' exercise) some pupils invoked the algorithm for doing long division and probably did so because of the way the exercise was set out. This led to calculation errors whereas those who reverted to the inverse operation of multiplication and used their 'times table' got it right. The lack of a presented context here meant pupils had to understand the process of division or at least to know it was the inverse of multiplication before they could solve the problem.

What was intended as a 'context free' question in the area of Data Handling (CFQ6) provided some insight into the confusion that can be caused by giving an unusual context to a familiar mathematical task. (Some evidence of this claimed familiarity lies in the fact that all 10 pupils succeeded in the parallel CQ3.) Wood(1988) stresses the need for learners to *recognise* a solution before they can generalise it. It would seem that some pupils could not generalise the procedures carried out successfully in dealing with a question related to a tally of birthdays with a situation where only numbers were involved. The fact that they were not counting something more aligned with the 'real world' but simply abstractions in the form of numbers themselves was enough to cause some either to try a different strategy or not to do the question at all. Any 'reality' linked with the pre-context of a tally chart had been lost in the counting of numbers.

CONCLUSION

Gagne(1977) states that learning is most effective when the learner knows what the goal of an activity is. What seems clear is that placing a mathematical problem in a context has the effect of supplying a goal for pupils in a way that a context-free mathematics problem does not. A context gives pupils something to *reason with* and also supplies them with a source of reassurance about the *reasonableness* of their answer. However, underlying the assessment of any mathematics in a presented context there lies the assumption that pupils will bring what Wood (1988) calls 'prior knowledge' to the situation in order for them to 'hear' or 'see' what they are shown. It remains open to question as to whether this is happening and whether questions set *allow* it to happen. Boaler (1993) reminds us that pupils "interact with the context of a task in many different ways and that this interaction is, by its nature, individual." If the assessment of mathematics is to continue to be carried out using contextualised questions, it is important that we explore how we can employ the elements within such problems to provide an *optimal context* and that pupils are not only enabled to show evidence of positive achievement in their mathematical learning but that they are rewarded for doing so.

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TABLE I

Correct responses (N=10)

Question	C	CFQ	Both	Neither	CQ only	CFQ only
Number	8	6	4	0	4	2
Shape & Space	2	3	2	7	0	1
Data Handling	10	6	6	0	4	0