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The effects of different types of written support on children's approaches to mathematical investigations

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The effects of different types of written support on children's approaches to mathematical investigations

Abstract
The study explored the effects of different types of written support on children's approaches to mathematical investigations. Five versions of an investigation were trialled with year six children and their responses were analysed using a coding framework. The versions of the activity varied according to the type of written support provided and the coding framework was designed to probe the way in which the children approached the investigation, as well as to explore their strategies as they worked through the task. The framework was based on Polya's four stages of problem solving: understanding the problem; devising a plan; carrying out the plan and looking back. Clinical interviews were carried out with a sample of the children to probe their thinking further and to explore in more detail how their approaches to the activity had been affected by the scaffolding within the task. The data were analysed to find out which types of support were more effective and why. Implications for teaching and assessment were also explored.

Introduction

The aim of the study was to find out if written support included in the investigation would make the task more accessible and also to explore how children's thinking was affected by different types of support. Another question raised in the study was whether it was possible to assess investigations not only according to whether the child achieved the correct solution, but by taking into account partial achievement and the strategies used by the child. Assessment of mathematical investigations is challenging since the nature of the activity is open-ended and by definition a variety of methods are acceptable. At level 4 of Attainment Target 1: Using and Applying Mathematics, the following skills and processes are listed,

> *'Pupils are developing their own strategies for solving problems and are using these strategies both in working within mathematics and in applying mathematics to practical contexts. They present information and results in a clear and organised way. They search for a solution by trying out ideas of their own.*
> (p.9 Attainment Targets, National Curriculum.)

Such criteria make assessment of investigations difficult and this raises the question of how we can reward positive achievement for more individual and personal approaches to problem solving. Although investigative skills and processes are included in the national curriculum they are not assessed separately in national tests. Such skills are embedded in the tests in the context of the other attainment targets. Given that we have an increasingly assessment-led curriculum, there is a need to consider how such an important area of mathematics can be assessed while ensuring validity and reliability. At the same time investigations which are designed for assessment purposes, need to be accessible and to provide effective support for children as they approach unfamiliar and open-ended tasks. It may be possible to consider 'generic' types of support, which can be transferred for use in a range of investigations.

The prompts which were included in the investigation in this study were designed to help the child to start the problem-solving process and not to teach them what to do. If the prompts provide too much support then the value of the task could be compromised.

This danger was described by Mason et al (1985),

> *'Specific "hints" remove the opportunity for you to do the thinking, and ask the important points, which are the impulses that produced those "hints". Furthermore, the whole nature of "hints" suggests a view of mathematics as a bag of tricks that have to be discovered or revealed.'* (p.115, 1985)

Garrard (1986) also recognised the problem and commented that,

> *It is important that children are encouraged to devise their own method of recording, rather than use a method imposed on them by the teacher.* (p.34.)

Gipps (1994) suggested that assessment techniques should be designed to focus on models of understanding that children construct for themselves, while admitting that scaffolded assessments create practical and administrative problems.

If prompts could be carefully designed to act as 'jumping-off points', rather than complete methods, then they could act in a facilitating way, enabling children to access problems and to select appropriate strategies to solve them. Ridgway (1988) defended scaffolding, pointing out that a lack of guidance could lead to 'painful methods of pupil explanation, partial success, modification, followed by more partial success. He suggested two features of investigative work which should be assessed, 'mathematical discoveries and the way they are presented,' (p.160,1988), although he agreed that, 'assessing [investigations] requires a good deal of skill.' (p.114,1988.) Investigative assessment tasks, and the criteria for assessment, need to be developed to take into account the dangers and difficulties which have been recognised. We can then work towards valid and reliable assessment and we can reward children for positive achievement as they approach mathematical problems.

The Investigation

There were five versions of the investigation and each version involved the same problem based on permutations, (see appendix A). Initially children were asked to find all possible ways of seating three people around a table and this was then extended to seating four people. Subsequently children were asked to review and justify their answers and then to search for patterns and make predictions. Once the investigation had been selected the five 'versions' were developed in the following way:

version 1	without added support
version 2	un-worked example - blank table
version 3	simpler worked example - pictorial
version 4	simpler worked example - table
version 5	written 'clue' to suggest a way to begin

Care was taken to ensure that there were no language difficulties in the questions and that ambiguities and confusion were avoided. These issues were checked by carrying out pilot activities which were trialled with year six children.

The Trials

Eleven schools of different sizes and locations were involved, 445 children each completed one version of the activity. Almost all of the children were assessed as working at levels 3 to 5 by their teachers. In each school, spiral allocation within gender groups was used to form quasi-random 'equivalent groups' of boys and girls allocated to the five versions of the activity. Class lists were used, split by gender, with the first boy given version 1, the second given version 2 and so on. This was then repeated for the girls in each class. Teachers supplied a teacher assessment level for mathematics for each child and this provided the data necessary for the validity of comparisons between groups to be checked.

Table 1. Numbers of children in each group

version	*1*	*2*	*3*	*4*	*5*
n =	*91*	*86*	*93*	*89*	*86*

The Coding Framework (See appendix B)

The coding framework was based on Polya's stages of problem solving, (1945). He identified the following four stages in problem solving and considered whether problem-solving could be taught.
1. understanding the problem
2. devising a plan
3. carrying out the plan
4. looking back

The aim of the framework was to analyse children's thinking and strategies as well as to consider partial achievement and the effects of support, rather than simply to find out whether a child had found the correct answer.

The framework was also informed by the analysis of scripts from the pilot trial of the activity. Children's responses were useful in providing appropriate categories within the frames for the main trial. The codes in frames 1 to 7 focused on whether the child had found the correct solution and provided information about the answers and the reasons why the child was satisfied with the answers given. Frames 8 to 10 gave information about whether the child had understood the problem, while frames 11 to 14 considered whether there had been a plan involving numbering, labelling and recording. Frames 15 to 19 focused on whether the plan had been carried through and these codes concentrated on whether strategies and recording systems had been used. Codes 20 and 21 dealt with reflection in terms of self-correction and checking. The codes were applied by four researchers after a co-ordination day which involved a check of rater to rater agreement as well as discussions about the categories within the frames.

The Interviews

The aim of the interviews was to explore in greater depth the reasons why children adopted certain approaches and the ways in which their strategies were affected by the type of support provided. The nature of the clinical interview technique allowed the child to reflect and allowed the interviewer to ask 'how' 'and 'why' questions in an attempt to probe mathematical thinking. Probes were developed to clarify reasons for strategies adopted, rather than to act as prompts to suggest possible responses e.g. 'How did you decide what to do next?'
Ginsberg (1981) described the clinical interview as,

> *'a flexible method of questioning intended to explore the richness of children's thought, to capture its fundamental activities...and enjoy the freedom to vary the form and nature of the questions....[and] follow up on interesting leads by asking new questions.'*(pp.4 – 5,1981)

A semi-structured interview and a paired interview were also piloted but the clinical interview was used in the main study since it proved to be more flexible and allowed the child more freedom of expression. A sample of 16 children was selected from three of the schools involved in the main written trials. The sample was selected so that all versions were represented and so that boys and girls of different abilities were included.

Findings

Written Trials: Coding Frameworks

The results of an analysis of variance (ANOVA), using teacher assessment levels as the controlling variable for ability, indicated that there were no significant differences in ability between the groups for each version. Therefore the groups were well-matched and the comparisons of results between versions were valid.
The results from the coding framework analysis can be found in Appendix C, tables 2 - 7.

- The initial problem involved permutations of three people. Overall there were no significant differences between versions in terms of finding a complete solution. However, boys were more successful with the written clue in version 5, with differences significant at the 5% level. When support was provided in tables, and as a written clue, children were more successful in *beginning* to solve the problem since for these versions fewer children gave no answer (see tables 2 and 3). Ability was not a significant factor when comparing results by version.

- Results were analysed for each version, including only the children who used the suggestion given in the support. There were no statistically significant differences between versions overall, by gender or ability, and so caution should be exercised when interpreting the results. The data suggested that where tables were used in the support, children were more successful in finding the correct solution and that more children gave 'reasonable' solutions when there was support in the question.

- More children identified the initial problem when support was provided. Differences between versions were not statistically significant overall but were significant for boys who benefited more from the worked example in a table and from the written clue (see table 4).

- Where a blank table (version 2) and a worked example with a picture (version 3) were provided children were more likely to adopt these methods for recording in the initial stage of the problem. The least popular method was the written clue. Differences were significant at the 1% level (see table 5). The impact was greater for children assessed at levels 3 and 4 and girls were more likely to adopt the suggested support than boys.

- Results were analysed for the initial problem to find out how many children used no strategy, how many attempted a strategy but did not carry it through and how many used a systematic strategy and carried it through to a complete solution. Differences between versions were statistically significant at the 1% level. When no support was provided 62% failed to use a strategy, whereas with a worked example in a table and with a written clue this percentage dropped to 37% and 34% respectively. Only 20% used a systematic strategy through to a complete solution with no support; this figure rose to 47% with the tabular worked example. These effects were greater for boys (see table 6).

- As the investigation progressed, the demand increased and the support was further removed from the question stem. In the latter part of the activity, where there were permutations involving four people, there were no significant differences between versions. This was the case by ability, by gender and for those who adopted the methods suggested in the support.

- In the later stage, as in the earlier stage of the investigation, fewer children found ‘unreasonable’ solutions (greater than the correct solution) where support was provided. Also, more children identified the problem with support (87%) than without (73%), although differences were not statistically significant.

- Overall the blank table was the approach most likely to be carried through to the second part of the task, while the written clue was least likely to be used in the later stage. For children teacher assessed at level 3, the worked example with pictures was least likely to be carried through with tabular support most likely. Differences were significant at the 5% level. Children at level 4 were most likely to carry through the pictorial worked example, while for those assessed at level 5 the worked example in a table was most likely.

- Results for using a strategy followed a similar pattern as in the earlier stage of the investigation. However, differences were not statistically significant either for the whole sample or by gender.

- Children were more likely to label or name their representations where a worked example in a table was provided as support. For children assessed at level 4 differences were significant at the 5% level (see table 7).

- More of the children who adopted the support given as a written clue corrected their answers, whereas fewer did so with a pictorial worked example. Differences were statistically significant at the 5% level.

- Only 8% of the whole sample used a second method to check their solutions and there were no statistically significant differences between versions.

- When asked how they knew that they had found all possibilities, most children answered that they had exhausted all possible solutions, relating this to the permutations. The evidence suggested that the type of support did not affect this reflective stage of the problem solving process.

- When asked to predict the number of permutations for 5 people, only 4% gave the correct answer. The numbers in each category were too small to judge the effects of the type of support. When asked how they had made their predictions, there were no significant differences between versions and as at the reflective stage, the results suggested that the type of support had not affected the hypothesising stage of the process.

Interviews

It was clear from the children’s comments that the cartoon illustration, which was present in all versions, had provided ‘unintentional’ support and had helped them to understand the investigation. Where no support was provided, comments suggested that further guidance and explanation were needed to help children to access the problem. The blank table and the pictorial worked example did not seem to have helped with understanding the problem. Where worked examples were given there was some confusion as children were looking for the question within the support. This could suggest that a worked example was an unfamiliar type of support, whereas the blank table was possibly more familiar. The written clue appeared to have helped with understanding the problem more than the other types of support.

At the planning stage the pictorial worked example seemed to be effective in providing a ‘jumping-off point’, although in some cases it restricted children’s choices and was interpreted as the ‘expected method’. Although children noted the use of tables in the support, it was clear from the interviews that tables were not always used as they could be ‘confusing’.

Most comments about the planning stage came from those with the written clue as they felt that the idea of labelling their representations made it easier and less confusing. Again there were cases where the support was interpreted as an instruction rather than a suggestion.

There were fewer comments about carrying out the plans than about understanding and planning. Comments ranged from those who felt that the suggestion was an instruction to those who did not seem to realise the effect of the support on the recording systems used. At the reflective stage there were few comments about the effects of support. One child, who had the worked example in a table, commented on checking his work, but did not relate this to the support provided. Two children, who had the written clue, felt that it had encouraged them to reflect on their answers.

Where there were pictures in the support the children commented on the context of people moving around a table. Children felt that using pictures for recording was too time-consuming and this was clearly an issue for them. Throughout the investigation one negative effect was that for many children the support was seen as 'compulsory' as it came from a 'higher authority', the teacher, and this may have prevented children from using more individual strategies.

Discussion and conclusions

The main questions explored in the study focused on whether the written support helped children to find complete solutions, which support was most popular and how the different types of support affected the stages of problem solving. The evidence suggested that support had greater impact on the understanding and planning stages, especially in the initial stages of the investigation. The effects on carrying out plans, reflection and hypothesising were limited. Overall the type of support did not significantly affect children's ability to find complete solutions, although the data suggested that tables were more effective and that all types of support led to more reasonable answers. The tables and the written clue did seem to help children to *begin* to solve the problem. Although the final answers were not affected, the support did help children to identify and access the initial stage of the problem. When pictorial support was provided children discussed their strategies in terms of people moving around a table, whereas with other types of support they seemed less able to adopt a mathematical procedure. The pictures seemed to introduce a 'real problem', and children's comments suggested that this may have helped them to understand what they were being asked to do.

The blank table and the pictorial example were most likely to be adopted from the support, possibly because these methods were more familiar. Tables were also the type of support most likely to be carried through to the second part of the investigation and to encourage labelling or naming of representations. Children working at levels 3 and 4 were more likely to adopt the suggested support, while those at level 5 were less likely to do so. This suggests that those at level 5 were more confident and therefore more likely to find their own individual strategies for solving the problem.

It was clear from the coding framework data and from the interviews that the support, which was at the beginning of the activity, had less impact as the children worked through the investigation. By the time children reached the reflective and hypothesising stages there were no significant differences by version and this could again be because the written support was at the beginning and any effects had been 'lost'.

The fact that the cartoon illustration was supportive highlights the fact that such 'unintentional effects' need to be considered carefully when developing questions, as there is a danger that unnecessary or unhelpful information could be introduced into the activity. Worked examples seemed to be less familiar than blank tables or written clues and the issue of 'familiarity' did affect children's ability to understand the initial problem.

The national curriculum states that one of the aims of mathematical investigations is to encourage children to develop and apply their own strategies as they search for solutions. When providing any support, one danger is that children may feel that ‘suggestions’ are in fact compulsory. Their comments in interviews reflected the concerns of Mason et al (1985), who recognised that support could be restrictive and could limit individuality.

Support did make a difference, although the type of support affected the stages of problem solving in different ways. All support seemed to make the investigation more accessible, by re-stating or clarifying the initial problem. The aim should be to provide ‘optimum’ support, which will not restrict individual thinking, but will provide a starting point for the child faced with an unfamiliar problem. Finding ‘optimum’ types of support is complicated by children’s individual learning styles. In a teaching situation children can be offered a range of methods, such as trying out simple examples, labelling representations and using standard recording systems. They can then select strategies for themselves. In written assessments the tasks are inevitably more ‘closed’ and the type of support may affect performance differently according to the individual’s skills and experiences. Supporting mathematical investigations is therefore more difficult in an assessment than in a teaching context.

Ridgway (1988) suggested that investigative work should be assessed in terms of mathematical discoveries and the way they are presented. In this study some aspects of investigative work were more easily assessed and coded than others. These included: methods of presentation; identification of the problem; recording systems and recognition of similar mathematics in different problems. Assessment and coding in other areas were more challenging e.g. use of strategies and reflection. It was clear from the written activities that children had problems explaining what they had done and this added to the problems when coding their justifications and explanations of their methods. It also raised the issue of validity, since there is a danger that we may be assessing writing rather than mathematical skills. Qualitative information was more accessible during the interviews rather than from the written scripts, which were open to interpretation. This illustrates a key problem for the assessment of investigations, since individual assessments are time-consuming and lead to problems of reliability.

Mathematical investigations are often highly contextualised and this makes the use of ‘generic’ support more difficult. The types of support used in the study had varying effects at the different stages of problem solving and effects also varied from child to child. These factors make it more difficult to devise transferable methods of assessment while ensuring reliable assessment in different contexts and with various types of support. Investigations encourage mathematical thinking and problem solving strategies, however, they create a range of problems for assessment. In mathematics, and in other subjects, the aim of scaffolding in assessment tasks is to facilitate children, enabling them to understand the tasks they face. However, when we include support within a task, the effects can be varied and complex and can create problems of interpretation and reliability. The issues raised while studying the effects of different types of written support on children’s approaches to mathematical investigations support Ridgway’s assertion that, ‘assessing [investigations] requires a great deal of skill’.

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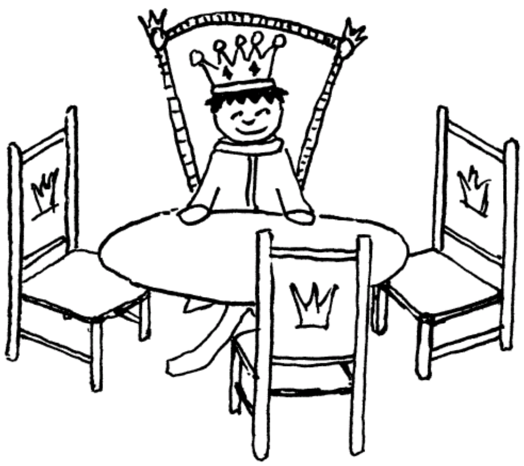
King Basil sits on his throne at the Round Table.



1. **There are 3 empty seats for his 3 bravest knights.**

You have to plan where the knights will sit.

King Basil and his throne do not move.

How many different ways are there?

Appendix A contd.

2. What if there were 4 empty seats for his 4 bravest knights?

How many different ways would there be?

3. How did you know you had found all the possible ways?

....

....

....

....

....

....

4. Can you guess how many ways there would be for

5 knights?

....

How did you make your guess?

...

...

...

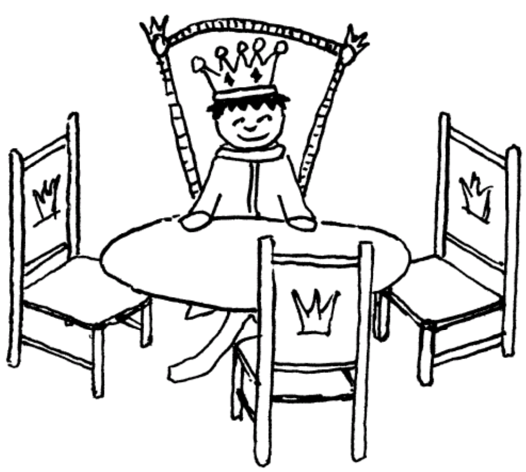
...

...

....

Appendix A contd. Version 2: blank table

King Basil sits on his throne at the Round Table.



1. There are 3 empty seats for his 3 bravest knights.

You have to plan where the knights will sit.

King Basil and his throne do not move.

How many different ways are there?

You could use a table like this to help you.

Seat 1	**Seat 2**	**Seat 3**

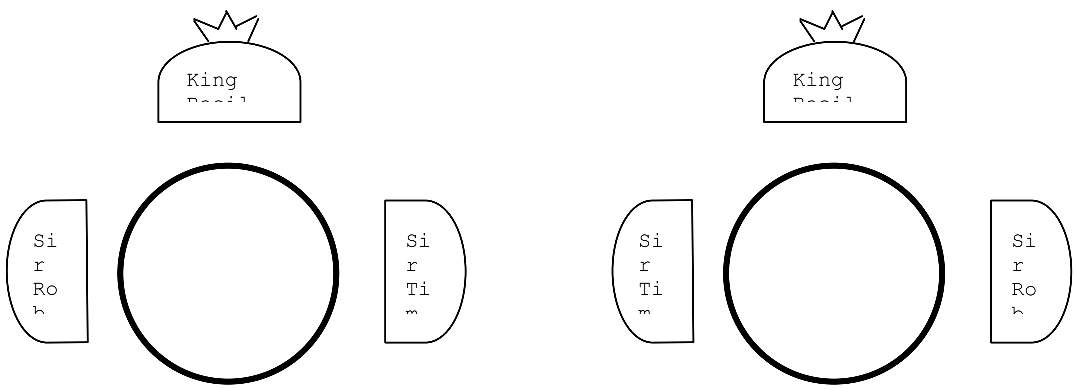
Answer on the next page

King Basil sits on a throne at the Round Table.

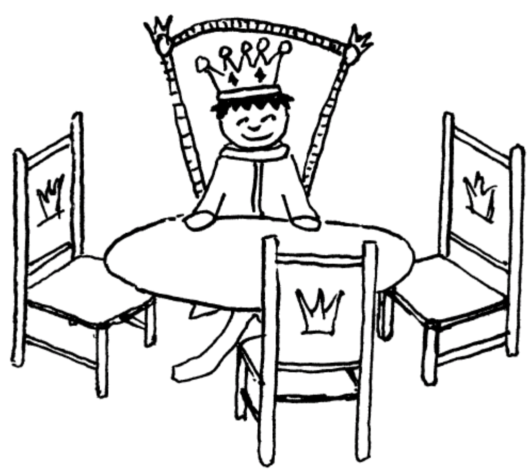
There are 2 empty seats for his 2 bravest knights, Sir Tim and Sir Rob.

King Basil thinks about how many different ways the 2 knights can sit around the table.

He decides that there are 2 different ways.



1. Now there are 3 empty seats for his 3 bravest



knights.
You have to plan where the knights sit.
King Basil and his throne do not move.

How many different ways are there?

Answer on the next page.

King Basil sits on his throne at the Round Table.

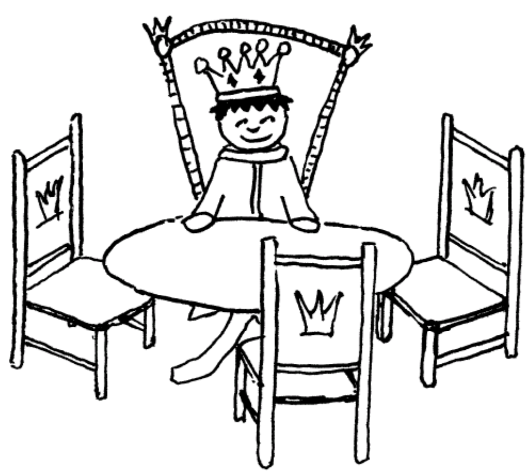
There are 2 empty seats for his 2 bravest knights, Sir Tim and Sir Rob.

King Basil thinks about how many different ways the 2 knights can sit around the table.

He decides that there are 2 different ways.

Seat 1	Seat 2
knight A	knight B
knight B	knight A

1. Now there are 3 empty seats for his 3 bravest



knights.

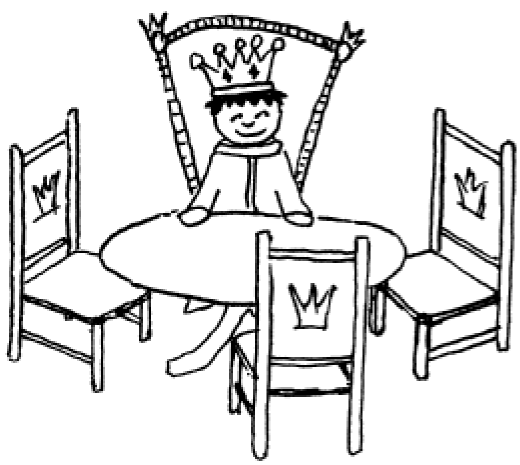
You have to plan where the knights will sit.

King Basil and his throne do not move.

How many different ways are there?

Answer on the next page.

King Basil sits on his throne at the Round Table.



1. **There are 3 empty seats for his 3 bravest knights.**

You have to plan where the knights will sit.

King Basil and his throne do not move.

How many different ways are there?

Clue!

You could try naming the three knights. Then write down in columns all the possible orders they can sit around the table.

Appendix B. Coding Framework

Candidate number

1	Q1: In how many different ways can 3 knights sit?	0 no answer given 1 incorrect 2 6	
2	How many ways did they find?	Enter number of ways	
3	Q2: In how many different ways could 4 knights sit?	0 no answer given 1 incorrect 2 24	
4	How many ways did they find?	Enter number of ways	
5	Q3: How did you know you had found all the possible ways?	0 no answer 1 answer relates to pictures 2 answer relates to permutations 3 answer relates to pattern 4 answer relates to being unable to find any more 5 answer relates to using a system 6 answer relates to method used helping/clarifying 7 didn't find them all as too many 8 other	
6	Q4: How many ways did they guess?	Enter number of ways	
7	Q4: Could you guess how many ways for 5 knights?	0 no answer at all 1 no answer to second part of question 2 'just a guess', but bigger number than for Q2 3 guess - reason makes no sense 4 explanation of how to get from solution for 4 knights to 5 knights 5 explanation of how to get from solution for 3 knights to 4 knights to solution for 5 6 explanation of how to get from the number of knights to the number in the solution 7 justified guess other than above 8 120 + explanation 9 not a guess, but incorrect reasoning	
8	Have they understood the problem in Q1?	1 no - no attempt to answer 2 no - misunderstood problem 3 yes - have attempted method which shows they understood question 4 unsure	U
9	Have they understood the problem in Q2?	1 no - no attempt to answer 2 no - misunderstood problem 3 yes - have attempted method which shows they understood question 4 unsure	U
10	Are there any errors made due to the language in Q1?	1 no 2 yes 3 not sure	U
11	Have they numbered / labelled the seats?	1 no 2 yes	P
12	Have they labelled / named the knights?	1 no 2 yes	P

Appendix B contd.

13	Have they designed a recording system in Q1?	1 no 2 yes - same as in support 3 yes - similar to support 4 yes - different from support 5 yes - no support given	P
14	Have they designed a recording system in Q2?	1 no 2 yes - same as in support and Q1 3 yes - similar to support and Q1 4 yes - same as support and different from Q1 5 yes - different from support and same as Q1 6 yes - different from support and different from Q1 7 yes - no support given and same as Q1 8 yes - no support given and different from Q1	P
15	Have they used a recording system in Q1?	1 no recording system / no working shown 2 unclear - writing on picture given in question 3 yes - list 4 yes - table 5 yes - pictures (as in question) 6 yes - diagrams (unlike those in question) 7 yes - other 8 more than one recording system used	C
16	Have they used a recording system in Q2?	1 no recording system /no working shown 2 unclear - writing on picture given in question 3 yes - list 4 yes - table 5 yes - pictures (as in question) 6 yes - diagrams (unlike those in question) 7 yes - other 8 more than one recording system used	C
17	Have they used a strategy in Q1?	1 no obvious attempt at strategy 2 attempt at strategy 3 yes - systematic 4 yes, but not carried through	C
18	Have they used a strategy in Q2?	1 no obvious attempt at strategy 2 attempt at strategy 3 yes - systematic 4 yes, but not carried through	C
19	Have they used the same strategy in Q2?	1 no 2 yes 3 similar 4 more than one strategy used 5 no strategy used in Q1	C
20	Have they self-corrected?	1 no 2 not necessary 3 yes changes to total number of permutations given 4 yes changes to labels in permutations 5 yes changes to method of recording 6 yes - other	L/R
21	Have they used more than one method in order to check their solution to Q1 or Q2?	1 no evidence 2 yes	L/R

Appendix C. Coding Framework Data

Table 2 % of children who answered Q1 correctly.

version	1	2	3	4	5	whole sample
no answer	7%	2%	7%	3%	2%	4%
incorrect	56%	67%	69%	67%	56%	63%
correct	37%	30%	25%	29%	42%	33%

Table 3 % of boys who answered Q1 correctly.

version	1	2	3	4	5	whole sample
no answer	9%	3%	9%			4%
incorrect	53%	70%	65%	67%	44%	60%
correct	37%	28%	26%	33%	56%	35%

Table 4 % of children who identified the problem in Q1

version	1	2	3	4	5	whole sample
no	24%	21%	16%	15%	13%	17%
yes	76%	79%	84%	85%	87%	83%

Table 5 % of children who used the same or similar recording system as that suggested by the support

version	2	3	4	5	whole sample
yes	79%	77%	62%	55%	68%
no	21%	23%	38%	45%	32%

Table 6 % of boys who used a strategy in Q1

version	1	2	3	4	5	whole sample
no strategy	70%	53%	61%	29%	29%	49%
attempt, but not carried through	21%	20%	11%	22%	37%	22%
systematic strategy	9%	28%	28%	49%	34%	30%

Table 7 % of children who labelled / named the knights

version	1	2	3	4	5	whole sample
no	43%	20%	23%	9%	17%	22%
yes	57%	80%	77%	91%	83%	78%