O Level

Mathematics

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UNIVERSITY OF CAMBRIDGE
LOCAL EXAMINATIONS SYNDICATE

SYLLABUSES

MATHEMATICAL SUBJECTS
1966 and 1967

GENERAL CERTIFICATE
OF EDUCATION

SCHOOL CERTIFICATE
(Oversea Centres only)

HIGHER SCHOOL CERTIFICATE
(Oversea Centres only)

There are two syllabuses for Ordinary Level and School Certificate Mathematics, the Alternative A syllabus and the Alternative B syllabus, and the intention is that they should be of equal weight.

The Alternative A syllabus is arranged in separate sections for Arithmetic, Geometry and Algebra. There will be one paper on each of these sections. Questions on Trigonometry will be included as an alternative to some of the questions on Arithmetic.

The Alternative B syllabus is an alternative syllabus in Mathematics as distinct from syllabuses in separate sections. There will be two papers and each paper may contain questions on any part of the syllabus. The Alternative B syllabus was introduced following the publication, in 1944, of the Jeffery report. This report was issued after discussions between representatives of the Examining Boards, the Teachers' Associations, and the Mathematical Association, and recommended an alternative syllabus in Mathematics as distinct from syllabuses in the separate sections. One of the aims of the authors of the new syllabus was to bring Mathematics more closely into relation with the life and experience of the pupil; another was to reduce the amount of formal geometry, items of trigonometry and practical geometry being added.

MATHEMATICS ALTERNATIVE A
ORDINARY LEVEL AND SCHOOL CERTIFICATE
SCHEME OF PAPERS

There will be three papers.

(a) *Arithmetic*. (1½ hours.) The paper will include, as alternatives to a few of the questions on Arithmetic, easy questions on numerical Trigonometry.

[*For special oversea papers see S.C. Regulations.*]

(b) *Geometry*. (2 hours.)

(c) *Algebra*. (1½ hours.) A limited number of questions may be set involving a knowledge of simple geometrical properties.

DETAILED SYLLABUS

Arithmetic

Candidates should be familiar with the British and metric systems of weights and measures and with the following monetary systems: pounds, shillings and pence; dollars and cents; francs and centimes. Addition, subtraction, multiplication and division applied to numerical calculations. Fractions and decimals. Proportion and proportional parts. Extraction of square roots. Averages, percentages, simple and compound interest; profit and loss. Elementary mensuration of the triangle, the circle, the rectangular block, the cylinder and the sphere. Problems on speeds. Graphs from numerical and statistical data. The use of logarithm tables.

Candidates may be required to give results to a specified degree of approximation, but the use of contracted methods of multiplication and division is not essential.

In the School Certificate examination, special question papers in which the currency of the country concerned is used in questions dealing with money, are provided for some areas (see School Certificate Regulations).

Trigonometry

Sine, cosine and tangent of acute and obtuse angles; use of the trigonometrical tables. Use of the sine and cosine formulae for a triangle and the formula $\frac{1}{2}bc \sin A$. One simple three-dimensional problem may be set.

Geometry

The paper in Geometry will contain questions on Practical and on Theoretical Geometry.

The questions on Practical Geometry will be set on the constructions contained in Schedule A (see pp. 6, 7), together with easy extensions of them. In cases where the validity of a construction is not obvious, the reasoning

by which it is justified may be required. Every candidate must provide himself with a ruler graduated in inches and tenths of an inch, and in centimetres and millimetres, a set square, a protractor, compasses, and a fairly hard pencil. All figures must be drawn accurately and distinctly. Questions may be set in which the use of the set square or of the protractor is forbidden.

The questions on Theoretical Geometry will consist of theorems contained in Schedule B (see pp. 7–10), together with questions upon these theorems, easy deductions from them, and arithmetical illustrations. Any proof of a proposition will be accepted which appears to the examiners to form part of a systematic treatment of the subject; **the order in which the theorems are stated in Schedule B need not be followed.** In the proof of theorems and deductions from them, the use of hypothetical constructions will be permitted.

Questions will be set on Schedules A (i), A (ii), A (iii) and B (i), B (ii), B (iii). The use of algebraical symbols and (in the solution of riders) of trigonometrical ratios is permitted; the use of trigonometry in theorems is not permitted.

Algebra

Elementary algebraic operations; formulae expressing arithmetical generalisation; change of subject of a formula; factors, fractions. The *use* of fractional and negative indices and the elementary theory of logarithms. Calculation by logarithms to base 10 with the use of four-figure tables. Solution of linear equations involving not more than two unknowns and quadratic equations involving only one unknown; the solution of simultaneous equations, one linear and one quadratic, involving two unknowns; and simple problems leading to such equations. The *use* of the remainder theorem; ratio and proportion; variation. Graphs and their simple applications. Arithmetical and finite geometrical progressions.

Geometry Schedules
Schedule A. (Practical)
A (i)

Bisection of angles and of straight lines.

Construction of perpendiculars to straight lines.

Construction of an angle equal to a given angle.

Construction of angles of 60°, 45°, and 30°.

Construction of parallels to a given straight line.

Simple cases of the construction from sufficient data of triangles and quadrilaterals.

Division of straight lines into a given number of equal parts or into parts in any given proportions.

A (ii)

Construction of a triangle equal in area to a given polygon.

Construction of tangents to a circle and of common tangents to two circles.

Construction of circumscribed, inscribed and escribed circles of a triangle.

A (iii)

Simple cases of the construction of circles from sufficient data.

Construction of a square equal in area to a given polygon.

Construction of a fourth proportional to three given straight lines and a mean proportional to two given straight lines.

Construction of regular figures of 3, 4, 6, or 8 sides in or about a given circle.

Schedule B. (Theoretical)

In this schedule candidates may be asked to prove any of the theorems in *italics*; they will not be asked to prove any of the theorems in ordinary type.

B (i)
Angles at a Point

If a straight line stands on another straight line, the sum of the two angles so formed is equal to two right angles; and the converse.

If two straight lines intersect, the vertically opposite angles are equal.

Parallel Straight Lines

When a straight line cuts two other straight lines, and
    (i) a pair of alternate angles are equal,
or (ii) a pair of corresponding angles are equal,
or (iii) a pair of interior angles on the same side of the cutting line are together equal to two right angles,
then the two straight lines are parallel; and the converse.

Straight lines which are parallel to the same straight line are parallel to one another.

Triangles and Rectilinear Figures

The sum of the angles of a triangle is equal to two right angles.

In a polygon of n sides, the sum of the interior angles is equal to $2n-4$ right angles.

If the sides of a convex polygon are produced in order, the sum of the angles so formed is equal to four right angles.

If two triangles have two sides of the one equal to two sides of the other, each to each, and also the angles contained by those sides equal, the triangles are congruent.

If two triangles have two angles of the one equal to two angles of the other, each to each, and also one side of the one equal to the corresponding side of the other, the triangles are congruent.

If two sides of a triangle are equal, the angles opposite to these sides are equal; and the converse.

If two triangles have the three sides of the one equal to the three sides of the other, each to each, the triangles are congruent.

If two right-angled triangles have their hypotenuses equal, and one side of the one equal to one side of the other, the triangles are congruent.

If two sides of a triangle are unequal, the greater side has the greater angle opposite to it; and the converse.

Of all the straight lines that can be drawn to a given straight line from a given point outside it, the perpendicular is the shortest.

The opposite sides and angles of a parallelogram are equal, each diagonal bisects the parallelogram, and the diagonals bisect each other.

If a pair of opposite sides of a quadrilateral are equal and parallel, it is a parallelogram.

The straight line drawn through the middle point of one side of a triangle parallel to another side bisects the third side.

The straight line joining the middle points of two sides of a triangle is parallel to the third side, and equal to one-half of it.

If there are three or more parallel straight lines, and the intercepts made by them on any straight line that cuts them are equal, then the corresponding intercepts on any other straight line that cuts them are also equal.

B (ii)

Areas

Parallelograms on the same base and between the same parallels are equal in area.

Triangles on the same or equal bases and of the same altitude are equal in area.

Equal triangles on the same or equal bases are of the same altitude.

In a right-angled triangle, the square described on the hypotenuse is equal to the sum of the squares described on the sides containing the right angle; and the converse.

Loci

The locus of a point which is equidistant from two fixed points is the perpendicular bisector of the straight line joining the two fixed points.

The locus of a point which is equidistant from two intersecting straight lines consists of the pair of straight lines which bisect the angles between the two given lines.

The Circle

A straight line drawn from the centre of a circle to bisect a chord which is not a diameter, is at right angles to the chord; conversely, the perpendicular to a chord from the centre bisects the chord.

There is one circle, and one only, which passes through three given points not in a straight line.

Equal chords of a circle are equidistant from the centre; and the converse.

The tangent at any point of a circle and the radius through the point are perpendicular to each other.

The tangents to a circle from an external point are equal.

If two circles touch, the point of contact lies on the straight line through the centres.

The angle which an arc of a circle subtends at the centre is double that which it subtends at any point on the remaining part of the circumference.

Angles in the same segment of a circle are equal; and, if the line joining two points subtends equal angles at two other points on the same side of it, the four points lie on a circle.

The angle in a semicircle is a right angle; and the converse.

The opposite angles of any quadrilateral inscribed in a circle are supplementary; and the converse.

B (iii)

Areas

The square on a side of a triangle is greater or less than the sum of the squares on the other two sides, according as the angle contained by those sides is obtuse or acute. The difference is twice the rectangle contained by one of the two sides and the projection on it of the other.

In any triangle, the sum of the squares on any two sides is equal to twice the square on half the third side together with twice the square on the median which bisects the third side.

The Circle

In equal circles (or, in the same circle) (i) if two arcs subtend equal angles at the centres, they are equal; (ii) conversely, if two arcs are equal, they subtend equal angles at the centre.

In equal circles (or, in the same circle) (i) if two chords are equal, they cut off equal arcs; (ii) conversely, if two arcs are equal, the chords of the arcs are equal.

If a straight line touch a circle, and from the point of contact a chord be drawn, the angles which this chord makes with the tangent are equal to the angles in the alternate segments.

If two chords of a circle intersect either inside or outside the circle the rectangle contained by the parts of the one is equal to the rectangle contained by the parts of the other.

Proportion: Similar Triangles

(Proofs which are applicable only to commensurable magnitudes will be accepted.)

If a straight line is drawn parallel to one side of a triangle, the other two sides are divided proportionally; and the converse.

If two triangles are equiangular their corresponding sides are proportional; and the converse.

If two triangles have one angle of the one equal to one angle of the other and the sides about these equal angles proportional, the triangles are similar.

If a perpendicular is drawn from the right angle of a right-angled triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle, and likewise the external bisector externally.

The ratio of the areas of similar triangles is equal to the ratio of the squares on corresponding sides.

MATHEMATICS ALTERNATIVE B
ORDINARY LEVEL AND SCHOOL CERTIFICATE

SCHEME OF PAPERS

There will be two papers (each $2\frac{1}{2}$ hours). Each paper may contain questions on any part of the syllabus, and the solution of any question may require knowledge of more than one branch of the syllabus; a high standard of accuracy will be expected. Each paper will consist of two sections. The first section will contain five questions, to be attempted by all candidates, on the more elementary parts of the syllabus. The second section will include a variety of questions and a choice will be allowed.

Unless the terms of a question impose specific limitations,

(a) a candidate may use any appropriate method;

(b) tables of logarithms, trigonometrical functions, squares and square roots, etc., may be used wherever they give the required degree of accuracy; a slide rule cannot be expected to give sufficient accuracy;

(c) T-squares, set squares, graduated rulers, diagonal scales, protractors and compasses may be used.

DETAILED SYLLABUS

Numbers

The ordinary processes of Arithmetic.

The British and Metric systems of weights, measures, and money; densities; temperature scales (centigrade and Fahrenheit); speeds.

Fractions, decimals; proportion and proportional parts; calculation of averages, percentage, and simple (but not compound) interest.

Use of common logarithms and square-root tables.

Significant figures.

(Questions may be set on the applications of these processes to problems of everyday life in the home and community including straightforward financial transactions, but such questions will not involve complicated operations or the knowledge of uncommon technical terms. The 'long rules' for the extraction of square roots and the determination of H.C.F. are not included in the syllabus.)

Mensuration

The rectangle, triangle, and figures derived from them, including easy extensions to three dimensions.

The circle (including the length of an arc in terms of the angle at the centre), cylinder, cone and sphere.

(Questions may involve knowledge of the altitude and centre of an equilateral triangle and the ratio of the sides of the right-angled isosceles triangle and the 30°, 60°, 90° triangle.)

Algebraic Symbols, Expressions and Equations

Statement of rules (e.g. area = length × breadth, temperature C/100 = (F − 32)/180) and generalisations of arithmetical relations (e.g. pressure × volume = constant) in symbols; the interpretation of statements given in symbolic form.

Evaluation of algebraic expressions; change of subject in a formula.

The use of indices.

(Only simple examples of fractional and negative indices will be set; candidates may be expected to understand the use of indices to express such numbers as $3·74 \times 10^8$ or $1·35 \times 10^{-6}$. Only simple manipulation of surds will be required.)

Common factors, factors of $(a^2 - b^2)$ and of such extensions as occur in mensuration. Factors of trinomial expressions. Simple fractions. Simple equations, quadratic equations, and simultaneous linear equations in two variables.

Graphs, Variation, Functionality

Graphs from numerical and statistical data.

Translation into symbols of relations such as 'y is inversely proportional to x', 'V varies as x^3', and their illustration by sketch-graphs.

The idea of a function of a variable and its graphical representation. Graphical treatment of the function:

$$y = Ax^3 + Bx^2 + Cx + D + E/x + F/x^2,$$

where not less than three of the constants A, B, C, D, E, are zero. The gradients of these graphs, determined by drawing.

Plane Geometry and Trigonometry

The following schedule is arranged in a definite sequence.* The THEOREMS form a logical backbone of the course, in relation to which the remaining definitions and properties included take their place. Candidates may be asked to prove any of the theorems in *italics*; they will not be asked to prove any of the theorems in ordinary type. Knowledge of the converses of the theorems (when valid) is expected, but their proofs will not be asked for. In proving any theorem, the preceding theorems (whether in italics or not) may be assumed and need not be proved; trigonometrical methods, where appropriate, may be used. Apart from the theorems in *italics*, candidates will not be expected to have learnt any formal proofs. If proofs of other properties covered by the syllabus are required, they will be set as riders. Candidates might, for example, be asked to prove as a rider on

* The positions in the sequence of the sections on Similarity, Area and The Circle may be varied.

congruent triangles that the diagonals of a kite are at right angles. Teachers are thus free to use intuitive or practical methods of establishing many of the properties covered by the syllabus. Riders will be straightforward and will be worded so as to make clear not only what is to be proved but also what is to be assumed.

Angles and Parallel Lines

Degrees, angles of elevation and depression, bearings.

The properties of adjacent and vertically opposite angles; the properties of parallel lines, corresponding, alternate, and conjoined angles.

Triangles and Parallelograms

THEOREM: *the sum of the angles of a triangle is two right angles.*

The property of the exterior angles of a triangle and of a polygon. Congruence. The isosceles triangle.

The properties of the angles, sides, and diagonals of the parallelogram, rhombus, rectangle, square, trapezium and kite.

THEOREM: if there are three or more parallel straight lines and the intercepts made by them on a transversal are equal, the intercepts made on any other transversal are equal.

Similarity*

THEOREMS: *if a straight line parallel to one side of a triangle meets the other sides, these sides are divided proportionally; and the converse.*

Similarity of triangles; definitions of the cosine, sine, and tangent of an angle, acute or obtuse.

THEOREM: *the sine rule for any triangle.*

Area*

THEOREM: *parallelograms on the same base and between the same parallels have equal areas.*

THEOREMS: *determining the areas of* the rectangle, *parallelogram, and triangle (including $\frac{1}{2}bc \sin A$).*

Pythagoras' theorem

THE THEOREM OF PYTHAGORAS: (*if the method of similar triangles is used, candidates must first prove that the perpendicular from the right angle to the hypotenuse makes two triangles similar to the whole*) *and the converse.*

THEOREM: *$a^2 = b^2 + c^2 - 2bc \cos A$ for any triangle.*

The Circle*

The symmetrical properties of chords.

THEOREM: *the angle which an arc of a circle subtends at the centre is double that which it subtends at any point of the remaining part of the circumference.*

* The positions in the sequence of the sections on Similarity, Area and The Circle may be varied.

THEOREM: *the angle in a semicircle is a right angle.*
THEOREM: *angles in the same segment are equal.*
THEOREM: *angles in opposite segments are supplementary.*
Perpendicularity of tangent and radius.
THEOREM: if two circles touch, the point of contact is on the line of centres.
THEOREM: the tangents to a circle from an external point are equal.
THEOREM: *if a straight line touch a circle, and from the point of contact a chord be drawn, the angles which this chord makes with the tangent are equal to the angles in the alternate segments.*

Three-dimensional Geometry

Parallel lines, parallel planes, normal to a plane, angle between a line and a plane, angle between two planes. Questions may be asked involving a knowledge of the forms of the cube, rectangular block, pyramid, tetrahedron, prism, wedge, circular cone, sphere and circular cylinder.
Knowledge of the following loci and of the method of intersection of loci:
Locus of points in two or three dimensions.
(a) which are at a given distance from a given point;
(b) which are at a given distance from a given straight line;
(c) which are equidistant from two given points.
Locus of points in two dimensions
(d) which are equidistant from two given lines;
(e) at which a given segment of a straight line subtends a given angle.

Applied Geometry

In this section great importance will be attached to neatness and accuracy. The use of a protractor and of set squares (e.g. in drawing parallels) will be allowed, unless expressly prohibited in any particular question. The use of parallel rulers will not be allowed. In drawing plans and elevations,* the use of drawing boards and T-squares will be allowed. Drawing paper will be provided.
Solution of triangles by the use of the sine and cosine rules and trigonometrical tables.
The following constructions, using straight edge and compasses only:
bisection of angles and straight lines;
construction of perpendiculars to straight lines, of an angle equal to a given angle, and of angles of 60°, 45°, and 30°;
simple cases of the construction from sufficient data of triangles, quadrilaterals, and circles, including circumscribed and inscribed circles of a triangle;

* In such questions, either 1st or 3rd angle projection may be used. A simple account of both these methods is given in a pamphlet entitled 'Plans and Elevations' which may be obtained from Syndicate Buildings, Cambridge, price 2*s*. 6*d*.

division of straight lines into a given number of equal parts or into parts in any given proportions.
Scale drawing.
Simple problems in three dimensions soluble by analysis into plane figures.
There will be a choice of questions involving the application of scale drawing or trigonometry to the following:
(i) simple map problems, scales, contour lines, slopes;
(ii) navigation—determination of position by two bearings, the nautical mile and the knot, triangle of velocities, drift, course, track;
(iii) latitude and longitude, small and great circles on a sphere;
(iv) heights and distances;
(v) construction of plans and eleuations.