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Cover feature

Our cover for this issue features a selection of images that have featured on the covers of *Research Matters* over the last 10 years. Can you spot the issue numbers and publication dates? The answers are provided on page 48.

If you would like to comment on any of the articles in this issue, please contact Sylvia Green – Director, Research Division. Email: researchprogrammes@cambridgeassessment.org.uk

The full issue of *Research Matters 20* and all previous issues are available from our website: www.cambridgeassessment.org.uk/our-research

Research Matters : 20

A CAMBRIDGE ASSESSMENT PUBLICATION

Foreword

This issue marks ten years of *Research Matters* – a period which has seen huge changes both in the qualifications landscape in England, and in international developments in assessment. Regulation in England has escalated in its complexity and reach, whilst in almost all nations, policy aspirations increasingly have centred on the outcomes of the large international surveys of educational attainment. Both of these tendencies have the risk of myopic preoccupation. Regulation necessarily is partial, otherwise the regulator becomes the de facto qualifications provider. The big international surveys focus on serial cross-sectional studies of certain ages, and have very specific measurement focus – specific expressions of knowledge, skills and understanding. Undue preoccupation with regulation and the surveys means that large areas of assessment theory, practice and development run the risk of being neglected. But it is essential that assessment research continues to support wide curriculum interests, that the economic and social function of assessment continues to be examined and developed, and that constant improvement and innovation in assessment is promoted. Looking back over the issues of the past ten years, it is heartening to see articles which engage fully with this wider agenda. *Research Matters* hasn't neglected regulation and transnational comparison; issues of standards-setting and standards-maintenance have been repeatedly examined, while the methods and findings of international comparative work have been extensively scrutinized. While some pundits have lamented the decline of evidence-based policy, I believe that the last ten years has seen exactly the reverse: an increase in demand, from policy makers and advisers, for well-grounded assessment research. Policy will most likely always tend to consider a wider range of interests and influences than solely the push from research, but many aspects of recent developments in national assessment and public examinations have been heavily research-driven. Removal of levels from national assessment, development of more dependable accountability measures, refinement of coursework in GCSE and GCE – all have been heavily influenced by well-grounded research. *Research Matters* has played a key role in getting summaries of research out early into the public domain, enhancing debate and development in assessment. With an international circulation list of 1,000+, we hope that it continues to support a rich, broad, and forensic discussion of key matters of method, analysis, equity and development.

Tim Oates, CBE *Group Director, Assessment Research and Development*

Editorial

Following the recent launch of Cambridge Mathematics, most of the articles in this 10th anniversary issue focus on Mathematics in a range of contexts. In his article, Gill investigates whether the equivalencies between qualifications assumed by the Universities and Colleges Admissions Service (UCAS) tariff are reasonably accurate or whether predictions could be improved. His findings have interesting implications for the way in which the current tariff measure could be improved. In the first of the Mathematics themed articles, Darlington compares and contrasts the mathematical skills required to answer examination questions from four different post-16 Mathematics qualifications. This research addresses some of the concerns raised in relation to preparedness for undergraduate study and the comparability of relevant qualifications.

Vidal Rodeiro, Sutch and Zanini continue the Higher Education theme. With the introduction of new qualifications, the withdrawal of some and the reform of others, their work sheds light on how current qualifications are used by young people to progress to HE in the UK. Rushton and Wilson consider transition from GCSE Mathematics to A level Mathematics or employment. They discuss the dual-purpose of the GCSE qualifications and the challenges that this poses in relation to skills that are necessary in different contexts. Given the current reforms taking place in Mathematics qualifications in the UK, they also consider the potential for alleviating problems in transition.

Munro's article on Statistics and Mechanics introduces an international perspective to the comparability of Mathematics qualifications. This research highlights differences between the A level (Mechanics and Statistics) and similar qualifications in other jurisdictions. International comparability has been a focus in the current UK reform programme and this work has implications for employers and universities who include Mathematics qualifications for recruitment and admissions purposes.

The final article continues the international theme with an analysis of the *Trends in International Mathematics and Science Study* (TIMMS) 2011. Zanini and Benton explore the link between teaching styles, curriculum and Mathematics achievement. They also address the issue of the taught curriculum and its assessment rather than the programmes of study which may not be taught.

Details of Cambridge Mathematics can be found at www.cambridgemaths.org/

Sylvia Green *Director, Research Division*

Using generalised boosting models to evaluate the UCAS tariff

Tim Gill Research Division

Introduction

The Universities and Colleges Admissions Service (UCAS) is a UK-based organisation providing the application process for almost all British universities. The UCAS tariff points system is used by universities to help select students for entry to their courses. Each grade in a qualification has a certain number of UCAS points allocated to it, which are then summed to provide an overall tariff points score for each student. The assumption made is that two students with the same UCAS tariff gained from different qualifications are of the same ability, or have the same potential to achieve at university.

This article uses a statistical technique known as generalised boosting models (GBMs) to evaluate the use of the UCAS tariff as a predictor of degree outcome. GBMs are able to analyse complicated interactions between large numbers of variables (such as UCAS tariff points from different qualifications) to produce more accurate predictions. By running GBMs on a set of data including degree class and UCAS tariff points in different qualifications, it is possible to make predictions about the degree class. If these predictions are no better (or only slightly better) than the predictions from using only the total UCAS tariff score then this would mean the UCAS tariff could not be improved by including the extra information – in other words, the equivalences between qualifications assumed by the UCAS tariff are reasonably accurate.

This investigation was also undertaken for different qualifications separately to see whether any effects found were different.

Data

The data for this research was provided by the Higher Education Statistics Agency (HESA)[1]. The data consisted of all full-time graduates who were 17–19 years old when they started a first degree (expected not to last more than three years) in the academic year 2010/11 in a UK HE institution, and completed it in the academic year 2012/13. Thus students entering for degrees lasting more than three years (e.g., in Medicine, Dentistry, Veterinary Science and in many language or Engineering courses) were excluded. Included in the database was information on the prior qualifications taken by students at Level 3, including type of qualifications, subjects, grades achieved and total UCAS tariff points. Where students re-sat an examination only, the highest grade was kept and only qualifications that were graded with at least a pass were included.

After some initial investigation of the data it was found that for some students the UCAS tariff included in the database did not match the tariff calculated (by the author) from their grades achieved in prior qualifications. This is likely to be because some students achieved grades in other, minor, qualifications that were not included in the prior qualifications database (e.g., Key Skills). These qualifications are likely to have, at most, only a small impact on degree performance, so it was decided to use the UCAS tariff calculated from the grades in the prior qualifications database as the basis for predicting degree performance. To prevent the distribution of tariffs having a very long tail, potentially distorting the analysis, students had their overall tariff capped at 700 (equivalent to 5 grade A*s at A level).

Students whose degree status was 'Classification not applicable' or 'Missing' were excluded from the data, as these students had dropped out or had not yet completed their degree. This meant that the total number of students included in the analysis was 83,468.

Method

The main aim of this research was to compare the accuracy of the predictions of final degree outcomes from students' UCAS tariff scores, with the predictions from a more complex model which takes into account which qualifications were taken and the combination of qualifications taken by students that contribute to the tariff scores. A GBM was used to generate the more complex predictions. Brief instructions for how to use a GBM in the current context now follow. For a more detailed explanation see Elith, Leathwick and Hastie (2008) and Ridgeway (2012):

1. Split the available data on all individuals into a training data set and a test data set. The training data set will be used to build the statistical model and estimate parameters. The test set will be used to evaluate the model and prevent over-fitting. That is, it is used to prevent the statistical model focussing on characteristics of the data that are unlikely to be repeated in future data sets.
2. Make an initial prediction of outcomes for all individuals in the dataset. In the context of this research this might be the overall probability of achieving a First-class honours degree (hereafter called a 'First') amongst all students.
3. Estimate some simple adjustments to the model to improve its predictive power[2]. For the model described here, this involves the use of regression trees. These work by searching for the partition of the data that leads to the greatest increase in predictive power. For example, the model might divide the data three ways; between students with 300 or more UCAS tariff points from A levels, those

1. Source: HESA Student Record 2010/11 and 2012/13. Copyright Higher Education Statistics Agency Limited 2013. HESA cannot accept responsibility for any inferences or conclusions derived from the data by third parties.

2. Predictive power refers to how accurately the model predicts the final degree outcome for each student.

with less than 300 UCAS points from A levels and those not taking any A levels. Then, within each subgroup, the model looks for further partitions of the data that improve the predictive power. The number of partitions allowed within the tree is pre-determined. The prediction within each subgroup then becomes the average outcome within the subgroup. For example, the overall probability of a First might be 0.20; after the first partition the probabilities of a First might be 0.30 for those with 300 or more A level UCAS points, 0.18 for those with fewer than 300 A level UCAS points and 0.15 for those not taking A levels.

4. Partially accept these adjustments to the predictions and update the model. Instead of accepting the adjustments from Step 2 completely, the model will only be adjusted by a fraction of the amount suggested. This fraction is known as the *learning* rate, and its value can be between 0 and 1, but is usually set to 0.01 or below. For example, if the suggested adjustment for those achieving 300 or more A level UCAS points is 0.1 (that is, an increase in probability of a First of 0.1), then if the learning rate is set to 0.01 the model would adjust the prediction by 0.001 (i.e., increase the probability to 0.201). The point of setting the learning rate to be so low is that, even if we are using many predictors, and even if we are considering the differences between small subgroups, it ensures that no individual adjustment results in the overall model predictions matching too closely to currently available data.
5. Return to Step 2 and repeat, using the adjusted predictions, for a specified number of iterations. Thus the model will again search for the best partitions. The number of iterations is pre-determined and is usually in the thousands.
6. Evaluate the number of iterations at which the model had the greatest predictive accuracy. Then apply the adjustments up to and including this iteration to any new data set to make predictions.

GBMs have been shown to improve the accuracy of predictions, compared with other predictive methods such as linear regression or neural networks (see Ridgeway, 2013). There are also specific reasons why the method may be particularly appropriate in the context of this research. Firstly, the models automatically handle missing data, which is useful in this situation with students taking different combinations of qualifications. Secondly, they automatically find the most important interactions between variables, rather than having to run many complex regression models in order to try and determine which interactions are important. Finally, they also have built-in mechanisms to avoid over-fitting of data, which is important when analysing complex data.

The variables included in the GBMs were the overall UCAS tariff points, the total tariff points achieved in each qualification and the mean tariff points achieved in each (relevant) qualification. The qualifications included were A level, Advanced Subsidiary (AS) level, A level (double), AS level (double), A level (9 unit award), Extended Project, International Baccalaureate (IB), BTEC Diploma, Certificate and Award, Oxford, Cambridge and RSA (OCR) National Extended Diploma, Diploma and Certificate, Cambridge Pre-U Certificate, Cambridge Pre-U Global Perspectives and Research (GPR) and Cambridge Pre-U Short Course. For some of the qualifications the mean tariff points score was not included, because all students taking the qualification took the same number of subjects (for example, Extended Project, Cambridge Pre-U GPR) and so it did not add any further information than the total tariff score.

The analysis compared the predictions from the GBM with the predictions from using the UCAS tariff only. Significant improvements in the predictive power from using the GBM would suggest that using the UCAS tariff to predict degree outcomes is not the ideal model. Two different predictions were made using each method: the probability of a student achieving a First; and the probability of achieving at least an Upper Second-class honours degree (hereafter called an 'Upper Second'). The UCAS tariff predictions were generated by a logistic regression with a smoothing spline. This allowed the relationship between predictor and outcome to vary from the standard log function for a logistic regression.

As well as an analysis of all students together, the predictions from the different models for those taking particular qualifications were compared. This was done to give an indication of how well aligned the tariff points are for different qualifications. For a particular qualification, if the predictions from the GBM are much better than those generated by using only the total UCAS tariff, this would suggest that the tariff points are not well aligned because knowledge of the qualification improved the predictive power of the model. This analysis was limited to three qualifications with large numbers of candidates; A levels, BTECs and the IB. Students were classified as follows: those taking only A levels and AS levels were categorised as 'A levels only'; those taking BTEC qualifications only were categorised as 'BTECs only'; those taking the IB were categorised as 'IB only'; all other students were categorised as 'Mixed'.

The reason for using the GBM method was to find out if predictions could be improved by including extra information, such as the different qualifications taken, the grades achieved and the combinations of qualifications. Whilst it would not be plausible to use such complex models in reality, it would be possible to change the tariff equivalencies for different qualifications. An analysis of the accuracy of tariff equivalencies for a number of qualifications is undertaken in a separate report (Gill, 2015).

Results

The first stage in using GBMs is to determine the best model. This involves changing a number of different factors that affect how the model runs; specifically the number of trees, the shrinkage factor and the interaction depth. Essentially, these determine how far the model searches in order to find the best outcome. By increasing the number of trees or the interaction depth the model will either investigate more trees, or more branches within each tree. Reducing the shrinkage factor means that a smaller proportion of the adjustments from each iteration will be applied before the next iteration, so the model updates at a slower rate. Changing these factors may improve the model outcomes, but beyond a certain point the improvements are too small to be worthwhile. A number of different models were run to determine at what level to set these factors to produce a good model within a reasonable time. This led to a selection of a model with 3,000 trees, a shrinkage factor of 0.01 and an interaction depth of 3 for both of the different predictions.

There are a lot of different variables feeding into the GBMs, so it is of interest to look at which of the variables had the most influence on the prediction. Table 1 presents the top 5 variables in order of relative influence, for the probability of a First, whilst Table 2 does the same for the probability of achieving at least an Upper Second:

Table 1: Relative influence of variables in GBM (predicting probability of achieving a First-class degree)

Variable	*Relative influence (%)*
A level mean	65.2
A level total	9.6
AS level mean	8.5
IB total	8.0
AS level total	3.1

Table 2: Relative influence of variables in GBM (predicting probability of achieving at least an Upper Second-class degree [2:1])

Variable	*Relative influence (%)*
A level mean	60.3
A level total	18.2
IB total	8.1
AS level mean	4.3
BTEC Diploma	2.5

Thus, according to the GBM, by far the most important variable in terms of predicting degree outcomes was the A level mean tariff points. This suggests that the current UCAS system, where achievement is based on total UCAS tariff points, could be improved by using a mean points score instead (at least in terms of A levels). The current system apparently over-values the performance of students who perform less well in a larger number of A levels, compared with students doing better in fewer A levels. However, it may be that admissions tutors are aware of this and therefore take account of it when making offers.

This effect is also illustrated in Figures 1 and 2, which show the relationship between the likelihood of achieving a First or at least an Upper Second (as measured by the log of the odds according to the GBM) and the value of the A level mean and A level total variables. The figures demonstrate that there is a fairly good linear relationship between the A level mean variable and the likelihood. However, for the A level total points variable, beyond a certain value the likelihood does not increase as the total increases (and actually falls in Figure 1).

To see whether the GBMs improved the prediction accuracy we compared the prediction of degree performance to the actual outcome, for the model using the UCAS tariff only and for the model using the

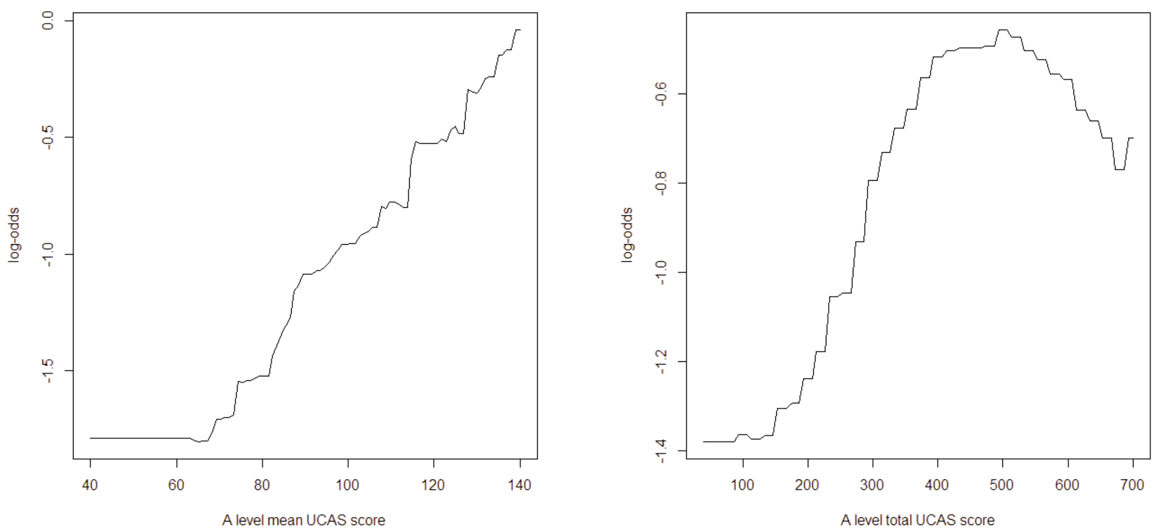

Figure 1: Log odds of achieving a First-class degree, for given values of A level/A level mean

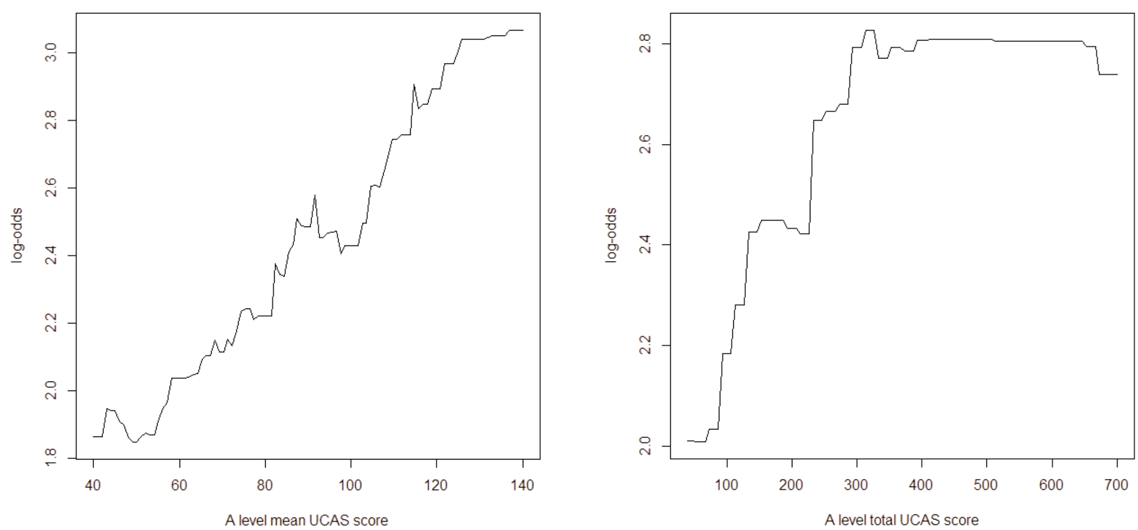

Figure 2: Log odds of achieving at least an Upper Second-class degree, for given values of A level/A level mean

GBM. We used two measures to evaluate how well the models predicted outcomes overall; the correct classification rate and the proportion of deviance explained.

The correct classification rate was calculated as the percentage of candidates where the model prediction of whether they would achieve, for example, a First (that is, whether their predicted probability was above 0.5) matched whether they actually achieved this. This measure is easy to understand but has some weaknesses. For example, this is a binary measure so it doesn't take account of whether a student was very close to being correctly classified (e.g., achieves a First, probability of a First of 0.49) or not (e.g., achieves a First, probability of a First of 0.10).

An alternative measure is the deviance. This is based on the likelihood of students achieving their actual outcome, given the model (it is in fact minus two times the log of this value). So if the model predicts a probability of achieving a First of 0.25 (i.e., unlikely to get a First) for a particular student and they do not achieve this, their likelihood will be 0.75 and their deviance will be -2*log(0.75)=0.575. However, if that student did achieve a First, the likelihood will be 0.25 and their deviance will be -2*log(0.25)=2.77. Therefore, the lower the level of deviance, the better the model is at predicting the outcome. The overall deviance was calculated by summing the deviance across all students. One advantage of using this measure is that different models can be compared, with a lower value indicating a better model fit. The final measure used here to compare different models was the percentage improvement in deviance of each model compared to the 'null' model, which just assigns the overall probability of achieving a First to all students (i.e., a model which is a very poor predictor of outcomes).

Table 3 presents, for the probability of achieving a First-class degree, correct classification rates and proportion of deviance for all students together and then for those taking only the listed qualification(s). It should be noted that using the UCAS tariff prediction, none of the students had a prediction of more than 0.5. Thus the correct classification rate was just the percentage of students who did not get a First (83.68% overall). This was also the case for the GBM prediction for students taking BTECs only or IB only. Thus, this measure tells us very little for these subgroups of students.

Table 3: Comparison of prediction accuracy of UCAS only and GBMs (probability of achieving a First-class degree)

Qualification	*Students*	*Correct classification*		*Proportion of deviance explained*	
		UCAS prediction	*GBM prediction*	*UCAS prediction*	*GBM prediction*
A level only	71,270	83.72	83.78	0.0428	0.0550
BTEC only	3,190	91.19	91.19	0.0713	0.0951
IB only	1,930	78.76	78.76	0.0505	0.0711
Mixed	7,060	81.17	81.49	0.0649	0.0968
All	**83,450**	**83.68**	**83.75**	**0.0458**	**0.0604**

For the analysis of all students together, the improvement in the deviance measure from the model using UCAS tariffs rather than the null model was 0.0458. This was slightly less than the improvement when using GBM (0.0604). Similar differences were found for students taking the separate qualifications, although the difference was greater for students taking a mix of qualifications and for BTEC only students.

The results for the probability of achieving at least an Upper Second are presented in Table 4. Note that neither method predicted any IB only students to get lower than an Upper Second, so the correct classification rate is just the percentage achieving at least an Upper Second (82.38%).

The correct classification rate using the UCAS tariff only was 75.71%, improving to 76.49% using GBM. There was a very small improvement in the correct classification rate for A level students and none at all for IB students. However, for BTEC students the correct classification rate was substantially higher using the GBM prediction (58.94%) than using the UCAS tariff only (52.95%). Using the GBM improves the proportion of deviance explained measure from 0.0789 to 0.1043 overall. For BTEC only students there was a large improvement in this measure, from 0.0731 to 0.1805. There was also a large improvement in this measure for students taking 'Mixed' qualifications, from 0.1132 to 0.1739.

Table 4: Comparison of prediction accuracy of UCAS only and GBMs (probability of achieving at least an Upper Second-class degree)

Qualification	*Students*	*Correct classification*		*Proportion of deviance explained*	
		UCAS prediction	*GBM prediction*	*UCAS prediction*	*GBM prediction*
A level only	71,270	76.43	76.82	0.0759	0.0919
BTEC only	3,190	52.95	58.94	0.0731	0.1805
IB only	1,930	82.38	82.38	0.0819	0.1236
Mixed	7,060	76.89	79.40	0.1132	0.1739
All	**83,450**	**75.71**	**76.49**	**0.0789**	**0.1043**

Conclusion

The research presented in this article has shown evidence that using a GBM to predict degree performance based on attainment in Level 3 qualifications produces more accurate results than using a model based on the overall UCAS tariff only. This is likely to be because the GBM is able to cope better with the complexity of the data, such as the different qualifications and combinations of qualifications taken by students that contribute to the tariff score. It is difficult to assess the size of the improvement in the prediction accuracy because the measure used (proportion of deviance explained) is not easy to interpret. However, it is possible to use this measure to make comparisons between different qualifications in terms of the levels of improvement in prediction accuracy.

Thus, the GBM produced larger improvements in predictive accuracy for students taking BTECs only and for students taking a mix of qualifications, than for students taking A levels or IB. One possible reason for this could be because the current UCAS tariff equivalencies for these qualifications are not well aligned with A level tariffs, and therefore knowledge of the qualifications (and of the combinations of qualifications) taken by students improved the predictions. An assessment of the equivalencies of the UCAS tariff for different qualifications is undertaken in a separate report (Gill, 2015).

It is interesting that the models indicated that the most influential measure in terms of predicting future performance was the A level mean, rather than the A level total score. This is likely to be because of an attenuation effect at the top of the tariff range, where getting higher

tariff scores by taking more qualifications is not indicative of higher ability levels (as demonstrated by Figures 1 and 2). For instance, students achieving 5 A* grades at A level (700 UCAS points) are probably not much more able than those achieving 4 A* grades (560 points).
This suggests that the current tariff measure, based on total points score could be improved by taking account of this in some way.

Finally, it is worth considering to what extent admissions tutors (particularly those with many years' experience) are aware of some of these issues and account for them when making offers to students. They may, for instance, take some account of the number of qualifications contributing to a student's UCAS tariff score, or they may value points scores gained from some qualifications more than scores gained from other qualifications. This should go some way to making up for any lack of equivalence between UCAS tariff scores for different qualifications.

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Post-16 Mathematics qualifications: Differences between GCE A level, International A level, Cambridge Pre-U and Scottish examination questions

Ellie Darlington Research Division

Introduction

This article describes the application of a taxonomy in order to compare and contrast the mathematical skills required to answer examination questions from four different post-16 Mathematics qualifications taken by students both in the UK and overseas: A levels and Advanced Subsidiary (AS) levels, International A and AS levels, Cambridge Pre-U, and Scottish Highers and Advanced Highers. Though the precise content and structure of the different qualifications differ slightly, they are all qualifications which should provide students with a sound basis for university study in Mathematics. All UK universities accept these qualifications as prerequisites for their Mathematics courses. It is therefore of interest to establish whether the questions asked in the assessments of these qualifications require the same kinds of mathematical skills. If there are notable differences among the qualifications, this could suggest that there might be corresponding differences in how well prepared students are for studying Mathematics at university.

In recent years the number of UK schools offering alternative qualifications to General Certificate of Education (GCE) A level has increased. This perhaps may be attributable to head teachers' diminishing confidence in the A level system, with 67 per cent of those surveyed by the Office of Qualifications and Examination Regulation (Ofqual) in 2014 reporting that constant changes to the A level system were of concern. Furthermore, 12 per cent of head teachers surveyed said that they thought that international qualifications such as the International Baccalaureate (IB) and the Cambridge Pre-U were more challenging than A levels. A levels have been criticised for being "oblique at measuring academic ability" (de Waal & Cowen, 2007, p.8), with mathematicians in Higher Education (HE) claiming that it is easy for A level Mathematics students to "'learn the exam' rather than the subject" (Higton et al., 2012, p.58).

Furthermore, concerns are regularly voiced by educational researchers and university admissions and teaching staff regarding the preparedness of new undergraduate mathematicians. For example:

- a restructure of the modular system in A level Mathematics in 2006 resulted in complaints that there was diminishing content (Bassett, Cawston, Thraves, & Truss, 2009; Porkess, 2003, 2006) and that the newer examinations were easier (Qualifications and Curriculum Authority, 2007);
- the modular system of examinations has been criticised for failing to test students' synoptic understanding of Mathematics (Hodgson & Spours, 2004; Quinney, 2008; Wilde, Wrighton, Hayward, Johnson, & Skerrett, 2006);
- some have commented that the A level does not prepare students well for undergraduate Mathematics (Smith, 2004);
- the Engineering and Physical Sciences Research Council (EPSRC) has claimed that "mathematical A-levels are not as rigorous as they used to be." (EPSRC, 2004, p.17);
- the value of the top grade has been questioned, as some stakeholders have claimed that it can be "…achieved through high levels of accuracy rather than extended mathematical reasoning." (Smith, Mitchell, & Grant, 2012, p. 30); and
- claims have been made that standards are falling in the A level, that higher grades are becoming easier to obtain (Coe, 2011; Lawson, 1997).

A rigorous analysis of the types of skills required to answer Mathematics examination questions in GCE A level and its equivalents should therefore be an important source of evidence for these various debates. In particular, it can help shed light on the validity of the assessments (e.g., Shaw, Crisp & Johnson, 2012) in the sense of whether the skills actually assessed match those that are claimed to be assessed; and it can help the various 'users' of these different qualifications (e.g., students, teachers, university admissions tutors) to understand any differences among them.

This article first describes the different Mathematics qualifications that were analysed, then describes the taxonomy used to classify the skills, before presenting and discussing the findings.

Mathematics qualifications

The following post-16 Mathematics qualifications were analysed in this research:

GCE A levels: This is the most common qualification taken by students aged 16–19 in England, Wales and Northern Ireland. A level Mathematics was the most popular subject in 2014, constituting 10.4 per cent of all A levels examined (Joint Council for Qualifications, 2014).

Most students take three A levels (Gill, 2014), choosing from a wide variety of subjects. They may stop after one year and earn an AS level by taking examinations in the units which were taught in the first year of the A level. In Mathematics, Applied units are available in three topics: Mechanics, Statistics and Discrete Mathematics[1] and students may choose to study a narrow or broad range of these topics (see Figure 1).

The qualifications are currently offered by five different awarding bodies, which are all accredited by Ofqual, the regulator of qualifications and assessments in England. Mathematics and Further Mathematics[2] each consist of six equally-weighted units (three of which constitute the AS level) which are individually examined. Further Mathematics may only be studied in addition to Mathematics, with the units in Further Mathematics building upon the knowledge of earlier units taken as part of A level Mathematics.

International AS and A levels: These are a very popular qualification all around the world and an increasing number of schools in the UK are beginning to offer them.

Cambridge International Examinations (Cambridge) offers A levels in Mathematics and Further Mathematics and the way in which students choose and take the International A level is much the same as with the GCE A level: students most often take three subjects from a wide variety of their choosing, the A level is studied over a two-year period, and AS levels are available. However, Cambridge A levels are assessed linearly, unlike the modular assessment in GCE A levels, and the number and content of the units is not the same (see Figure 1).

Cambridge Pre-Us: This is a relatively recent qualification, which currently has a small number of candidates, although it is continuing to grow.

Launched in 2008, the Cambridge Pre-U is a post-compulsory qualification which is aimed at those students wishing to go on to tertiary study (see University of Cambridge International Examinations, 2012).

The Cambridge Pre-U Mathematics is divided into three components, each with one two-hour examination of equal weighting.

Though its uptake is small, it is recognised by UK universities and an increasing number of institutions across the globe. Steinberg and Hyder (2011) describe the Cambridge Pre-U as being among the best international qualifications, with some arguing that it is more demanding than GCE A levels, partly because of its linear (as opposed to modular) assessment structure (University and Colleges Admissions Service [UCAS], 2008a). It has also been found to act as a good predictor of degree outcome (Gill & Vidal Rodeiro, 2014).

Scottish Highers and Advanced Highers: This is the most common qualification for students in Scotland. Students aged 16–19 study for Highers and, sometimes, Advanced Highers, typically studying four or five subjects for Highers over the course of one year. In 2012, 86 per cent of students doing five or more Highers took Mathematics as one of their subjects, with nearly 5,000 doing Advanced Higher Mathematics, and over 18,000 doing Higher Mathematics (Nuffield Foundation, 2013, p.5).

Higher Mathematics, which only covers topics in Pure Mathematics, consists of three compulsory progressive[3] units which are assessed by the means of two terminal examinations. Advanced Highers in Mathematics and Applied Mathematics are available, and are each assessed by the means of one terminal examination based on three compulsory progressive units (Scottish Qualifications Authority, 2010).

Advanced Highers are considered to be equivalent in standard to the first year of undergraduate study in that subject at Scottish universities, where typical degree programmes take a year longer to complete than in the rest of the UK. It is possible for students with Advanced Highers (or A levels) to skip the first year of undergraduate study if they wish. Consequently, the Highers are generally viewed by universities as approximately equivalent to AS level, and Advanced Highers to A level (The Association of Graduate Careers Advisory Services [AGCAS] Scotland, 2008; Munro, 1998; UCAS, 2008b).

Though all of the four aforementioned qualifications are accepted by universities as prerequisites to study undergraduate Mathematics, they do not all necessarily follow the same structure or examine exactly the same content. Figure 1 shows how the different qualifications have different compulsory and optional elements, and how the total number of examined elements differs.

MATH Taxonomy

A number of taxonomies are available for analysis and classification of Mathematics questions according to a set of criteria. *Bloom's Taxonomy of Educational Objectives* (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956) is perhaps the most famous example of this. However, whilst Bloom describes levels of learning and is not subject-specific, the *MATH Taxonomy* (Smith et al., 1996) is a modification of it for the context of undergraduate Mathematics, and can be used to describe the skills required to complete a task. It classifies skills according to three broad groups (A, B, C), which have two or three subgroups (see Table 1). It makes no claims to describe the level of difficulty of a question; that is, a Group A question might be more difficult than a Group C question.

1. Referred to in some specifications as Decision Mathematics.
2. Only the awarding body AQA offers A level Statistics, with Oxford, Cambridge and RSA (OCR) Examinations offering an AS level in Statistics.
3. That is, each unit builds upon the content of previous units.

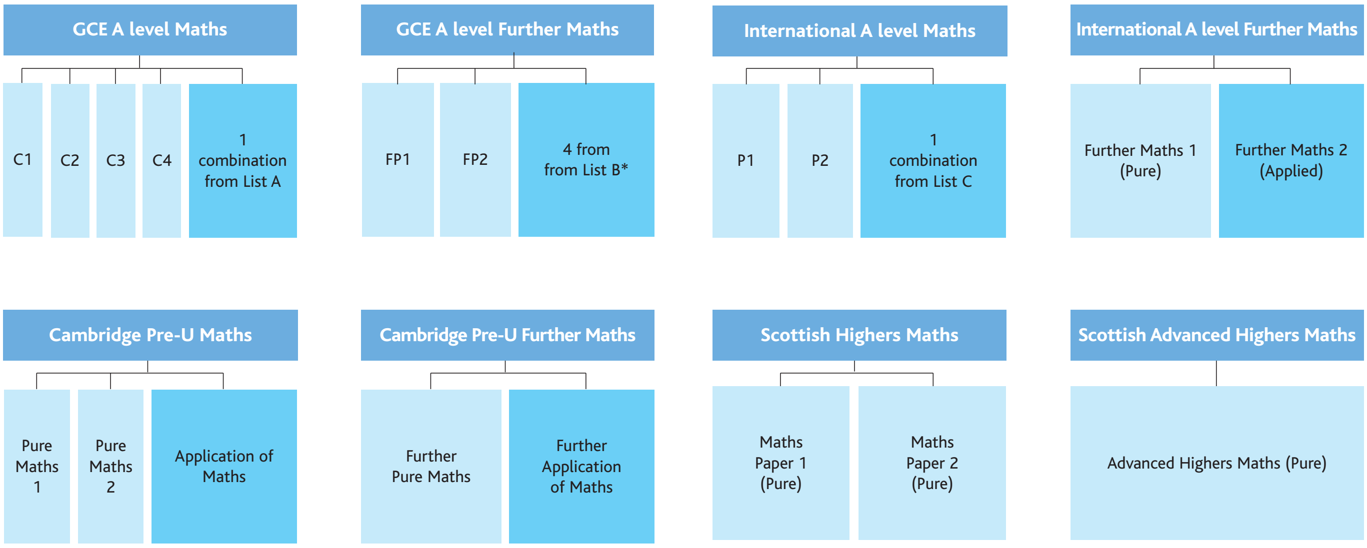


Figure 1: Different structures of post-16 Mathematics qualifications

** Students may not take a unit in Further Mathematics that they have already taken in Mathematics.*

List A M1+M2, D1+D2, S1+S2, S1+D1, M1+S1, M1+D1
List B FP3-4, M1-5 (where available), S1-4, D1-2
List C M1+S1, M1+M2, S1+S2
Where 'C' is Core Pure Mathematics, 'P' is Pure Mathematics, 'M' is Mechanics, 'S' is Statistics, 'D' is Discrete Mathematics and 'FP' is Further Pure Mathematics.

Table 1: Groups of mathematical skills according to the MATH Taxonomy

Group	*Subgroup*	*Outline*
A	Factual knowledge and fact systems (FK&FS)	Factual recall and routine procedures
	Comprehension (Comp)	
	Routine use of procedures (RUOP)	
B	Information transfer (IT)	Using existing mathematical knowledge and techniques in new ways
	Application in new situations (AINS)	
C	Justifying and interpreting (J&I)	Application of conceptual knowledge to construct mathematical arguments
	Implications, conjectures and comparisons (IC&C)	
	Evaluation	

Group C skills have been found to be associated with students who have deeper understandings of the material (Malabar & Pountney, 2002), and associated with university-level Mathematics (Barnett, 1990; Pountney, Leinbach, & Etchells, 2002).

Existing work suggests that GCE A level Mathematics examinations rely heavily on Group A tasks (Darlington, 2013a, 2014; Etchells & Monaghan, 1994), as do undergraduate Mathematics examinations (Ball, Smith, Wood, Coupland, & Crawford, 1998; Darlington, 2013a, 2013b, 2014, 2015; Smith et al., 1996). This is by no means a phenomenon confined to the UK. For example, work by Crawford (1983, 1986) and Crawford, Gordon, Nicholas, and Prosser (1993) found that new undergraduate mathematicians in Australia had very little prior experience of Group C tasks.

Examples of A level Mathematics and Further Mathematics, undergraduate Mathematics and university entrance examination questions associated with each group and subgroup in the *MATH Taxonomy* may be found in Darlington (2013a, 2013b, 2014, 2015). Table 2 gives examples of questions from GCE A level, International A level, Cambridge Pre-U and Scottish Highers and Advanced Highers papers in Mathematics and Further Mathematics which would fit into each category and subcategory.

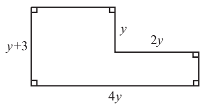
Sample

Question papers from A level Mathematics and Further Mathematics were analysed so as to give an indication of the nature of the skills required at the two different levels. Analysis was conducted at the subquestion level. For maximum contrast, the introductory Pure Mathematics unit was analysed from the Mathematics qualifications, along with the most 'advanced' Pure Mathematics unit from Further Mathematics (see Table 3). All analyses were conducted on the five[4] most recent publicly-available question papers from all awarding bodies, where applicable, at the time of analysis.

Pure Mathematics units were selected rather than Applied Mathematics units because: (1) the different qualifications had different emphases on Applied Mathematics, and as such the content was not comparable; and (2) Applied Mathematics units were not available for some of the qualifications (see Figure 1). Furthermore, it is highly likely that the majority of marks in Applied Mathematics units would be skewed towards Group B due to its focus (see Table 1).

4. Only four Cambridge Pre-U papers were available at the time of analysis.

Table 2: Examples of questions in each MATH Taxonomy subgroup

Category	*GCE A level*[5]	*International A level*	*Cambridge Pre-U*	*Scottish Highers*
GROUP A				
FK&FS	Sketch the curve $y = \frac{1}{x}$			Write down the derivative of $\sin^{-1}x$.
Comp	The curve $y = -\sqrt{x}$ is stretched by a scale factor of 2 parallel to the x-axis. State the equation of the curve after it has been stretched.	The function f is defined by $f:x \mapsto 2x^2 - 12x + 1|3$ for $0 \leq x \leq A$, where A is constant. State the value of A for which the graph of $y = f(x)$ has a line of symmetry.	Let $I_n = \int_0^{\alpha} \tanh^2 \theta \, d\theta$ for $n \geq 0$, where $\alpha > 0$. Given that $\alpha = \frac{1}{2}\,\mathrm{In}\,3$, evaluate I_0.	A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 3$ and $u_0 = 1$. What is the value of u_2?
RUOP	Express $\sqrt{18} - \sqrt{2}$ in simplified surd form.	Find the first 3 terms in the expansion of $\left(2x - \frac{3}{x}\right)^5$ in descending powers of x.	Find the equation of the line passing through the points $(-2,5)$ and $(4, -7)$. Give your answers in the form $y = mx + c$.	For what value of λ is $\begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ -1 & \lambda & 6 \end{pmatrix}$ singular?
GROUP B				
IT	Sketch the curve $y = 9x^2 + 18x - 7$, giving the coordinates of all intercepts with the axes.	Sketch the curve $y = (x - 2)^2$.	Sketch, on a single diagram, the graphs of $y = e^{\frac{1}{5}x}$ and $y = x$ and state the number of roots of the equation $e^{\frac{1}{5}x} = x$.	Describe the loci in the complex plane given by $\lvert z + i \rvert = 1$
AINS	A rectangular tile of length $4y$ cm and width $(y + 3)$ cm has a rectangle of length $2y$ cm and width y cm removed from one corner as shown in the diagram.  Given that the perimeter of this tile is between 20 cm and 54 cm, determine the set of possible values of y.	A television quiz show takes place every day. On day 1 the prize money is \$1000. If this is not won the prize money is increased for day 2. The prize money is increased in a similar way every day until it is won. The television company considered the following two different models for increasing the prize money. Model 1: Increase the prize money by \$1000 each day. Model 2: Increase the prize money by 10% each day. On each day that the prize money is not won the television company makes a donation to charity. The amount donated is 5% of the value of the prize on that day. After 40 days the prize money has still not been won. Calculate the total amount donated to charity (i) if Model 1 is used (ii) if Model 2 is used	The curve C has Cartesian equation $x^2 - xy + y^2 = 72$. Find the exact area of the region of the plane in the first quadrant bounded by C, the x-axis and the line $y = x$. Deduce the total area of the region of the plane which lies of the plane which lies inside C and within the first quadrant.	The radius of a cylindrical column of liquid is decreasing at the rate of $0.02\ ms^{-1}$, while the height is increasing at a rate of $0.01\ ms^{-1}$. Find the rate of change of the volume when the radius is 0.6 metres and the height is 2 metres.
GROUP C				
J&I	The variables x and y satisfy the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}$ Explain briefly why there is no particular integral of either of the forms $y = ke^{3x}$ or $y = kxe^{3x}$.	The function g is defined by $g:x \mapsto 2x^2 - 12x + 13$ for $x \geq 4$. Explain why g has an inverse.	Let $f(x) = x^2$ and $g(x) = 7x - 2$ for all real values of x. Give a reason why f has no inverse function.	Prove by induction that, for all positive integers n, $\sum_{r=1}^{n}(4r^3 + 3r^2 + r)$ $= n(n$ $= 1)^3$
IC&C	w denotes the complex number $\cos\frac{2}{5}\pi + i\sin\frac{2}{5}\pi$ Write down a polynomial equation of degree 5 which is satisfied by w.	For the series $\sum_{n=1}^{N} \frac{4n + 9}{(n + 2)(n + 3)(2n + 3)(2n + 5)}$ find the sum to infinity.	The cubic equation $x^3 + x^2 + 7x - 1 = 0$ has roots α, β, γ. State what can be deduced about the nature of these roots.	Let n be a natural number. For each of the following statements, decide whether it is true or false. If true, give a proof; if false, give a counterexample. A If n is a multiple of 9 then so is n^2 B If n^2 is a multiple of 9 then so is n.

Note: No examples of Group C's 'Evaluation' questions could be found in any of the papers analysed.

5. All GCE A level examples taken from OCR question papers.

Table 3: Question papers analysed

Qualification	*Introductory Pure Mathematics Question Paper*	*'Advanced' Pure Mathematics Question Paper*
GCE A level	Core Pure 1	Further Pure 3
Cambridge International A level	Pure Mathematics 1	Further Pure Mathematics 1
Cambridge Pre-U	Pure Mathematics 1 Paper 1	Further Pure Mathematics
Scottish Higher/ Advanced Higher[6]	Higher Mathematics Paper 1	Advanced Higher Mathematics

Note: See Figure 1, which illustrates the selection of these particular examinations.

GCE A level question papers from the different awarding bodies were treated together as one group because all awarding bodies are regulated by Ofqual, and a number of studies have found that there are no differences between the awarding bodies' papers in terms of difficulty. For example, Taverner (1996) compared students' A level Information Systems (ALIS) scores[7] with their A level results in order to see whether there were any differences between awarding bodies, and found no significant differences.

Six examples of analysis for each qualification were checked by a Mathematics education specialist, and there were no disagreements in the classifications of skills for those questions.

Results

For each question paper, the proportions of marks awarded for Group A, B and C tasks were calculated, and averaged across the qualification to enable comparisons to be made.

The results of this analysis are given in two sections: (1) for qualifications equivalent to GCE A level Mathematics C1, and (2) for qualifications equivalent to GCE A level Further Mathematics FP3.

Data for GCE qualifications and the associated analysis are taken from Darlington (2015).

A level Mathematics equivalent

For all of the qualifications analysed, the majority of the marks awarded in the question papers were for Group A skills (see Figure 2), of which 88.5% were routine uses of procedures.

Figure 2 shows that, with 75.43% of marks, Scottish Highers put less focus on Group A skills than all of the other qualifications. The International A level had the highest proportion of marks awarded for Group A skills (94.13%). Scottish Highers awarded substantially more marks for Group C skills (16.57%) than GCE (2.25%) and International (0.52%) A levels and the Cambridge Pre-U (0.31%). It is unsurprising that there were relatively few marks awarded for Group B skills in all of the qualifications (ranging from 5.33% of the marks in the International A level to 8.47% in the GCE A level) because Pure Mathematics examinations were analysed, whilst Group B skills are more associated with Applied Mathematics.

6. Introductory question paper from Higher Mathematics, and Advanced paper from Advanced Higher Mathematics.
7. ALIS tests are run by the Centre for Evaluating and Monitoring (CEM) at the University of Durham. The scores act as performance indicators for post-16 students, using data from GCSE grades and CEM's baseline tests.

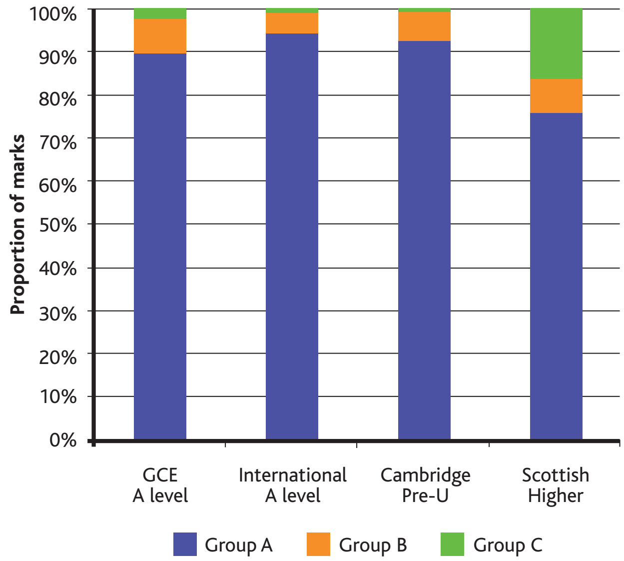


Figure 2: Contrasts in question type composition in qualifications equivalent to A level Mathematics C1

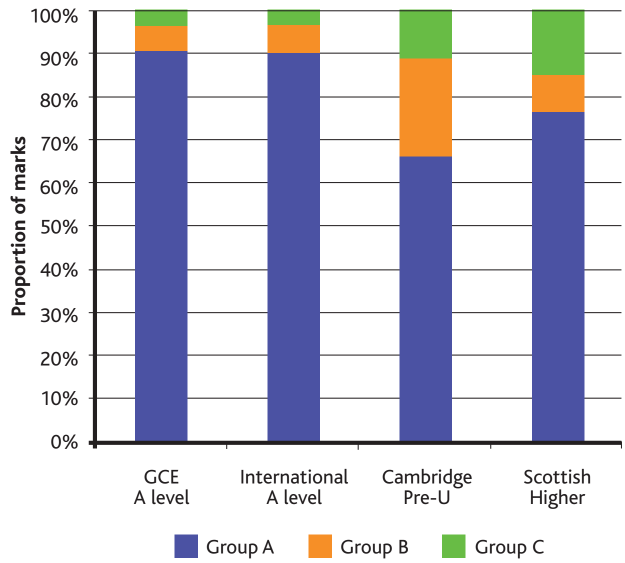


Figure 3: Contrasts in question type composition in qualifications equivalent to A level Further Mathematics FP3

A level Further Mathematics equivalent

As with qualifications equivalent to GCE A level Mathematics, the majority of marks in question papers equivalent to GCE A level Further Mathematics FP3 were for Group A skills (see Figure 3).

Both the GCE (90.6%) and the International A levels (89.45%) awarded more marks for Group A skills than Scottish Advanced Highers (76.6%) and the Cambridge Pre-U (66.04%). The converse could be said for Group C skills, where the A levels awarded fewer marks than the other qualifications. The Cambridge Pre-U awarded significantly more marks for Group B skills (23.34%) than the other qualifications.

Discussion and Conclusion

Analyses conducted for the purpose of this article revealed that the majority of marks awarded in examinations at both the C1-equivalent and FP3-equivalent level were for Group A skills. That is, the majority of questions required students to demonstrate an ability to answer questions which could be prepared for by doing drill-style practice, something perhaps aided by the apparent frequency of the topics and similar types of questions posed year-to-year (see Darlington, 2013a). This was the same for all qualifications, though it was more extreme in some instances than others. Specifically, the concentration of Group A skills appeared to be higher in both Mathematics A levels than Scottish Highers, and higher in A level Further Mathematics than both Scottish Advanced Highers and the Cambridge Pre-U.

It should be noted that the GCE A level C1 examinations can be taken after just one school term of learning, whereas the Cambridge A level and Scottish Highers are taken at the end of one year of study, and the Cambridge Pre-U after two. Therefore, it *might* be possible that the non-GCE A level students may be better-practiced with certain techniques and therefore more freely-able to use this Mathematics when eventually assessed later on in their study of the qualification. Consequently, there may be more scope for assessment to assess a wider range of skills.

Whilst this research might suggest that A levels, Cambridge Pre-Us and international qualifications may not place an emphasis on students demonstrating certain Group C skills, this does not necessarily have to be interpreted as a criticism. Not only do many students do these qualifications out of interest or as a service subject for Science or Social Science degrees, but developing and marking Group B and Group C tasks can be time-consuming and challenging for examiners and teachers (Leinch, Pountney, & Etchells, 2002). However, one could question the validity of the assessment objectives (AOs) of A level Mathematics, as current guidelines describe AO2 as:

> *Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form*. (Ofqual, 2011, p.12)

As AO2 is supposed to constitute at least 30 per cent of the overall marks for the qualification, and shares a similar meaning to the definition of Group C skills (see Table 1), this brings into question whether A level Mathematics effectively examines that particular AO. However, it should be noted that this analysis using the *MATH Taxonomy* was conducted on a subquestion level, whereas AOs are categorised on a mark-by-mark basis. Hence, a question classified here as Group C may, in reality, reward students for a number of AOs depending on the mark allocation. Therefore, comparisons between AOs and the groups in the *MATH Taxonomy* can only be crude – AOs refer to what is assessed and what is rewarded by the mark scheme, whereas the *MATH Taxonomy* refers to the skills required to answer the questions.

The differences in the mathematical skills assessed between these qualifications should be read with caution. Whilst 30 GCE A level question papers were analysed, only four or five question papers from the other qualifications were subjected to the same analysis due to limited availability. Furthermore, Scottish universities have different teaching structures to those in the rest of the UK in accordance with the different secondary school examinations there, meaning that Highers do not serve exactly the same purpose for universities in Scotland as A levels do for English universities. The reader should not necessarily interpret the data as meaning that any of these qualifications are 'better' or 'worse' than the others, but recognise that some differences do appear to exist between them.

However, this is perhaps an opportunity to recognise the value of Mathematics admissions tests and extension papers such as the *Sixth Term Examination Papers* and the *Advanced Extension Award*, as described by Darlington (2015), rather than any apparent shortcoming of post-16 qualifications. However, although problems with access to prepare for and take these assessments mean that these should not be seen as a 'solution' to the gap between the skills assessed at A level and university (Darlington 2014, 2015).

GCE A level Mathematics and Further Mathematics are currently undergoing revisions which will reportedly see them involve more problem solving in examinations, as well as restructures to units, syllabuses and content. Quite what impact this will have in terms of the skills required to answer examination questions remains to be seen; however, the research here suggests that GCE A level Mathematics and Further Mathematics questions are not vastly different to some of the alternatives available in both the UK and overseas.

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Progressing to Higher Education in the UK: The effect of prior learning on institution and field of study

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Introduction

Students applying to study a course in a Higher Education (HE) institution have to make two choices: what subject to study and at which institution. These decisions are influenced by a range of different factors, for example their personal interests, their socio-economic background and, in particular, their prior qualifications and performance. In fact, Hoelscher, Hayward, Ertl and Dunbar-Goddet (2008) showed that the educational background of students is the factor with the greatest influence. This clearly makes sense as some qualifications (and, in some cases, specific subjects) are a necessary condition for studying a course in a specific HE institution.

Young people progressing to HE hold a wide range of qualifications and combinations of qualifications. In England, the vast majority of learners at Level 3[1] (usually aged 16–19) still take 'traditional' academic qualifications such as AS/A levels in schools or colleges (e.g., Department for Education [DfE], 2013a). However, the government's commitment to widening participation has encouraged the growth of more and different pathways to HE study. For example, AS/A levels are supplemented with or replaced by other academic qualifications such as the Cambridge Pre-U, the International Baccalaureate (IB) or the Extended Project. Recently, there has been an increase in learners taking more applied or vocational qualifications such as Applied AS/A levels, Advanced Diplomas, Oxford, Cambridge and RSA Examinations' (OCR) Nationals and, particularly, Business and Technology Councils' (BTECs) (e.g., Hayward & Hoelscher 2011; UCAS, 2012; Gill, 2013)[2].

In the last decade, there has been some research on how different educational pathways lead to different kinds of institutions and different subjects. This body of research includes themes such as the status of non-traditional qualifications, the transition from vocational education and training to HE, and the imbalance of different types of qualifications across HE institutions and courses. For a broad review of the literature in this area see Vidal Rodeiro, Sutch and Zanini (2013). However, new qualifications that aim to prepare learners for study at university have been introduced quite recently, some qualifications have been withdrawn, and others are being comprehensively reformed. It is therefore crucial to better understand how current qualifications, both academic and vocational, are used by young people to progress to HE. Understanding the use of different pathways for progression should enable fairer and more transparent admissions to HE.

The main aim of this work was, therefore, to provide detailed quantitative evidence to shed light on the above topic. Specifically, the research focused on the following issues:

1. Understanding the range of qualifications and combinations of qualifications held by learners aged 16–19 who progressed to different types of HE institutions to study different subjects. It should be noted that, to date, some work has been carried out at a subject level to understand which subjects studied at age 16–19 facilitate progression to HE courses (Russell Group, 2012; Vidal Rodeiro & Sutch, 2013).
2. Identifying the HE destinations (both institutions and subjects) of learners holding different types of qualifications and of learners with a mixed economy of qualifications.

Data and methods

Data

The data for the analyses carried out in this article was provided by the Higher Education Statistics Agency (HESA)[3]. It covers all full-time, first year undergraduates aged 17–19, domiciled in England, studying at UK universities in the academic year 2011/12. In particular, this dataset includes information on the students' qualifications prior to starting the HE course, the courses studied and the institutions where the students were enrolled. Alongside this, detailed student-level information such as gender, socio-economic background and previous institution was also available.

In this research, the HE institutions were considered in 'mission' groups. The following university groups were considered: Russell Group, 1994 Group[4], University Alliance and Million+ Group. Universities that have not joined any of these groups were included in a separate group, labelled as 'Other'. The Russell and the 1994 groups consist of research intensive and highly selective institutions. The University Alliance and the Million+ Group are constituted by the newest universities and colleges, which are usually recruiting universities or universities with former 'polytechnic' status. A full list of members of each group can be obtained from the groups' websites.

For each student, information on up to three subjects of study and the subject percentage (i.e., the relative contribution of that subject to the university degree) was provided. The subject of study was aggregated into 20 broad subject areas and analyses were carried out at this level. It should be noted that the subject area relates to the principal subject of study. For degrees with more than one subject (e.g., balanced combinations or triple honours) it corresponds to the subject with the largest percentage. If a student took a balanced combination or a triple honours degree in three different subject areas, then the subject area was 'Combined'.

1. Each regulated qualification in England has a level between Entry Level and Level 8. Qualifications at the same level are of a similar demand or difficulty. To find out more about qualification levels see http://www.ofqual.gov.uk/help-and-advice/comparing-qualifications/.
2. Information about the Level 3 qualifications considered in this research can be found here: http://www.accreditedqualifications.org.uk/qualification-types-in-the-uk.html.
3. Source: HESA Student Record 2011/12. Copyright Higher Education Statistics Agency Limited 2012. HESA cannot accept responsibility for any inferences or conclusions derived from the data by third parties.
4. The 1994 group dissolved in November 2013.

In this research, the following Level 3 prior qualifications were considered: AS and A levels; Double Award AS and A levels (also known as Applied AS/A levels); BTEC; Extended Project; Free Standing Mathematics Qualification (FSMQ); International Baccalaureate (IB) Diploma; OCR National; Advanced Diploma; Progression Diploma; Cambridge Pre-U.

Methods

The issues researched in this article were addressed, in the first instance, through descriptive analyses. Subsequently, an assessment of the universities and courses in which the different prior qualifications were over- or under-represented was made using odds ratios derived from multilevel logistic regressions. The regression analyses differ from the descriptive analyses because they take into account students' characteristics when looking at the probability of attending a specific university or pursuing a specific course.

Multilevel models were proposed due to the hierarchical or clustered structure of the data (as students were grouped within schools). Detailed discussions of the implementation and outcomes of the multilevel logistic regression can be found in Goldstein (2011).

For the purpose of the regression analyses presented in this article, the dependent variables for the models were: 1) enrolment in a university; and 2) studying a course in a subject area. The independent or explanatory variables were: gender, prior educational institution, socio-economic status and prior learning.

Prior learning was categorised in two different ways:

- Candidates were classified as having the following types of prior qualifications, and no other qualifications alongside: A level; IB; Cambridge Pre-U; BTEC; OCR National.
- Candidates were classified as having A levels *plus* one other type of mainstream prior qualification, as follows: A levels only; A levels plus Extended Project; A levels *plus* Cambridge Pre-U Principal Subject; A levels plus Cambridge Pre-U GPR[5]; A levels *plus* BTEC; A levels *plus* OCR National; A levels *plus* Double Award A level.

The focus of this research was on the association between prior qualifications and the dependent variables, once background characteristics of the students had been taken into account. Therefore, only the odds ratios for the prior qualifications variables are discussed.

Results

The first part of the Results section focuses on which mainstream Level 3 qualifications are most commonly held by first year English undergraduates in different types of HE institutions and courses. To that end, Table 1 shows the numbers and percentages of students who were enrolled in a UK university with the different prior qualifications listed previously[6].

The most popular mainstream qualifications held by undergraduates at HE institutions were A and Advanced Subsidiary (AS) levels, with around 86% of the first year undergraduates having at least one A level. BTECs, with 14% of the first year undergraduates having at least one qualification of this type (Award, Certificate or Diploma), were the second most popular qualification, followed by the Extended Project (6%). Around 2% of the first year undergraduates had OCR Nationals (Certificate, Diploma or Extended Diploma) or Double Awards at AS and A level. Other academic qualifications such as the IB or the Cambridge Pre-U were held by less than 1% of the first year undergraduates.

It should be noted that the percentages in Table 1 add to more than 100% because students can hold more than one type of prior qualification when entering HE. Indeed, Table 2, which shows the percentage of students with different combinations of just two qualifications, highlights that just over 28% of students entering HE with A levels had only A levels and a further 66% of those entering with A levels had one or more AS levels. The next most common qualification also held by those with A levels was the Extended Project (7%). Table 2 also shows that OCR Nationals were taken more in combination with other qualifications (e.g., A and AS levels) than BTECs. Furthermore, a very high percentage of students with Cambridge Pre-U qualifications had at least one A level. Only 8% of the students who obtained Cambridge Pre-U Principal Subject qualifications held no other types of qualifications. Interestingly, only 12% of the students with a Cambridge Pre-U GPR qualification obtained a Cambridge Pre-U qualification in a principal subject. Finally, the qualification most frequently taken in isolation was the IB (92% of the IB students had no other qualification). This was followed by the BTEC Diploma and the OCR National Extended Diploma, which are equivalent to three A levels, and were taken in isolation by 73% and 71% of the students respectively.

Table 1: Numbers and percentages of students with each mainstream prior qualification

Prior qualification		*Number of students*	*Percentage*
A level		214,230	85.6
AS level		145,430	58.1
IB		2,270	0.9
Cambridge Pre-U GPR		165	0.1
Cambridge Pre-U Principal Subject		815	0.3
Extended Project		16,080	6.4
Free Standing Mathematics		595	0.2
Advanced Diploma		585	0.2
Progression Diploma		65	0.0
A level (Double)		3,480	1.4
AS level (Double)		160	0.1
A+AS level combined		70	0.0
BTEC	All types	35,195	14.1
	Award	7,005	2.8
	Certificate	6,115	2.4
	Diploma	24,015	9.6
OCR National	All types	3,780	1.5
	Certificate	2,600	1.0
	Diploma	1,090	0.4
	Extended Diploma	305	0.1

Due to the large number of possible combinations of prior qualifications, and in order to look at a mixed economy of qualifications, students were classified as having pursued one of the three following programmes of study:

- **Academic:** Students obtained one or more of the following qualifications: AS/A level, IB, Cambridge Pre-U, Extended Project, Free Standing Mathematics.

5. Global Perspectives and Research.
6. Numbers of students have been rounded to the nearest multiple of 5 throughout this article and percentages calculated on groups which contain 52 or fewer individuals were suppressed, following HESA's rounding strategy.

Table 2: Combinations of prior qualifications ~ for row i and column j, percentage of students with qualification i that also have qualification j

	A level	*AS level*	*IB*	*Cambridge Pre-U GPR*	*Cambridge Pre-U Principal Subject*	*Extended Project*	*Free Standing Mathematics*	*Advanced Diploma*	*Progression Diploma*	*A level (Double)*	*AS level (Double)*	*A+AS level combined*	*BTEC Award*	*BTEC Certificate*	*BTEC Diploma*	*OCR National Certificate*	*OCR National Diploma*	*OCR National Extended Diploma*
A level	28.1	65.8	0.0	0.1	0.3	7.1	0.3	0.1	0.0	1.3	0.1	0.0	2.0	1.2	0.5	0.9	0.3	0.0
AS level	96.9	1.0	0.0	0.1	0.3	8.1	0.3	0.1	0.0	1.0	0.0	0.0	1.5	1.1	1.1	0.8	0.3	0.0
IB	3.3	3.1	92.3	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.1	0.0	0.0
Cambridge Pre-U GPR	100.0	63.8	0.0	0.0	12.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Cambridge Pre-U Principal Subject	90.4	48.0	0.0	2.5	7.6	4.7	0.4	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0
Extended Project	94.3	73.3	0.0	0.0	0.2	0.8	0.3	3.0	0.4	1.0	0.0	0.0	2.2	0.9	1.3	0.8	0.3	0.1
Free Standing Mathematics	91.9	74.7	1.2	0.0	0.5	8.9	0.7	0.2	0.0	0.7	0.0	0.0	2.0	1.0	6.2	0.7	0.5	0.0
Advanced Diploma	48.0	19.6	0.0	0.0	0.0	82.9	0.2	1.5	0.0	0.5	0.0	0.0	19.1	1.5	1.2	2.9	0.3	0.0
Progression Diploma	29.2	41.5	0.0	0.0	0.0	87.7	0.0	0.0	3.1	0.0	0.0	0.0	7.7	3.1	0.0	0.0	0.0	0.0
A level (Double)	78.9	43.6	0.0	0.0	0.0	4.6	0.1	0.1	0.0	7.7	0.2	0.1	3.4	0.6	0.7	1.6	0.3	0.0
AS level (Double)	67.3	42.6	0.0	0.0	0.0	2.5	0.0	0.0	0.0	3.7	6.2	0.0	9.3	3.7	9.3	1.9	0.0	0.0
A+AS level combined	88.2	41.2	0.0	0.0	0.0	4.4	0.0	0.0	0.0	4.4	0.0	1.5	5.9	2.9	1.5	1.5	0.0	0.0
BTEC Award	68.2	35.5	0.0	0.0	0.0	5.8	0.2	1.8	0.1	1.9	0.2	0.1	11.4	9.7	4.9	3.8	1.3	0.1
BTEC Certificate	42.6	25.8	0.0	0.0	0.0	2.5	0.1	0.2	0.0	0.4	0.1	0.0	10.0	33.3	1.0	2.2	0.4	0.0
BTEC Diploma	4.8	6.9	0.0	0.0	0.0	0.9	0.2	0.0	0.0	0.1	0.1	0.0	1.2	0.2	73.3	0.4	0.1	0.0
OCR National Certificate	79.2	45.2	0.1	0.0	0.0	5.3	0.2	0.7	0.0	2.3	0.1	0.0	9.6	5.4	3.6	4.0	2.6	0.1
OCR National Diploma	54.1	34.2	0.0	0.0	0.0	4.8	0.3	0.2	0.0	1.0	0.0	0.0	7.5	2.0	2.9	6.0	19.8	0.0
OCR National Extended Diploma	8.2	7.8	0.0	0.0	0.0	2.9	0.0	0.0	0.0	0.0	0.0	0.0	1.6	0.0	0.0	0.7	0.0	71.2

(Note: The shaded diagonal cells show the percentage of students with qualification i that have no other qualification types)

- **Vocational:** Students obtained one or more of the following qualifications: Double Award AS/A level, BTEC, OCR National.
- **Mixed:** Students obtained a Diploma (Progression or Advanced) or a combination of academic and vocational qualifications.

Figure 1 shows the percentages of students progressing to HE through the different programmes of study. Although the percentages of university students having followed vocational and mixed programmes of study have been growing in the last few years (see, for example, Hayward & Hoelscher (2011)), the majority of the first year undergraduates in the academic year 2011/12 had followed an academic programme of study (80%). Approximately 11% of the first year undergraduates had followed a vocational programme and the remaining 9% followed a mixed one.

Figure 2 shows the percentages of students who progressed to HE through the different programmes of study by university mission group. The highest percentages of students having followed an academic programme of study were in universities of the Russell Group (96%), followed closely by universities in the 1994 Group (90%). The lowest percentages of students with an academic programme were in universities of the Million+ Group (67%). The highest percentages of

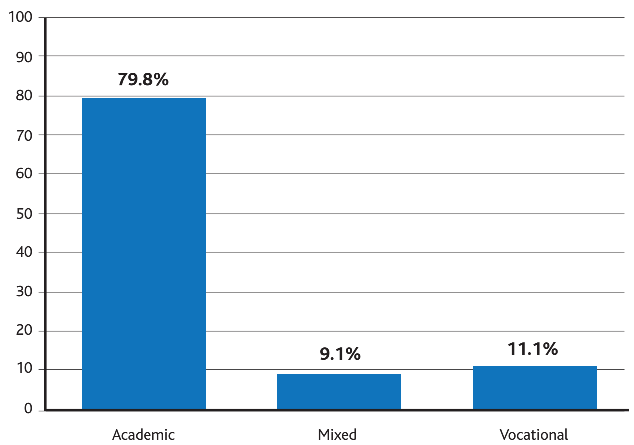

Figure 1: Percentages of students progressing to HE through different programmes of study

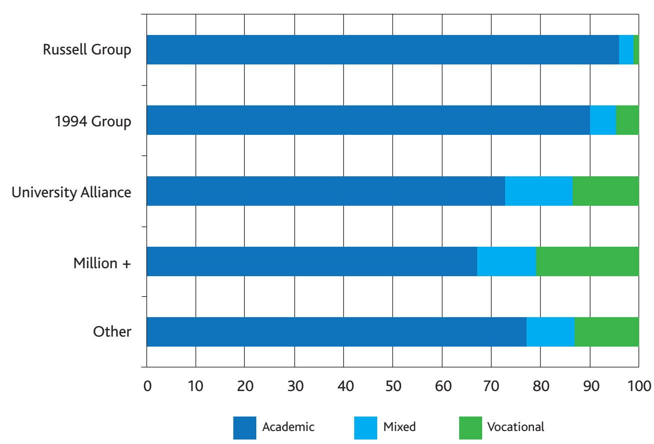

Figure 2: Percentages of students progressing to HE through different programmes of study, by type of university

students with a vocational programme of study were in universities of the Million+ Group (21%), followed by universities in the University Alliance (14%). Unsurprisingly, the lowest percentages of students having followed a vocational pathway into university were in the universities of the Russell Group (1%). It is worth noting that, in the Russell Group universities, the percentage of students with a mixed programme of study was higher than the percentage of students following a vocational one (3% vs. 1%).

Table 3 shows the percentages of students who progressed to HE through the different programmes of study by the field of study (subject area) at university. It shows that the highest percentages of students progressing from an academic programme of study were in subject areas related to Languages, in Historical and Philosophical Studies, Medicine and Dentistry and Physical Sciences. Conversely, the highest percentages of students progressing from a vocational programme of study were in Creative Arts and Design, Education, Technologies and Veterinary Sciences, Agriculture and related subjects.

Table 3: Percentages of students progressing to HE through different programmes of study, by subject area

University subject area	*Programme of study*		
	Academic	*Mixed*	*Vocational*
Architecture, Building and Planning	78.2	9.5	12.3
Biological Sciences	78.1	8.6	13.3
Business and Administrative Studies	71.0	13.8	15.2
Creative Arts and Design	68.8	10.9	20.2
Eastern, Asiatic, African, American and Australasian Languages, Literature and related subjects	97.0	2.1	0.9
Education	67.1	15.8	17.0
Engineering	82.9	7.1	10.0
European Languages, Literature and related subjects	96.4	3.5	0.1
Historical and Philosophical Studies	95.7	3.9	0.4
Law	86.0	9.3	4.7
Linguistics, Classics and related subjects	94.1	5.5	0.4
Mass Communications and Documentation	74.6	12.6	12.9
Mathematical and Computer Sciences	69.6	13.0	17.4
Medicine and Dentistry	97.8	2.2	0.0
Physical Sciences	92.4	4.7	2.9
Social Studies	85.6	7.5	6.9
Subjects Allied to Medicine	75.0	11.2	13.8
Technologies	65.5	11.5	23.0
Veterinary Sciences, Agriculture and related subjects	73.6	5.3	21.1
Other/Combined	85.1	7.8	7.1

To complement the analyses presented so far, the remainder of this section focuses on individual prior qualifications (namely, A level, IB, Cambridge Pre-U, BTEC and OCR Nationals) and shows the universities and fields of study in which they are over- or under-represented using odds ratios derived from multilevel logistic regressions.

An odds ratio represents the factor of increase in the odds of attending a university (or studying a subject) when the value of a categorical independent variable changes from the baseline to a specified category or when the value of a continuous independent variable increases by a specified unit. An odds ratio greater than 1 indicates an increase in the likelihood of attending a university (or studying a subject), with a greater odds ratio indicating a greater likelihood. Conversely, an odds ratio less than 1 indicates a decrease in the likelihood of attending a university (or studying a subject), with a smaller odds ratio indicating a smaller likelihood. Finally, an odds ratio equal to 1 indicates an equal likelihood of attending a university (or studying a subject).

Tables 4 and 5 present the odds ratios for prior qualifications in comparison to A levels. The reference group, A levels only, is not shown in the tables, as all values for the odds ratios would be 1.

Table 4 shows that students who followed a full IB programme prior to entry at university were more likely to study in a Russell Group university or in a university member of the 1994 Group than those who followed a more traditional pathway and studied A levels only (higher likelihood in a Russell Group university). On the other hand, IB students were less likely to study in universities of the Million+ Group or the University Alliance (lower likelihood in a Million+ Group university). There was a relatively small number of students progressing to university with only Cambridge Pre-U qualifications (see Table 1). However, those who progressed were much more likely to study in a Russell Group university than the students holding any other prior qualifications. Similarly to IB students, Cambridge Pre-U students were under-represented in universities of the Million+ Group or the University Alliance. The opposite pattern was found for students holding BTEC qualifications or OCR Nationals.

Table 4: Type of university – odds ratios for mainstream prior qualifications in comparison to A levels

University mission group	*Prior qualification*			
	IB	*Cambridge Pre-U*	*BTEC*	*OCR National*
Russell Group	**2.98**	**4.25**	**0.18**	**0.10**
1994 Group	**1.63**	0.37	**0.46**	**0.26**
University Alliance	**0.38**	**0.07**	**1.09**	1.15
Million +	**0.31**	0.03	**1.55**	**2.17**
Other	**0.73**	0.57	**1.21**	0.96

Note: Candidates have only the stated qualification
Note: Significant odds ratios at the 0.05 level are presented in bold type

Table 5 shows that having an Extended Project qualification alongside A levels significantly increased the probability of attending a university in the Russell or 1994 groups. Similarly, holding a Cambridge Pre-U GPR qualification alongside A levels (and also, to some extent, holding a Cambridge Pre-U qualification in a principal subject) increased significantly the probability of attending a university in the Russell Group (increases in the probability of attending a university in the 1994 Group were not statistically significant). In contrast, having an OCR National or a BTEC qualification alongside A levels decreased the likelihood of attending the more competitive universities (Russell Group and 1994 Group) but increased the likelihood of attending universities in the Million+ Group and in the University Alliance.

Tables 6 and 7 present, in the form of odds ratios, the likelihood of studying a university course in a specific subject area of a student with a non-traditional background (academic and/or vocational qualifications) compared with that of a student with a traditional academic qualification (A levels only). As above, the reference group, A levels only is not shown in the tables.

Table 5: Type of university – odds ratios for combinations of prior qualifications in comparison to A levels only

University mission group	*Prior qualifications*					
	A level + Extended Project	*A level + Cambridge Pre-U*	*A level + Cambridge Pre-U GPR*	*A level + Double A level*	*A level + BTEC*	*A level + OCR National*
Russell Group	**2.16**	**1.64**	**2.15**	-	**0.12**	**0.23**
1994 Group	**1.09**	1.24	1.45	-	**0.47**	**0.38**
University Alliance	**0.64**	**0.58**	0.67	1.56	**2.20**	**1.73**
Million +	**0.62**	**0.54**	**0.46**	2.40	1.00	**1.44**
Other	**0.82**	**0.61**	0.76	0.46	0.97	0.94

Note: Significant odds ratios at the 0.05 level are presented in bold type

Table 6 shows that students who followed a full IB programme prior to entry to university were significantly more likely to study courses in the areas of Languages and Literature than those who studied A levels only. IB students were also significantly more likely to study Medicine and Dentistry, Historical and Philosophical Studies and were also significantly more likely to study courses in the areas of Law, Physical Sciences or Social Studies. Conversely, they were significantly less likely than A level students to study courses in the subject areas of Creative Arts and Design, Education, Mathematics and Computer Sciences, Business and Administrative Studies, Engineering, Subjects Allied to Medicine, Mass Communications and Documentation and Veterinary Sciences, Agriculture and related subjects.

Table 6: Subject of study – odds ratios for mainstream prior qualifications in comparison to A levels

University subject area	*Prior qualification*			
	IB	*Cambridge Pre-U*	*BTEC*	*OCR National*
Architecture, Building and Planning	1.23	1.02	0.93	**0.26**
Biological Sciences	1.10	0.01	**1.71**	0.92
Business and Administrative Studies	**0.60**	0.35	**1.63**	**2.73**
Creative Arts and Design	**0.36**	2.60	**1.18**	**0.53**
Eastern, Asiatic, African, American and Australasian Languages, Literature and related subjects	**2.12**	-	**0.16**	-
Education	**0.48**	0.22	**2.29**	**2.18**
Engineering	**0.75**	0.30	**0.76**	**0.31**
European Languages, Literature and related subjects	**1.89**	**6.08**	**0.02**	-
Historical and Philosophical studies	**1.59**	1.39	**0.05**	**0.08**
Law	**1.39**	1.09	**0.35**	**0.45**
Linguistics, Classics and related subjects	**1.46**	**4.70**	**0.04**	-
Mass Communications and Documentation	**0.41**	-	**0.90**	**1.68**
Mathematical and Computer Sciences	**0.43**	1.29	**1.52**	**2.35**
Medicine and Dentistry	**1.70**	0.26	-	-
Physical Sciences	**1.31**	0.86	**0.30**	**0.10**
Social Studies	**1.24**	1.12	**0.66**	0.96
Subjects Allied to Medicine	**0.68**	-	**1.38**	**2.33**
Technologies	0.80	-	**1.83**	1.08
Veterinary Sciences, Agriculture and related subjects	**0.44**	-	**1.72**	0.35
Other/Combined	**1.35**	0.84	**0.72**	**0.67**

Note: Candidates have only the stated qualification
Note: Significant odds ratios at the 0.05 level are presented in bold type

There were no statistically significant differences between the students holding only Cambridge Pre-U qualifications and only A level qualifications in the majority of the subject areas. The only significant differences appeared in the European Languages, Literature and related subjects and the Linguistics, Classics and related subjects areas. In those two subject areas, Cambridge Pre-U students were much more likely than A level students to be pursuing a course.

Regarding the vocational qualifications (BTECs or OCR Nationals) the most extreme differences were found in some academic subject areas such as Languages, Historical and Philosophical Studies or Physical Sciences. For example, the likelihood of someone holding just BTEC qualifications entering a course in the area of European Languages was more than 50 times lower than for a student with A levels. BTEC and OCR National students were also significantly less likely than A level students to study in the areas of Engineering, Law or Social Studies. However, these vocational students were over-represented in Biological Sciences, Creative Arts and Design (BTEC students only), Business and Administrative Studies, Mathematical and Computer Sciences, Education and Subjects Allied to Medicine. It should be noted that the majority of these university subject areas correspond with BTEC sectors and/or OCR National subjects and therefore it is not surprising that students with these prior qualifications were more attracted to them.

Table 7 shows that students holding an Extended Project qualification alongside their A levels were more likely to study Medicine and Dentistry than students without it. It could be the case that in competitive courses such as these, the Extended Project had been used to differentiate among very high achieving candidates at A level. These students were also significantly more likely to study a degree in the following subject areas: European Languages, Literature and related subjects, Historical and Philosophical Studies, Linguistics, Classics and related subjects, Law, Physical Sciences and Veterinary Sciences, Agriculture and related subjects.

Students holding Cambridge Pre-U qualifications alongside A levels were significantly more likely to study courses in the areas of Languages and Literature than those who studied A levels only. Students with A levels and Cambridge Pre-U qualifications were also over-represented in Historical and Philosophical Studies and Creative Arts and Design. In contrast, they were less likely than students holding A levels only to pursue courses in the subject areas of Biological Sciences, Medicine and Dentistry, Engineering, Physical Sciences, Subjects Allied to Medicine and Veterinary Sciences, Agriculture and related subjects. It is worth pointing out here that the choice of university degree might also depend on the subject of the prior qualification and, in the case of the Cambridge Pre-U,

Table 7: Subject of study – odds ratios for combinations of prior qualifications in comparison to A levels only

University subject area	*Prior qualifications*				
	A level + Extended Project	*A level + Cambridge Pre-U*	*A level + Cambridge Pre-U GPR*	*A level + BTEC*	*A level + OCR National*
Architecture, Building and Planning	0.93	0.96	2.33	1.14	0.98
Biological Sciences	1.05	**0.58**	1.21	**1.11**	**0.75**
Business and Administrative Studies	**0.57**	0.71	**0.40**	**1.83**	**1.85**
Creative Arts and Design	**0.61**	**1.80**	0.93	**1.65**	0.90
Eastern, Asiatic, African, American and Australasian Languages, Literature and related subjects	1.05	**2.56**	4.02	**0.09**	**0.43**
Education	**0.65**	0.52	0.33	**1.75**	**1.84**
Engineering	**0.87**	**0.40**	0.46	**0.56**	**0.73**
European Languages, Literature and related subjects	**1.18**	**2.74**	0.35	**0.31**	**0.31**
Historical and Philosophical studies	**1.61**	**1.45**	**2.51**	**0.29**	**0.44**
Law	**1.39**	0.83	0.51	**0.75**	0.79
Linguistics, Classics and related subjects	**1.58**	**2.02**	1.34	**0.34**	**0.60**
Mass Communications and Documentation	**0.78**	0.67	0.88	**1.28**	**1.74**
Mathematical and Computer Sciences	**0.83**	0.79	0.37	1.12	**2.58**
Medicine and Dentistry	**2.25**	**0.61**	2.12	**0.03**	**0.08**
Physical Sciences	**1.16**	**0.52**	1.39	**0.31**	**0.38**
Social Studies	1.01	1.06	1.14	**0.71**	**0.79**
Subjects Allied to Medicine	**0.75**	**0.29**	0.67	**0.87**	0.94
Technologies	**0.63**	0.48	1.33	**1.99**	1.02
Veterinary Sciences, Agriculture and related subjects	**1.40**	**0.12**	-	1.04	0.62
Other/Combined	**1.10**	**1.53**	1.07	0.92	**0.74**

Note: Significant odds ratios at the 0.05 level are presented in bold type

the most popular principal subjects in the June 2012 examination series (DfE, 2013c) were Literature in English, History, Mathematics, French, Economics and Philosophy and Theology, which supports the relationships reported above.

There were no statistically significant differences between the students holding A levels and Cambridge Pre-U GPR qualifications and those holding only A level qualifications in the majority of the subject areas. The only significant differences appeared in the areas of Historical and Philosophical Studies and Business and Administration Studies.

For students holding A levels and one of the vocational qualifications (BTECs or OCR Nationals) the most extreme differences were found in some academic subject areas such as the Languages, Historical and Philosophical Studies or Physical Sciences. For example, the likelihood of a student holding a BTEC alongside the A levels entering a course in the area of European Languages was around 4 times lower than for a student with A levels only. Students with BTEC and OCR Nationals alongside A levels were also significantly less likely to study in the areas of Engineering, Law, Medicine and Dentistry or Subjects Allied to Medicine than students with only academic qualifications. However, students holding BTECs or OCR Nationals alongside their A levels were significantly more likely to study for a degree in Biological Sciences (BTEC students only), Creative Arts and Design, Business and Administrative Studies, Education, Mass Communications and Documentation, Mathematical and Computer Sciences (OCR National students only), Social Studies and Technologies. These subject areas also attracted students with BTECs and OCR Nationals only (see Table 6) so it seems that when a student has a combination of A levels and vocational qualifications, the latter might be driving the choice of subject at university.

Note that the odds ratios for the combination of A levels and Double A levels were not included in Table 7, as there were no statistically significant differences with A levels only, and in many of the subject areas there was not enough data to allow for comparison.

Conclusions and discussion

In a rapidly evolving qualifications system it is crucial to better understand how qualifications, both academic and vocational, are used by students for progression, in particular to HE. This article aimed to provide quantitative evidence to show how different types of qualifications and combinations of qualifications channelled learners in particular directions. Note that the nature of this study does not allow drawing causal relationships between specific qualifications and students' participation in HE, as there might be other factors not included in the analyses that have a direct impact on progression to a HE institution or field of study.

We considered data covering first year undergraduates aged 17–19, domiciled in England, studying at UK universities in the 2011/12 academic year. Considering data on undergraduates did not allow us to study the determinants of progression to HE, but enabled us to focus on university participation in terms of institution attended and subject chosen for the students who did progress. It was not possible to identify, for example, whether students with vocational qualifications failed to apply to prestigious or highly selective institutions or whether they applied but were not accepted. Furthermore, it should be taken into account that the data on prior learning provided information about the qualifications achieved by students who accessed HE but not about the qualifications actually required by the HE institution.

Overall, the current research showed that prior qualifications, and combinations of prior qualifications, are represented in different proportions in HE and particularly in the different institution types and fields of study. This could be cause for concern as the prior learning of the students might steer them towards universities and courses that could bring fewer economic benefits or provide a disadvantage in the labour market. For example, Chevalier and Conlon (2003) reported that prestigious institutions provide higher financial returns to their graduates; specifically, even after accounting for personal characteristics, graduating from a Russell Group institution adds between 0% and 6% to a graduate's earnings compared to graduating from a modern university, for example those in the University Alliance or Million+ Group. More recent studies (e.g., Bratti, Naylor & Smith, 2005; Walker & Zhu, 2005; Greenwood, Harrison & Vignoles, 2011; Walker & Zhu, 2011) showed that there is a large heterogeneity in wages by degree subject area with Health, Science and Social Sciences graduates earning more than Humanities, Education and Arts graduates. Furthermore, Walker and Zhu (2013) revealed that after certain controls were factored in, male graduates of Russell Group universities earn 3% more than students in post-1992[7] universities.

In more detail, the outcomes of this research showed that AS and A levels were the most popular mainstream qualifications held by undergraduates at HE institutions. In fact, in the academic year 2004/05 the vast majority of university entrants (almost 81%) held AS/A level qualifications (Connor, Banerji & Sinclair, 2006) and our research showed that just below 86% of the students starting in 2011/12 did so as well. However, the percentage of full-time entrants holding A levels only has been decreasing in the last few years (UCAS, 2012) and, as shown in this research, it reached 28% in 2011/12.

The highest percentages of students with A levels were in universities of the Russell Group, where high A level grades usually dominate entry requirements. Furthermore, students with other academic qualifications (e.g., Cambridge Pre-U or IB) were also more likely to go to Russell Group or 1994 Group universities than to other types of universities. Previous research (HESA, 2011) has already shown that students with an IB Diploma were more likely than A level students to study at high ranking institutions.

Regarding some of the recently introduced academic qualifications, this research showed that having an Extended Project qualification or a Cambridge Pre-U GPR qualification alongside AS/A levels significantly increased the probability of attending a university in the Russell or 1994 groups. These qualifications, which require research and autonomous working, have been praised by universities, especially competitive ones, as they allow the development of independent research skills needed for undergraduate study. It is therefore not surprising that they provide 'better' access to competitive universities.

On the topic of the field of study in the HE institutions, the highest percentages of students with A levels were in subject areas related to Languages, Historical and Philosophical Studies, Medicine and Dentistry and Physical Sciences. This finding was supported by research carried out by Connor et al. (2006), who found that A levels were over-represented in the Humanities and also in Physical Science, Law and Social Studies. Similarly, Hoelscher et al. (2008) reported that the likelihood of a student with a traditional academic background (A levels) studying Medicine and Dentistry was more than 25 times higher than that for a student with other types of qualifications.

7. The *Further and Higher Education Act 1992* ended the divide between universities and polytechnics in the UK. The former are known as pre-1992 HE institutions; the polytechnics and those more recently obtaining degree-awarding powers, such as colleges of HE, are named as post-1992 HE institutions (Hayward & Hoelscher, 2011).

Similarly to A level students, those holding Cambridge Pre-U (Principal Subject) qualifications or an IB Diploma were more likely to study Languages, Historical and Philosophical Studies, Linguistics, Classics and related subjects and Social Studies, and less likely than average to study Science subjects, with the exception of Medicine and Dentistry. IB and Cambridge Pre-U students were particularly unlikely to study Creative Arts and Design or Mathematical and Computer Sciences. However, the university subject choices of these students may reflect patterns of uptake or provision of Cambridge Pre-U or IB subjects in schools.

On the topic of progression from vocational backgrounds, previous research by Connor et al. (2006) showed that there was an uneven distribution of vocationally qualified entrants to full-time degree courses across HE institutions and they represented a relatively small proportion of the overall intake. The latter seems to be changing, with percentages of university students having followed vocational and mixed programmes of study growing in the last few years (see, for example, Hayward and Hoelscher (2011) or UCAS (2013)).

BTEC qualifications, particularly the BTEC Diplomas, have become valued and respected qualifications and are a popular option within Further Education (FE) and HE. In fact, the current research has shown that BTECs are the second most popular qualification held by undergraduates at HE institutions in the UK. However, its popularity varied by type of institution and subject.

Regarding the type of institution, our research showed that the highest percentages of students with BTEC qualifications were in universities of the Million+ Group, followed by universities in the University Alliance. As expected, the lowest percentages of students with these qualifications were in universities of the Russell Group. Those findings support previous studies by Schwartz (2004) and Hoelscher et al. (2008), among others, who have shown that students with non-traditional qualifications typically progressed to post-1992 institutions and colleges of HE. This could be partly related to the type of courses offered by each group of institutions. In fact, Carter (2009) argued that vocational progression routes are often best developed in the newer parts of the HE sector. Many post-1992 universities, FE and HE colleges have rich experience in developing learning programmes and recruitment procedures that are tailored to the needs of vocational learners. This research confirms somewhat the above argument as the most popular destinations for candidates with vocational qualifications, and in particular BTECs, were HE institutions in the University Alliance and Million+ Group, which are constituted by the newest universities and colleges.

Regarding the subject of study, and as pointed out previously by the Higher Education Funding Council for England (HEFCE) (2007), the most popular fields of study at university for students with BTECs were closely aligned to BTEC subject areas.

Similarly to students holding BTEC qualifications, students with other vocational qualifications (OCR Nationals and Double Award AS/A levels) were found to be more likely to attend a University Alliance or Million+ institution than other types of institutions.

The results in this research support the hypothesis that students with more academic backgrounds are more likely to go to universities in the Russell and 1994 groups, and those holding vocational qualifications are more likely to study in other types of universities (e.g., universities in the University Alliance or in the Million+ Group). One reason for this could be that the more applied/vocational subjects are over-represented in some types of institutions and, for example, candidates with vocational backgrounds are more attracted to those types of subjects. Therefore their university choices are determined by their subject choices. In fact, vocational students, who usually come from 'average' or 'more deprived' backgrounds, might be taking applied or more vocational subjects (e.g., Business and Administrative Studies, Law and Mathematics and Computer Science, or Subjects Allied to Medicine) at HE to allow them to quickly (i.e., soon after graduation) join the labour market.

This research has not looked at work based learning (WBL) and the progression to HE of learners with WBL qualifications, such as apprenticeships. One of the reasons for this relates to the fact that quantitative information or uptake data on these types of qualifications is difficult to access and in many cases it is incomplete (Seddon, 2005).

Recent policy developments and further research

The research reported in this article is not longitudinal but is rather a snapshot of the distribution of prior qualifications in HE in one academic year, 2011/12. However, during recent years there have been many changes in education and assessment in England, particularly relating to Level 3 qualifications and university admissions policies, which could potentially have an impact on the current situation.

Firstly, new qualifications have been introduced at Level 3 that aim to prepare learners for study at university, some qualifications have been withdrawn and other qualifications are being comprehensively reformed. The uptake of these qualifications will probably fluctuate and therefore patterns of entry to university of undergraduates holding them could also vary in the next few years.

Secondly, the uptake of vocational qualifications had increased in the years previous to our research. However, as a result of the *Wolf Review of Vocational Education* (Wolf, 2011), the government announced a reform to performance tables to remove the 'perverse incentives' which could have pushed young people into qualification routes that did not allow them to progress into FE. Following this decision, some vocational qualifications were removed from the league tables and others were reviewed. There are also plans to raise the status of vocational courses in sixth forms and colleges in England with the introduction of a Technical Baccalaureate (DfE, 2013b). This qualification will be taught at a level of difficulty meant to show that pupils are able to carry out 'complex and non-routine' skills, on a par with A levels and will become a league table performance measure from 2017. Those two reforms may have an impact on the provision and uptake of vocational qualifications in schools in the coming years and therefore on the distribution of students with these qualifications in HE institutions and subjects.

Thirdly, from the academic year 2012/13, students attending universities in the UK have been charged higher university tuition fees. The cost of each individual course is decided by the university offering it but, while it was originally claimed that £9,000 was the maximum amount universities could charge and that very few would decide to go that high, over a third of universities are charging the full amount. It has been claimed by Universities UK[8] that this could affect the governments' commitment to increasing social mobility, and students from low socio-economic backgrounds might find themselves with restricted options.

Similarly, a report by the Higher Education Policy Institute concluded that changes to tuition fees will make it far more difficult for bright students from poorer backgrounds to attend Britain's elite universities (Thompson and Bekhradnia, 2011). However, a report from HEFCE (2013) looking into the impact of the 2012 reforms in HE has shown that 'the current evidence suggests that the reforms have not made young people from disadvantaged areas less likely to study full time'. This report also found indications that students from all backgrounds are more likely to choose courses in clinical subjects and Science, Technology, Engineering and Mathematics and less likely to choose Arts, Humanities or Social Science courses, potentially due to their higher returns. Therefore, the changes in university tuition fees might be causing shifts in entries to particular groups of subjects.

In the light of the current changes, it would be advisable to replicate this research in a few years' time to explore how HE choices have changed and which factors have impacted on those choices. Furthermore, if data on performance prior to university were available, analyses could also control for prior achievements at Level 3 (or before) and comparisons between students who have similar 'academic ability' could be drawn.

8. Universities UK is the voice for universities in the UK. It has 134 members which include virtually all the universities in the UK and some colleges of HE.

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Teachers' and employers' views on the transition from GCSE Mathematics to A level Mathematics or employment

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Introduction

The General Certificate of Secondary Education (GCSE) is a British qualification taken by 16-year-olds at the end of compulsory formal schooling in England, Wales and Northern Ireland. Whilst students must continue with some form of education beyond this, it does not have to be full-time education, nor must it continue within a school or college. Therefore, the qualification marks a transition, and the results from it may be used as entry requirements for further study and employment.

Mathematics is one of the core GCSE subjects, and students are required to study the subject until the end of Key Stage 4 (KS4), when they are approximately aged 16. There is no requirement for students to take a qualification in Mathematics, but almost all students do. In 2011/12, 97.3 per cent of students at the end of KS4 took a qualification in Mathematics; 93.7 per cent of the KS4 students took a GCSE in the subject (Gill, 2013a).

GCSE Mathematics is important because it represents the end of students' compulsory Mathematics learning. Despite suggestions that all students should continue studying Mathematics beyond this point, it is currently only a requirement for those students who have not 'passed' GCSE Mathematics (i.e., those who have not gained a grade C or above) to continue studying to do so. Therefore, the qualification needs to accurately assess students' competence in Mathematics. It is also a requirement for continuing to study Mathematics at A level, so needs to reflect the skills that are necessary for starting that qualification. Finally, it may be necessary for studying Science subjects at A level; a good grade in GCSE Mathematics is often required if students wish to take A level Physics (Gill & Bell, 2013).

Despite being required for entry to further study and employment, there is a history of research that identifies problems in using GCSE Mathematics for both purposes. The transition to A level is seen to be problematic (Mendick, 2008; Hernandez-Martinez et al., 2011). Many students who start A level courses drop out during their first year or after their AS results (Mendick, 2008; Noyes & Sealey, 2012) and there is widely believed to be a gap between the Mathematics that is necessary to pass a GCSE and the Mathematics that students need to be able to do to start A level (Brown, Brown & Bibby, 2008; Noyes & Sealey, 2011). Schools have adopted two approaches in order to combat the gap and avoid high drop-out rates. Most schools require high grades for entry onto A level Mathematics courses. Students often have to have achieved a grade B at GCSE or even a grade A in order to be accepted for the course (Mendick, 2008; Noyes & Sealey, 2012; Hernandez-Martinez et al., 2011). The grades required for Mathematics may be higher than those used for other subjects (Mendick, 2008). Additionally, some schools run extra courses, or set work for students to complete between finishing GCSEs and starting A levels so that their Mathematics is of the standard that is needed for the A level course (Noyes & Sealey, 2011).

Whilst the notion of a gap between GCSE and A level Mathematics appears to be widely acknowledged, there is little research that has investigated what the nature of the gap is. Instead, most of the research focuses on students' reasons for dropping out (e.g., Noyes & Sealey, 2012; Mendick, 2008). A few studies do mention problematic content areas. Algebra and algebraic manipulation skills are identified as being problematic for students at the start of A level (e.g., Wiliam, Brown, Kerslake, Martin & Neill, 1999; Noyes & Sealey, 2011; Hernandez-Martinez et al., 2011). The range of dates from studies that identify this issue suggests that this is not a new problem with GCSE Mathematics. Trigonometry was also identified as being problematic in one study (Wiliam et al., 1999), but as it was carried out on courses that date back almost 20 years, these findings may no longer be relevant.

GCSE Mathematics is also important for employment. The majority of students entering employment will not have studied Mathematics beyond GCSE. Many students do not take A levels – in 2013, approximately 60 per cent of students did not study for any A levels[1]. Even amongst those studying A levels, only 12.4 per cent took A level Mathematics in 2013 (Department for Education [DfE] & Truss, 2013). Employers are known to value mathematical skills even for non-numeric jobs (Confederation of British Industry [CBI], 2013; UK Commission for Employment and Skills [UKCES], 2012; Advisory Committee on Mathematics Education [ACME], 2011; CBI, 2010). Numeracy, which is defined by the CBI as "confidence with the handling of numbers, general mathematical awareness and its application in practical contexts" (CBI, 2010, p.2), is part of GCSE Mathematics. Employers are particularly interested in potential recruits' numeracy levels: 50 per cent of employers consider numeracy levels as part of the recruitment process when employing school and college leavers (CBI, 2013). Therefore, it is important that GCSE Mathematics prepares students adequately, particularly in areas of numeracy.

A large scale survey showed that 56 per cent of employers think that five A* to C GCSE passes, including English and Mathematics, are the best indication of numeracy skills needed in the workplace (CBI, 2012). However, there is a perceived issue with the level of numeracy required by school-leavers for employment (CBI, 2013). Employers are dissatisfied with numeracy skills and some have to provide further training in this area (CBI, 2010; CBI, 2012). This may be caused by a difference between the Mathematics that is taught in schools and the numeracy skills that are required by employers; there is concern that employees are not good at applying the Mathematics skills that they learnt at school (ACME, 2011). However, some employers believe that the gap between completing GCSE Mathematics and entering employment also contributes to poor numeracy skills (ACME, 2011; CBI, 2010). This problem may be compounded by early GCSE entries, as the time between studying Mathematics and entering employment would be larger. Gill (2014a) found that almost 38 per cent of students entered at least one GCSE early (in Year 10). Whilst it was not found to affect students' chances of obtaining a grade C, it may have affected their ability to perform to their potential (Gill, 2013b).

The CBI carried out a survey in 2010 that focused upon employers' views on numeracy levels, and identified that employers had concerns about employees' numeracy skills. They identified the following areas as necessary for the workplace (CBI, 2010):

- Carrying out mental arithmetic (without using a calculator)
- Interpreting and responding to quantitative data
- Calculating percentages and interpreting their significance
- Working comfortably with fractions, decimals and ratios (the ability to use a formula is also 'highly desirable')
- Awareness of different measures and converting between them
- Checking potentially rogue results and calculation errors
- Having a basic understanding of odds and probabilities.

The current study aimed to build on the work that had been carried out previously, and to identify the areas of Mathematics that were problematic for students who had just completed GCSE Mathematics. It also aimed to discover whether there was any overlap in the skills that were considered to be problematic as preparation for A level and those considered to be problematic as preparation for employment. It uses responses from a larger survey of teachers and employers to consider three research questions:

1. What areas of Mathematics are GCSE students well/poorly prepared in?
2. What teaching is needed to bring students up to the standard for starting A level Mathematics?
3. What Mathematics training do employers run for school leavers?

Method

As part of the Oxford, Cambridge and RSA (OCR) Examinations Mathematics GCSE redevelopment work, stakeholders including schools/colleges and employers, were consulted about various issues affecting GCSE Mathematics. Researchers within the Research Division at Cambridge Assessment developed two questionnaires for this consultation work: one for schools/colleges, the other for employers. The researchers worked with the OCR Mathematics Redevelopment Team to identify and prioritise the issues and research questions that were considered useful for redeveloping the qualification. Questions were then drafted and reviewed by the researchers. The completed questionnaires were piloted by ten Mathematics teachers and five employers. A final draft of each questionnaire was then put into a web-based format for online completion. The teachers' questionnaire was sent to 2,085 schools and colleges offering OCR A level Mathematics qualifications. Participants in the employers' questionnaire were recruited by personal and institutional links, targeting of key professional roles, and snowball sampling (asking participants to pass the questionnaire onto colleagues in other organisations.) A total of 143 questionnaires were distributed to employers, of which 35 were returned.

Four questions from the teachers' questionnaire were considered to be useful for this study:

1. How well does GCSE/Cambridge International General Certificate of Secondary Education (IGCSE®) Mathematics prepare your students for A level/Advanced Subsidiary (AS) level Mathematics in terms of the following [16 areas of Mathematics were listed]
2. Which of the above areas (or other areas) would benefit from greater emphasis or greater depth at Key Stage 4 to aid progression to A/AS level Mathematics?
3. Are extra/recap lessons needed at the start of A level to bring students up to the level required?
4. If so, what content do they cover?

1. No published statistic is available on this. In mid-2013, the Office for National Statistics (ONS) (2015) estimated there to be 650,210 18-year-olds. Gill (2014b) reports that 260, 087 Year 13 students (i.e., students aged 18) took at least one A level in 2013. Using these two figures shows that approximately 60% of 18 year olds did not take any A levels in 2013.

Four questions from the employers' questionnaire were considered to be useful for this study:

1. What GCSE grade do you think provides good evidence that new employees have sufficient levels of Mathematics skills to work confidently, effectively and independently?
2. How useful would you consider the following mathematical skills and content to be for new employees? [12 areas of Mathematics were listed[2]]
3. Are there any specific skills that you think new employees lack when they have completed a GCSE Mathematics course and which are key to their successful transition from education to employment?
4. What is the purpose of any training that you currently provide for new employees who have already completed a GCSE Mathematics course?

Findings

The teachers' questionnaire was emailed to 2,085 schools and colleges offering OCR A level Mathematics qualifications. One hundred and seventy-nine schools responded; a response rate of 8.6 per cent. Responses were received from a range of different school types (see Table 1), although they were not representative of the proportions of each school type found nationally.

Table 1: Institution types within the questionnaire sample

Institution type	*Count*	*Percentage of responses*
Comprehensive	79	44.4
FE Institution	19	10.7
Independent	34	19.1
Secondary Selective	16	9.0
Sixth Form College	16	9.0
Other (please specify)	14	7.9

The employer questionnaire was sent out to 143 employers. Thirty-five employers responded; a response rate of 24.5 per cent. The majority (54 per cent) came from businesses that employed more than 250 employees, and they represented a wide range of employment sectors.

Teachers' questionnaire

Teachers considered that students were prepared adequately for AS/A level courses in most areas of Mathematics. However, several areas were identified where GCSEs were considered not to prepare students well. These were:

- Proof (68%)
- Unstructured problem solving (54%)
- Familiarity with other technology (47%)
- Algebraic fluency (44%).

In contrast, few teachers thought that students were unprepared in five areas:

- Sequences and patterns (12.4%)
- Appropriate uses of calculator (9.6%)
- Geometry (9.6%)
- Data handling (8.5%)
- Measures (5.1%).

When asked which areas would benefit from being taught in greater depth in KS4, a large number of responses were provided. The most frequent comments related to algebra or algebraic fluency (61%). For example:

> *Algebraic understanding. The GCSE exam requires very little understanding to gain the top grades and thus the issue at A level.*
>
> *I suggest all of them but in particular algebra and multistep problem solving. You can now get a B with very little algebra this is unacceptable...*
>
> *Greater fluency in algebra from grades C upwards (as this is our minimum entry requirement for AS level) – current boundaries allow students to gain C or B grades without actually having solid skills in algebra, which is insufficient for AS transition.*

However, two respondents were concerned about the effect that the increased emphasis on algebra would have on those students who were less strong or not intending to continue to study Mathematics. For example:

> *More emphasis on algebra would aid progression, but hinder those not wishing to proceed to A/AS.*
>
> *Algebraic manipulation – but then this would disadvantage those students who are not ace mathematicians.*

Other areas which were identified by ten or more respondents included: problem solving (18%); proof (18%); functions and graphs (18%); coordinate geometry (12%); and surds (7%).

> *...There also needs to be more emphasis on core skills such as surds and indices.*
>
> *Whilst I realise that Mathematics is a developing subject, an inclusion of a greater level of formal geometry would certainly help. It gives the students a better understanding of the concept of proof.*

The majority of respondents (86%) offered extra lessons for some or all of their students at the start of A levels (see Figure 1).

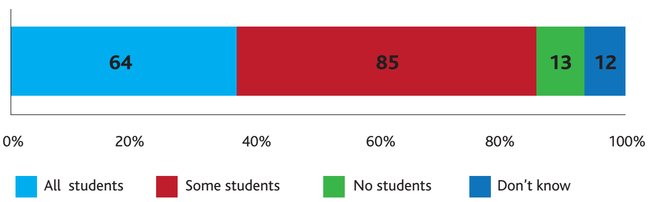

Figure 1: Extra lessons required at the start of A level Mathematics

Many of the respondents (130) provided detail of the content of these lessons, indicating that a variety of topics were covered. Most of these respondents focussed the sessions on particular knowledge and skills,

2. These areas of Mathematics were different to the ones listed in the teachers' questionnaire for aiding progression to KS4.

although a few (2%) tailored help for individual students. In general, the most common areas addressed in these sessions were:

- Algebra (80%)
- Surds (26%)
- Quadratics (17%)
- Graphs and functions (15%)
- Indices (14%)
- Solving equations (12%[3]).

Most of the respondents indicated that they taught generic algebra skills, but some of them focused on particular areas of algebra that students found difficult. For example:

> *General algebraic manipulation and the solving of linear, quadratic and simultaneous equations.*
>
> *Factorising. Expanding brackets. Simultaneous equations.*
>
> *Algebra, algebra and algebra, and the basic concepts that are connected to sim[ultaneous] equations, factorising, dealing with brackets either way – geometry a bit too.*

Other respondents mentioned areas of Mathematics that were needed for B grade GCSE students who were studying AS/A level Mathematics. For example:

> *Algebra is needed for the lower ability students who take on A level with a grade B at GCSE.*
>
> *Fractions, indices, surds, quadratics, simultaneous equations. It's not that this isn't covered in Core 1, but that the grade B students don't pick it up quickly enough to apply skills to work on coordinate geometry and calculus.*
>
> *The B grade students are encouraged to attend some extra classes after the exams. The "old money" intermediate students. We mainly cover the algebra topics such as quadratics, sketching graphs, we cover some trig and indices and surds.*

However, even the top grade GCSE students at some schools were given extra help. For example:

> *We re-do all the algebraic work and the surd work. We have found the A* target students are typically at an E grade of understanding if we do not do this.*
>
> *Some need it because they can achieve an A at GCSE with 65 per cent and little algebraic ability. They think they are good at Maths but not surprisingly bomb at A level because it is so algebraic.*

One respondent suggested that it was the gap between GCSE and AS that caused the problem, rather than the content of the GCSE course or the students' GCSE grades.

> *This is more because of the long time gap between them finishing lessons in Year 11 (before study leave and exams) and then coming back in 6th form. We cover basics of algebra, algebraic fractions, solving equations etc., surds, coordinate geometry.*

Another suggested that it was due to a lack of top level material on the GCSE exam papers.

> *We prepare our students very well for the GCSE exam, however there is not enough of the A/A* material on the exam. Our students get very good results and as a consequence think they are better at Maths than they actually are. A few students then decide to take A-Level Maths and are not really up to it!*

Employers' questionnaire

The majority of the employers who responded to the survey felt that a grade B in GCSE Mathematics provided good evidence that new employees could work confidently, effectively and independently (see Table 2). Very few required a higher grade at GCSE, but some indicated that a C would be adequate. Whilst there was an opportunity to choose grades below C, none of the employers chose these grades.

Table 2: GCSE grade providing good evidence of Mathematics skills to work confidently, effectively and independently

*A**	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*Don't know*	*None of the above*
8.6%	8.6%	42.9%	22.9%	0%	0%	0%	0%	2.9%	14.3%
3	3	15	8	0	0	0	0	1	5

The employers felt that almost all of the mathematical skills they were asked about were useful for new employees to have. The most useful skills were:

- Effective and appropriate use of ICT packages (spreadsheets, charts) (97%)
- Proficiency with quantity and number (97%)
- Ability to understand the principles behind calculations (94%)
- Ability to make meaningful estimates (94%)
- Using diagrams, charts and tables (91%).

Employers responded that the least useful of all the skills listed was use of symbolic notation, but even that skill was regarded as useful or very useful by 51 per cent of employers.

Very few employers responded to the question about skills that new employees lacked, so it was not possible to draw firm conclusions about the skills that were lacking but considered important for employment. Three areas were included by more than one respondent: mental arithmetic and basic skills; understanding the magnitude of numbers; and using Mathematics in context:

> *Basic Maths skills – times tables especially.* (Real Estate Activities sector employer).
>
> *Non-technical staff I would expect a basic level of arithmetic.* (Professional, Scientific and Technical Activities sector employer).
>
> *Having a feel for numbers and understanding what the calculations mean.* (Aerospace, Manufacture and Design sector employer).

Despite the low number of respondents identifying problematic skills, approximately half of the respondents said that they provided some form of Mathematics training for new employees. This was slightly less likely to be remedial training than job specific training, or training which built on existing knowledge (see Figure 2).

3. Whilst quadratics and solving equations are part of algebra, they were coded separately as many respondents listed them as separate topics.

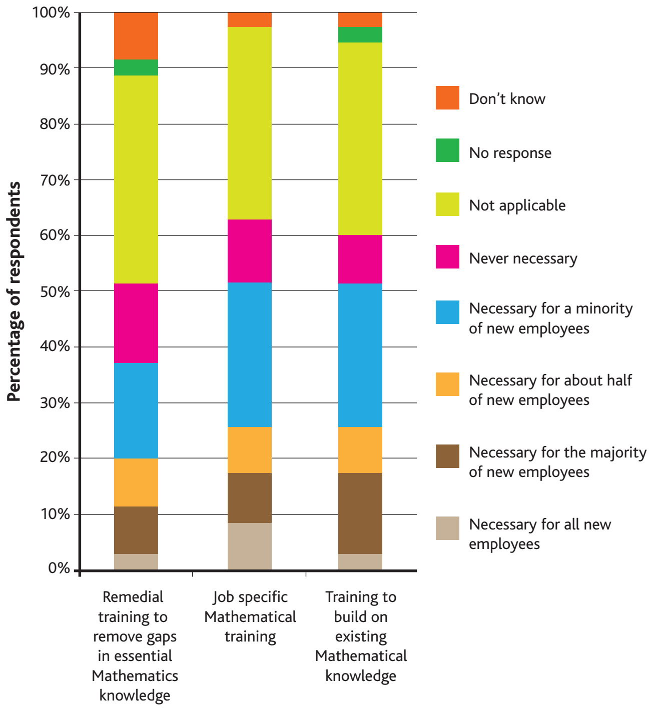

Figure 2: Purpose of extra Mathematics training provided to new employees with GCSE Mathematics qualification

Discussion

The clearest finding from the teachers' survey was that students' algebraic skills were considered weak by the majority of teachers, and students were considered to be underprepared in this area. These results confirm the findings from earlier research, which identified algebra skills as problematic for students beginning A/AS level Mathematics courses (e.g. Wiliam et al., 1999; Noyes & Sealey, 2011), and suggests that it is an on-going problem. Teachers were not asked why this area was problematic, but a few teachers made suggestions when answering other questions. One said that it might be the length of time between finishing GCSE examinations and starting A/AS level courses that caused the problem. Other respondents suggested that it might be because there was relatively little content related to this on the examination papers and that this meant that students were able to get the grades necessary to start A level study without understanding the more advanced GCSE algebra content.

Currently, many of the respondents compensate for the weakness in algebra skills by including them in extra lessons or work that is given to students before the start of their A level courses. However, a large number of them thought that algebraic skills would benefit from being taught in greater depth at KS4. Including more of this content may cause a problem, as the GCSE course also has to be suitable for learners aiming for lower grades, or those students not planning on continuing to study Mathematics post-16. Several of the teachers were concerned about the effects on these learners if specifications (and examination papers) contained a greater emphasis on algebra.

The tension between amending specifications to provide better preparation for A level and ensuring that they were appropriate for students with lower grades who were not intending to continue can also be seen in other problematic areas of content. Many teachers included work on surds and indices in their extra lessons, and they also suggested that students were poorly prepared in proofs. All these skills are important for A level, but they are less relevant for other students. Increasing the emphasis on these areas may mean that other topics that are important for employment are not given the emphasis that they require.

Responses to the employers' survey suggested that an emphasis on a very different set of skills is needed if GCSEs are to prepare students better for the workplace. Employers valued proficiency with number, understanding the principles behind calculations and making meaningful estimates. All these skills are much more closely aligned with the numeracy skills that previous research (e.g. CBI, 2010) has shown that employers value.

The only areas that both teachers and employers identified as weak were students' skills using ICT/Other Technology, and their ability to interpret graphs, functions, charts and tables. The differences between employers' and teachers' views may have arisen because different areas of Mathematics are necessary for progression to further study and employment. Many jobs require numeracy, indeed the CBI (2010) reports that employers want all of their employees to be numerate. However, algebra skills are less commonly required in workplaces, particularly in roles which do not require qualifications in Mathematics beyond GCSE level. In contrast, A/AS level Mathematics courses rely on good algebra skills more than they do upon good numeracy skills.

Approximately half of the employers responding to the questionnaire provided some form of Mathematics training for new employees. This is higher than the 18 per cent reported in the CBI (2010) numeracy survey, but this could be due to the low number and less representative nature of the employers responding to the questionnaire.

Limitations

The response rate for the teachers' questionnaire was low, although it was comparable to similar questionnaires which were sent out to English and Science teachers. This means that the responses cannot be generalised to a broader teaching population. However, given the limited literature that is available in this area, it provides a starting point for researchers, qualifications developers and the teaching community to understand areas of GCSE Mathematics that may be problematic for students.

Whilst the response rates for employers were higher, they still only represent a very small proportion of the employers within England and therefore it is also necessary to be cautious about making inferences from a limited sample. However, the results do add to the findings from the large scale surveys that have been carried out and enable researchers and qualifications developers to see whether the findings from the older studies are still relevant.

Conclusion

This study has shown that there are areas of Mathematics in which the transition from GCSE to A level Mathematics is problematic. These areas generally correspond to the ones that have been identified within previous studies. It has also found that employers also think that some areas of Mathematics are not being covered thoroughly enough at GCSE. Generally, the areas that employers are concerned about are different to those that are considered to be problematic for the transition to A level. Any additional content which would enable GCSEs to prepare students better for A levels is likely to mean less emphasis on an area that employers consider to be important and vice versa. Therefore, whilst this study has shown that there is support for amending the GCSE content, and has identified the content areas that should be considered, it is not possible to ascertain which of these areas should be covered in greater detail.

Postscript

Since this work was carried out, the updated subject content and assessment objectives for GCSE Mathematics have been published (see DfE, 2013). This document contains the detail that all awarding bodies need to include in their specifications for the reformed GCSEs, although additional content can be added by awarding bodies to increase the breadth and depth of their qualifications.

Several areas of subject content have been added to the specification, including extra algebra at both the Higher and Foundation tier, additional work on graphs, and extra number skills which should increase students' proficiency when working with quantities. However, the new content does not necessarily cover the entirety of the areas that had been identified as problematic. There may be other areas of these topics which teachers and employers would have liked to be included, but which were not. Furthermore, the inclusion of topics does not ensure that all students are taught these topics, since some are only included for the Higher tier. Students who are not likely to enter the Higher tier are unlikely to study this content.

There are also several areas identified in the study, such as tables and ICT skills, which appear not to have received any extra content. Awarding bodies may include these areas in the extra breadth and/or depth that they are allowed to add within their specifications, but this will not necessarily happen. It is likely that students will continue to be underprepared in these areas when the new GCSE specifications are taught.

In January 2014 a new "Core Mathematics" qualification was announced. This qualification is a Level 3 qualification, and is targeted at students who have achieved a grade C in GCSE Mathematics. It aims to "provide a sound basis for the mathematical demands that students will face at university and within employment across a broad range of academic, professional and technical fields" (DfE, 2014a, p8), by assessing students' skills in applying GCSE Higher tier Mathematics content to authentic situations, and developing further mathematical skills and knowledge.

A level Mathematics is also being redeveloped for first teaching in September 2016. The new subject content and assessment objectives for A level Mathematics (DfE, 2014b) were published in December 2014, after this analysis had been carried out. Several new areas of content have been added. The algebra content has been expanded, and some areas that were originally only included at Foundation level are now included at Higher level as well. There is also a more content for graphs, functions and charts, some of which overlaps with algebra. Within the areas that employers identified, there are more numeric proficiency skills. Making meaningful estimates is also covered in greater depth. However, the new content does not necessarily cover the entirety of the areas that had been identified as problematic. There may be other areas of these topics that teachers/employers wanted included and have not been added. Nor does it ensure that students are taught these areas. There are also several areas which appear to have no additional content. Awarding bodies may include these areas in the extra breadth and/or depth that they are allowed to add within their specifications, but this will not necessarily happen.

It is possible that changes to the core Mathematics content at A level (which relies on most of the areas that teachers suggested were inadequately prepared) may mean that the transition from GCSE to A level Mathematics is less problematic. Additionally, the redevelopment of both GCSEs and A levels at a similar time means that awarding bodies should be able to ensure a smoother transition between the two qualifications.

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Statistics and Mechanics: Comparing the Applied Mathematics of international Mathematics qualifications

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Introduction

In this article, we report on data collated as part of a large-scale study investigating how A level Mathematics and Further Mathematics prepare students for the mathematical demands of university study in a range of subjects. We investigate and compare the applied mathematical content (Mechanics and Statistics) in a range of international Mathematics qualifications and conclude that the A level has notable differences to similar qualifications in other jurisdictions. In particular, the existing modular structure at A level introduces significant variability into the mathematical backgrounds of students studying what is theoretically the same qualification. Although this problem will be rectified by the introduction of prescribed content from 2016, two other differences emerged during this investigation. First, whilst Mechanics content at A level is primarily studied in Mathematics and/or Further Mathematics, in nearly every other jurisdiction this content is studied within the Physics course. Secondly, there appears to be no international consensus about what statistical content is taught at this level. These findings may have implications for ongoing reform at A level, particularly with respect to the applied content in Further Mathematics, and may also prove interesting for employers and universities with a global reach who currently use Mathematics qualifications for admissions or recruitment purposes.

Background

As part of ongoing qualification reform in England and Wales, A level Mathematics and Further Mathematics are being reformed for first teaching in 2016. The reforms have significant implications for the structure and content of post-compulsory Mathematics in the UK. All A levels are moving from a modular to a linear system, meaning that students will be required to take all of their examinations at the end of the two-year course, rather than throughout as is currently the case. Additionally, the AS level and the A level are being 'decoupled'. The A level is currently a two-year course; students sit examinations during the first year which contribute to their final A level grades which also counts as a qualification in its own right (the Advanced Subsidiary or 'AS' level). However, in the reformed A levels, the AS level will become a stand-alone qualification and will no longer count towards a student's overall A level grade.

Students are able to study two Mathematics A levels: Mathematics and Further Mathematics[1]. The four main awarding bodies (AQA, Oxford, Cambridge and RSA (OCR) Examinations, Pearson Edexcel, and WJEC) all offer their own versions of both A levels, and students and schools are able to select which awarding body's specification they would like to study. Currently, there is a great deal of flexibility in the structure of both subjects, particularly in relation to the applied content. Further Mathematics must be studied alongside or after A level Mathematics, and its content builds on material covered in Mathematics.

In A level Mathematics, students sit four 'Core' Pure Mathematics modules, and two 'Applied' modules. The Applied modules can be chosen from Mechanics, Statistics and Decision Mathematics, and students can either take one module from two different strands, or multiple modules from the same area (e.g., Mechanics 1 and Statistics 1, or Mechanics 1 and 2). For example, a student interested in studying Engineering or Physics at university may be encouraged to specialise in Mechanics, whilst a prospective Biologist or Social Scientist may choose only Statistics modules (Lee, Harrison, & Robinson, 2007; A Level Content Advisory Board, [ALCAB] 2014). However, students are rarely able to choose their own modules as these decisions are predominantly made by the school/college. Schools/colleges often lack the resources to offer different modules for individual students and instead tailor their module

1. Two awarding bodies also offer an AS/A level in Statistics, but these are taken by very few students.

selection to the needs of the overall cohort. Consequently, students are often restricted to studying a mixture of modules, usually Statistics 1 and Mechanics 1 or Statistics 1 and Decision 1, rather than specialising in a particular area of Applied Mathematics.

In A level Further Mathematics, students must study two Further Pure Mathematics modules, and an additional four modules which can be a mixture of supplementary Further Pure modules and Applied modules. As the Applied modules have prerequisite modules in the same strand, the more advanced Applied modules, Statistics 3-4 and Mechanics 3-5, can only be studied in Further Mathematics. Consequently, students and teachers currently have considerable freedom regarding which modules they study at A level, which causes a degree of variability in the mathematical background of students when they reach university. This is exacerbated by the fact that awarding bodies often include different content in different modules, meaning that students taking the same modules in different specifications may not necessarily have covered the same content (see the National HE STEM Programme Wales, 2012, for a closer examination of differences in content division between awarding bodies).

However, in the reformed A level Mathematics, 100 per cent of the content will be prescribed by The Office of Qualifications and Examinations Regulation (Ofqual) (see Department for Education, 2014, for details). Additionally, this prescribed content includes both Statistics and Mechanics content, meaning that all students will have some grounding in both of these areas regardless of their school or specification. However, in Further Mathematics, only half of the content will be prescribed. The awarding bodies are able to decide what additional content they will include, which is likely to be applied as all of the prescribed content in Further Mathematics is Pure Mathematics material. This content does not necessarily need to follow the Statistics/Mechanics/Decision Mathematics framework, as awarding bodies may decide to introduce more specialised content such as 'Mathematics for Economists' or 'Mathematics for Biologists'. Nevertheless, it is likely that uptake of such specialised modules would be very low as most schools/colleges would be unable to afford to offer modules to individual students, and thus it is likely that awarding bodies will instead choose to offer optional Mechanics, Statistics or Decision content in Further Mathematics. Although the introduction of prescribed content will ameliorate the situation, there will therefore still be some variability in the mathematical backgrounds of students who have studied Further Mathematics, particularly with reference to Applied Mathematics.

Consequently, the changes to post-compulsory Mathematics qualifications in the UK and the development of the new specifications provide an opportunity to consider how the applied content of the existing A levels compare to their international counterparts.

Method

Mathematics qualifications from a number of international jurisdictions were analysed in order to ascertain the Applied Mathematics content of those qualifications most aligned with A level Mathematics and Further Mathematics. Mathematics qualifications from Alberta (Canada), Hong Kong, New South Wales and Victoria (Australia), Singapore, Scotland and the United States were investigated.

These jurisdictions were chosen for their similarity to A level, in terms of being used as an entrance qualification for undergraduate study and the age at which students sit their final examinations. Additional factors were the presence of an extension Mathematics course or additional content for the most able students, and the availability of curriculum materials in the English language. Specifications and content outlines were used to make judgements, obtained from awarding body and Ministry for Education websites. However, it should be noted that not all relevant documents may have been publicly available, or in the English language, and therefore there may have been information relating to these qualifications that was not used in this study (see Elliott, 2013, for more information about the limitations of comparability studies).

Analysis of applied content was separated into Mechanics and Statistics. Decision Mathematics was not investigated in the current study as it will not be included in the reformed A levels due to its perception as a 'soft' option (ALCAB, 2014). Although awarding bodies may well incorporate some Decision Mathematics into Further Mathematics, this investigation was restricted to the areas of Applied Mathematics that will become compulsory in the reformed A levels. Nevertheless, Decision Mathematics remains an interesting area for future comparison.

It was noted where particular topics occurred and in which module or course they appeared. Mathematics qualifications considered to be equivalent to A level Mathematics or Further Mathematics were used. However, during the course of the study it became apparent that Mathematics qualifications in the majority of the jurisdictions investigated did not include any Mechanics content. Upon closer examination, this content was found to be incorporated into Physics courses. Consequently, instances where a Mechanics topic occurred in a comparable Physics qualification are depicted in the results.

Qualifications

GCE A level – United Kingdom

In the UK, students take A levels at age 18. Their grades are typically used to gain entry to university and/or employment. Students usually take three or four subjects for the full two-year course and two Mathematics qualifications are available: A level Mathematics and A level Further Mathematics[2]. As described above, A levels are currently modular, and students must sit a mixture of core and applied units in both Mathematics qualifications.

For the purposes of this study A level Mathematics and Further Mathematics specifications offered by the four main awarding bodies were investigated: those offered by AQA, OCR, Pearson Edexcel and WJEC. It should be noted that OCR has two A level Mathematics/Further Mathematics specifications: the standard specification and the course developed in collaboration with the Mathematics in Education and Industry (MEI) organisation, which offers different content and unique optional units such as 'Further Pure with Technology' and 'Numerical Methods'. Both of these courses were investigated. All modules from all five specifications were analysed.

A level specifications analysed: AQA (2013), OCR (2013b), OCR MEI (2013a), Pearson Edexcel (2013), WJEC (2013).

2. Students can also gain an additional AS/A level in Additional Further Mathematics if they take enough units, but very few students choose to do so.

Alberta High School Diploma – Alberta, Canada

Students in Alberta sit the Alberta High School Diploma (HSD) in Grades 10–12 and graduate when they are 17–18 years old. Students must earn 100 credits across all modules, with compulsory modules in English Language Arts, Social Studies, Mathematics and Science.

As Mathematics is compulsory, there are three streams for students to choose from depending on ability. For the purposes of this study we focused on the most advanced stream: the '-1 course' sequence. This course consists of three units: 10C, 20-1, and 30-1. This sequence is also compulsory for students wishing to study Mathematics 31, an additional unit which extends knowledge of calculus and introduces some practical applications.

There is very little applied content in the main three modules, but Mathematics 31 includes three electives in Applied Mathematics: (1) Applications of Calculus to Physical Sciences and Engineering; (2) Applications of Calculus to Biological Sciences; and (3) Applications of Calculus to Business and Economics. There is also some more basic Statistics in the less-advanced Mathematics 'sequences'.

Alberta HSD courses analysed: 10C, 20-1, 30-1, Mathematics 31, Physics 20-30 (Alberta Education, 1995; 2008; 2014).

Hong Kong Diploma of Secondary Education – Hong Kong

The Hong Kong Diploma of Secondary Education (HKDSE) is taken by students in their final year of secondary school, when aged 16–17. Students take four core subjects in Chinese Language, English Language, Mathematics and Liberal Studies, as well as two or three elective subjects of their choice. Mathematics is unique amongst HKDSE subjects in that it contains two parts. In addition to the Compulsory part, it also contains an 'Extended part' for the most able students. The Extended part is designed to cater for students who intend to "...pursue further studies which require more mathematics; or follow a career in fields such as natural sciences, computer sciences, technology or engineering." (Curriculum Development Council and The Hong Kong Examinations and Assessment Authority [HKEAA], 2014a).

There is no Mechanics in either the Compulsory or Extended part, although there is some Statistics in both parts. Consequently, the Physics course was also investigated for the Mechanics comparisons.

HKSDE courses analysed: Mathematics (Compulsory, Extended), Physics (Curriculum Development Council and The HKEAA, 2014a; 2014b).

Higher School Certificate – New South Wales, Australia

Students in New South Wales (NSW) take the Higher School Certificate (HSC) at the end of Year 12, aged 17–18, making the HSC broadly equivalent to A level. The HSC consists of Preliminary courses, taken in Year 11, and HSC courses, taken in Year 12. Mathematics is not a compulsory subject in the HSC but there are a wide range of Mathematics courses available to choose from, dependent on ability and planned future progression into Higher Education (HE). For the purposes of this study only Mathematics Extension 1 (ME1) and Mathematics Extension 2 (ME2) are investigated.

There is very little statistical content in either ME1 or ME2, although there is some statistical content in the less advanced courses. However, there are Mechanics topics in both units.

HSC modules analysed: Mathematics Extension 1, Mathematics Extension 2. (Board of Studies New South Wales 1997; 2011a; 2011b).

Advanced Highers – Scotland

Advanced Highers (AHs) are the highest qualification offered by the Scottish Qualifications Authority (SQA), and are classified as a SQA Level 7 qualification. AHs contain more content and require more advanced skills than their counterparts, Highers (SQA Level 6), and are thus considered to be closer to the A level in terms of difficulty (see Johnson and Haywood, 2008, for a closer comparison of the demands of Advanced Highers and A level). Consequently, they are taken by the most able students, with 13,316 students entered for AHs in 2013 (SQA, 2013).

There are two AH courses in Mathematics available to students: Mathematics and Applied Mathematics. Mathematics is the considerably more popular subject – 3,314 students entered in 2013 compared to 361 for Applied Mathematics (SQA, 2013). Students taking Applied Mathematics are required to also take AH Mathematics, making up approximately 10.9 per cent of the total entries for this course. Whilst AH Mathematics has the larger number of candidates, it does not cover any Mechanics or Statistics and therefore Applied Mathematics is the qualification of focus for the comparisons in this study. Nonetheless, the low entry size for this course should be noted as the majority of AH students will thus not have studied any Applied Mathematics.

In Applied Mathematics, students take one core module and then choose between Mechanics and Statistics. It is important to note that, from 2015, the SQA will be splitting the Applied Mathematics AH into new separate Statistics and Mechanics courses. However, as these new qualifications are yet to be assessed it was felt to be more appropriate to compare the current qualifications, in line with the *Cambridge approach to comparative evidence from other jurisdictions* (Elliott, 2013).

AH modules analysed: Statistics 1-2, Mechanics 1-2, Mathematics 1-3 (SQA 2004; 2007; 2010).

Singapore-Cambridge GCE A level – Singapore

Students in Singapore sit Singapore-Cambridge A levels, designed and administered in collaboration with the Ministry of Education and Cambridge International Examinations. Singapore A levels are offered at three levels of study: H1, H2 and H3, with H3 being the most advanced (Ministry of Education Singapore, 2013).

For the purposes of this study only H2 and H3 Mathematics are investigated, due to their closer comparability with UK A level Mathematics and Further Mathematics than H1. H3 covers Pure Mathematics content in more depth than H2 whilst also introducing some more Mechanics content through the 'Differential Equations as Mathematical Methods' strand. Statistics is only included in the H2 course.

Singapore GCE A level courses analysed: Mathematics H2, Mathematics H3, Physics H2 (Singapore Examinations and Assessment Board [SEAB], 2013a, 2013b; 2013c).

Advanced Placement exams – United States and Canada

Advanced Placement (AP) exams are administered by the College Board and are taken primarily by students in the United States and Canada, although they may be taken by students around the world. They are not part of compulsory education but are instead taken by students intending to progress to HE. Although they are unlikely to feature in admissions requirements for US universities, which focus more on the Scholastic Aptitude Test (SAT), they may positively affect applications for scholarships. They can also be used to earn college credit and to enter Higher level college courses (College Board, 2014a).

There are four AP courses Mathematics or Mathematics-related courses available: Calculus AB, Calculus BC, Statistics and Physics C (Mechanics). For the purposes of this study, only Physics C (Mechanics) will be considered. This is because whilst it is a Physics course, it is the only AP course which contains any Mechanics content. Additionally, AP Statistics is classified as a Group B subject by the University and Colleges Admissions Service (UCAS), meaning that it is not considered to be comparable to A level in terms of demand (UCAS, 2006). Consequently, AP courses are only used in the Mechanics comparisons.

AP courses analysed: Physics C (College Board, 2014b).

Victorian Certificate of Education – Victoria, Australia

In Victoria, students sit the Victorian Certificate of Education (VCE) as the culmination of their secondary school education. The VCE is different in structure and in content to the HSC in New South Wales. In order to attain the VCE students must take a minimum of 16 units over Years 11 and 12. There are 12 Mathematics units in total available to students taking the VCE which are designed to satisfy a range of different abilities and needs.

For the purposes of this study we concentrated on the most advanced courses: Mathematical Methods Computer Algebra Systems (CAS) 1–4, Further Mathematics (FM) 3+4, and Specialist Mathematics (SM) 3+4. However, it is important to recognise that the Further Mathematics units should not be directly equated with A level Further Mathematics; this is because whilst there is some overlap in content between the two, a VCE student would need to study the additional Specialist Mathematics and Mathematical Methods units to cover the same range of content.

Further Mathematics in the VCE has a statistical focus, whilst Mechanics content is concentrated in Specialist Mathematics. There is also some more basic statistical content in the less advanced units.

VCE courses analysed: Mathematical Methods (CAS) 1–4, Further Mathematics 3+4, Specialist Mathematics 3+4, Physics (Victorian Curriculum and Assessment Authority [VCAA], 2010; 2012).

Results and Discussion

Mechanics

Mechanics is one of the two predominant areas of Applied Mathematics studied at this level (the other being Statistics) and forms the basis for further study in Engineering and the Physical Sciences, as well as Mathematics. Consequently, the Mechanics content of international qualifications has implications for the preparedness of students for a range of undergraduate courses, not just Mathematics.

In order to compare the Mechanics content of Mathematics qualifications, the specifications and assessment materials of the qualifications already outlined were utilised. The topics which recurred most often were listed, as well as which specific modules or units they were included in (see Table 1).

Table 1: Mechanics content

Topic	*AQA*	*OCR*	*OCR MEI*	*Pearson Edexcel*	*WJEC*	*AP*	*AH Applied Maths*	*Alberta*	*Hong Kong*	*NSW*	*Singapore*	*Victoria*
Vectors	C4|FP3|M1	C4|FP3	C4|M1	C4|FP3|M1	M2	Physics C	M1	Physics 20	Extension	ME2	Maths H2	SM 3+4
Kinematics	M1|M2	M1	M1	M1		Physics C	M1	31	Physics	ME2	Physics H2	GM 1+2
Newton's laws of motions applied to problems involving force	M1	M1	M1	M1	M1	Physics C	M1	Physics 20	Physics	ME2	Maths H3	Physics
Force as a vector	M1	M1	M1	M1	M2	Physics C	M1	Physics 20	Physics	Physics	Physics H2	SM 3+4
Work, energy and power	M2	M2	M2	M2	M2	Physics C	M2	31	Physics		Physics H2	Physics
Impulse and momentum in two dimensions	M1|M3	M3	M2	M1	M1(1D)	Physics C	M2	Physics 30	Physics	Physics	Physics H2	Physics
Centre of mass	M2	M4	M2	M3	M1	Physics C		31			Physics H2	Physics
Simple harmonic motion	M5	M3	M3	M3	M3	Physics C	M2	31		ME2	Physics H2	
Elastic springs and strings (inc. Hooke's law)	M2	M3	M3	M3	M2|M3		M2	31			Maths H3	Physics
Equilibrium of rigid bodies	M2	M2|M3	M2	M3	M1|M3	Physics C	M1		Physics			SM 3+4
Rotation of a rigid body	M4	M4	M4	M5		Physics C	Physics					
Stability and oscillations	M5	M4	M4	M3|M4		Physics C	M2	Physics 20		ME1	Physics H2	
Moment of inertia	M4	M4	M4	M5		Physics C	Physics	31	Physics	Physics		
Relative motion	M3	M4	M1	M4	M2	Physics C	M1				Physics H2	Physics
Angular motion	M2	M4	M4	M5	M2	Physics C	M2			ME2		
Linear motion under a variable force	M2	M3	M4	M3			M2	31				GM 1+2
Problems involving variable mass	M5		M4	M5				31			Maths H3	
Coefficient of restitution (elastic/inelastic collision)	M3	M2	M2	M2		Physics C*		Physics 30*	Physics*			
Motion of a projectile	M1	M2	M1	M2	M2	Physics C	M1	Physics 20	Physics Ext.	ME1	Physics H2	Physics
Moments	M2	M2	M2	M1	M1		M2		Physics	Physics	Physics H2	
Circular motion:												
Uniform motion in a circle	M2	M2	M3	M3	M2	Physics C	M2	Physics 20	Physics Ext.	ME2	Physics H2	Physics
Motion in a vertical circle	M2	M3	M3	M3	M2	Physics C			Physics Ext.			Physics

* denotes that elastic and inelastic collisions are covered but knowledge of the coefficient of restitution is not required

However, it should be noted that there are limitations to this approach: noting if and where a topic occurs does not address how much depth is given to these topics or the required level of understanding. Additionally, the occurrence of certain Mechanics topics in Physics courses means that the level of underlying Mathematics required when studying these topics is uncertain. For example, whether students are expected to have knowledge of calculus when studying Physics, and if so, is this taught within the Physics course or must students also study the Mathematics course? A deeper analysis of these factors is beyond the remit of the current study, which intends to highlight which Applied Mathematics topics students study in these jurisdictions, but would be an interesting avenue for future research. It should also be borne in mind when considering the results.

Results

The following Mechanics topics are covered by all qualifications under consideration and occur in the less advanced A level Mechanics modules (M1 and M2):

- Vectors
- Kinematics
- Newton's laws of motion applied to problems involving force
- Force as a vector
- Work, energy and power
- Impulse and momentum in two dimensions[3]
- Motion of a projectile
- Uniform motion in a circle[4].

There are also several topics that are covered in M1 and M2 of all the A level specifications, but not in all of the other qualifications, even when Physics courses are included:

- Moments
- Equilibrium of rigid bodies[5].

There are no topics that are included in the majority of international qualifications but not in A level.

Excluding the above topics, there is a reasonable amount of variation between specifications. However, it should be noted that in the international qualifications investigated, all topics are compulsory. Conversely, there is no guarantee that an A level Mathematics or Further Mathematics student will have studied *any* of the Mechanics topics, especially those occurring in the more advanced modules (M3–5).

Discussion

Through examination of Table 1, it initially appears that existing A levels cover considerably more Mechanics content than their international counterparts. However, the optionality of the modular structure has significant implications for the content an A level student will actually study. If a student studies all four Mechanics modules (or all five if they are taking the AQA or Pearson Edexcel specifications), they will certainly have covered Mechanics in greater depth than students taking other qualifications.

However, whilst it is technically possible for a student to take four or five Mechanics modules if they are taking the full A levels in Mathematics and Further Mathematics, entry numbers are very low for the more advanced modules. In June 2013 for the OCR specification, there were only 696 entries for M3 and 126 for M4, compared to the 6,216 candidates certificating for either A level or AS Further Mathematics. For the MEI specification, there were 717 entries for M3 and just 86 for M4, in comparison to the 6,302 candidates certificating for this specification in either A level or AS Further Mathematics. Additionally, M4 is often self-taught by students either out of personal interest or as advanced preparation for university study. Consequently, the number of schools/colleges routinely offering these modules is likely to be even lower than these figures suggest.

This suggests that only a very small minority of students studying Further Mathematics study the more advanced Mechanics modules. By contrast, M1 is the second most popular Applied module in both specifications, and M2 is the fifth most popular. It is therefore assumed that the majority of A level Further Mathematics candidates will have studied M1 and M2 only[6]. When this is taken into account, the Mechanics content of A levels is more in line with the international qualifications considered, but only when comparable Physics courses are also included.

These findings are corroborated by other research in this area. Leppinen (2008) found that 65% of first year undergraduate Mathematics students had studied at most two Mechanics modules at A level; 28% had studied three, with just 7% studying four or more. Ward-Penny, Johnston-Wilder, and Johnston-Wilder (2013) have found further evidence to suggest that uptake of the higher Mechanics modules is low, with less than half of all schools in their sample offering M3 for Further Mathematics students. Only 11% offered M4, and 61% of these were independent or grammar schools. Whilst the optionality inherent in A levels introduces flexibility for both teachers and students and allows them to study modules of particular interest, it introduces a degree of uncertainty for universities and consequently students' preparedness for tertiary study. Research indicates that this uncertainty has implications for undergraduate study: in a study of Engineering departments at 19 universities, Lee et al. (2007) found that only 17% of the lecturers they surveyed knew which Mechanics modules their students had studied within A level Mathematics. As a result, 58% of the lecturers assumed a level of Mechanics knowledge that their students did not in fact have.

Nevertheless, for all of the other qualifications under consideration, vectors were the only Mechanics-related content always covered within even the most advanced Mathematics courses. The rest (excluding AHs) incorporate a significant proportion of their Mechanics topics into their Physics qualifications. This suggests that there is a difference between what the UK and other jurisdictions perceive to be an appropriate place for this content.

It is difficult to know the rationale behind different jurisdictions' decisions about where to include specific content. However, it may be partially due to the non-modularity of these international qualifications. Whilst there are often a range of Mathematics courses to choose from, as in Alberta and Australia, all topics within a course or sequence are usually compulsory, unlike in A levels. Furthermore, where there is some optional content, as in the HKDSE, the content is usually compulsory for the most able students (i.e., those who would be most likely to take A level Further Mathematics if they were resident in the UK). This therefore necessarily

3. Only covered in one dimension by WJEC.
4. Covered in M3 for the MEI and Pearson Edexcel courses.
5. Covered in M3 for Pearson Edexcel.
6. This is true of all A level specifications apart from WJEC, which only offers three Mechanics units. Because of this, an A level Further Mathematics student will have had to study M1–3, unless they opted to study an additional Further Pure unit.

entails a reduction in content when compared to A level as all topics must be taught and studied. This may lead awarding bodies or Ministries for Education to incorporate any additional Mechanics content into their Physics courses in order to allow adequate time for study.

However, the difference in content may be due to a difference in belief about just how mathematical Physics qualifications should be. There is some evidence to suggest that A level Physics does not contain an appropriate amount of Mathematics to fully prepare students for Physics or related degrees, unless they also study at least A level Mathematics. The Institute of Physics (2011) found that this caused two problems: prospective undergraduates did not expect there to be so much Mathematics in their Physics course, and students struggled to apply Mathematics in physical contexts or understand where mathematical concepts might be useful to solve Physics problems. These problems were attributed to the lack of Mathematics in A level Physics by both lecturers and undergraduates. The fact that UK universities usually require both A level Physics and Mathematics for admission to undergraduate Physics and Engineering courses is perhaps indicative of this.

Nevertheless, the comparisons made in this study indicate that this appears to be a problem particular to A level, with other jurisdictions ensuring that their Physics courses are suitably mathematical. Whilst this means that students taking these qualifications must study both Mathematics and Physics, students in the UK also need to take both subjects to be similarly prepared. It could be argued that if students take qualifications in both Mathematics and Physics, they are likely to encounter the same content regardless of which subject it occurs in. However, the reforms to A level Mathematics and Further Mathematics offer an interesting opportunity to consider why these differences arise and why the A level differs so much to its international counterparts. Although the changes mean all A level Mathematics students will study some Mechanics, the perceived disconnect between Mathematics and Physics may well continue.

Statistics

Statistics is the second branch of Applied Mathematics investigated in this study. As well as further study in Mathematics, knowledge of Statistics is required in undergraduate courses that entail quantitative analysis, such as Psychology, Economics and other Social Sciences. The statistical content of international qualifications thus has implications for students in a variety of university courses, including some that may be traditionally considered non-mathematical.

All of the qualifications outlined above, excluding AP exams, are included for analysis. The most common topics were listed and compared, as well as which specific modules or units they occur in (see Table 2).

As well as Statistics content, probability is also included for comparison here. Whilst probability and Statistics are definitively not the same area of Mathematics, they are often included in the same areas or units of Mathematics qualifications and both are therefore included in this study. It should also be noted that some statistical content is often included in Social and Biological Science qualifications at this level (e.g., in A level Psychology) but investigation was restricted to Mathematics courses only.

Results

Initial data collation indicated that there was very little statistical content in either the Alberta '-1 course' sequence or the NSW HSC's Maths Extension 1 and 2. Consequently, comparisons were expanded to include the less advanced Mathematics courses for these qualifications. Where a topic occurs in one of these courses, it is clearly marked in Table 2.

There appears to be very little consensus as to either which or how many topics are included in the Statistics components of the international qualifications under consideration. Excluding the first three topics (Measures of central tendency, Measures of dispersion, and Visual presentation of data) as they are usually included as a review of earlier qualifications, the most common topics are:

- Binomial distribution
- Normal distribution
- Poisson distribution
- Product moment correlation coefficient
- Central Limit Theorem
- Conditional and independent events (probability)
- Expectation algebra.

The following topics are covered in the majority of the international qualifications but the minority of A level specifications:

- z-score[7]
- Permutation and combination[8].

Discussion

Once again, it initially appears that A levels cover the most Statistics content. However, as with Mechanics, the modular structure has implications for the content an A level Further Mathematics student will actually study. Whilst S1 is the most popular Applied module and S2 is the fourth (in both the OCR and MEI specifications), entries for S3 and S4 in 2013 were even lower than for the corresponding Mechanics modules. For the OCR specification, there were 399 entries for S3 and 82 for S4, whilst in the MEI specification there were 500 entries for S3 but just 28 for S4. Consequently, it is highly likely that an A level Further Mathematics student will have studied S1 and S2 only[9]. Although this data is for the OCR and MEI courses only, the small entry numbers for these modules are corroborated by Ward-Penny et al. (2012), who found that only 18% of sampled schools/colleges offered Further Mathematics students S3, with just 4% offering S4.

There are two key findings from these Statistics comparisons. First, which may not be immediately obvious from Table 2, is that the way in which Statistics is taught in other jurisdictions is notably different to A level Mathematics. In other jurisdictions it is predominantly taught in a practical way and involves the handling and collection of real data. This is particularly the case in Victoria and Hong Kong, where students have to undertake statistical coursework. Conversely, the existing A level specifications currently teach Statistics in a largely theoretical manner, with no requirement for students to handle real data. However, the statistical content in the reformed A levels will focus on practical applications of Statistics and real, large data sets, with compulsory content on hypothesis testing and sampling methods (ALCAB, 2014). This has been developed in consultation with the Royal Statistical Society with the belief that handling real data sets will prove to be more engaging and useful for students. Developing skills in hypothesis testing and

7. AQA only.
8. OCR, MEI and WJEC only.
9. As with Mechanics, the WJEC specification only has S1-S3 so a Further Mathematics student taking the course to A2 will have had to study all three Statistics modules unless they opt to study an additional Further Pure module.

Table 2: Statistics content

Topic	*AQA*	*OCR*	*OCR MEI*	*Pearson Edexcel*	*WJEC*	*AH Applied Maths*	*Alberta*	*Hong Kong*	*NSW*	*Singapore*	*Victoria*
Statistics:											
Measures of central tendency†	S1	S1	S1	S1	S1	S1	30 - 3		GM		FM 3+4
Measures of dispersion†	S1	S1	S1	S1	S1	S1	20 - 2	Compulsory	GM		FM 3+4
Visual presentation of data†		S1	S1	S1	S1		20 - 3		GM		FM 3+4
Binomial distribution	S1	S1	S1	S2	S1	S1		Extended	ME1	H2	CAS 3+4
Normal distribution	S1	S2	S2	S1	S2	S1	20 - 2	Extended	GM	H2	CAS 3+4
Poisson distribution	S2	S2	S2	S2	S1	S1		Extended		H2	CAS 3+4
Sampling methods		S2	S3	S3		S1		Compulsory	GM	H2	
Hypothesis testing	S2|S3	S2	S1	S2	S2|S3	S1				H2	
t-test (one- and two-tailed)	S2	S3	S1|S3	S2	S2	S1				H2	
Product moment correlation coefficient	S1	S1	S2	S1	S2	S2				H2	FM 3+4
Non-parametric tests (Wilcoxon/Mann-Whitney)		S4	S3			S2					
Least-squares regression line	S1	S1	S2	S1	S3					H2	FM 3+4
Chi-squared test for goodness of fit	S4	S3	S3	S3		S2					
Chi-squared test for use in contingency table	S2		S2	S3		S2					
Type I and II error	S2	S2	S4	S4							
Central Limit Theorem	S1	S2	S3	S3	S3	S1		Extended		H2	
"Uses and abuses"/applications							20 - 2	Compulsory			FM 3+4
Geometric distribution	S4	S1						Extended			
t distribution	S4		S3	S4	S3	S2					
Bernouilli trials and Markov chains			FP3		S1			Extended			CAS 3+4
Confidence intervals	S1|S2|S3	S3	S3	S3	S2	S1 | S2	20 - 2	Extended			
z-score	S3					S1	20 - 2	Extended	GM		FM 3+4
Probability:											
Calculation of expectation and variance	S2	S1	S1	S1	S1	S1		Extended		H2	
Conditional and independent events	S1	S1	S1	S1	S1	S1	30 - 2	Compulsory	GM	H2	CAS 1
Expectation algebra	S1	S3	S3	S1	S1	S1		Compulsory		H2	CAS 2
Probability generating functions		S4	S4								FM 3+4
Moment generating functions		S4	S4								
Estimators	S4	S4	S4	S3|S4	S3			Extended			
Probability density function	S2	S2	S3	S2	S1						CAS 3+4
Cumulative distribution function	S2		S3	S1	S1						CAS 3+4
Bayes' Theorem	S3	S4	D2		S1	S1		Extended			
Permutation and combination		S1	S3		S1		30 - 1	Compulsory		H2	CAS 2

† These topics are usually covered before A level or equivalent qualifications. Their inclusion here denotes a review for the teaching of more advanced Statistics.

Topics displayed in yellow occur in less advanced Mathematics courses.

sampling may also prove useful for students who intend to progress to undergraduate courses that require statistical but not necessarily mathematical knowledge, such as Psychology and the Social Sciences.

The second key finding is the number of international qualifications that contain very little Statistics. The Singapore GCE A level and HKDSE are the only other qualifications to have compulsory Statistics content, with further Statistics being available in Module 2 of the Extended part in the HKDSE. Conversely, even when the less advanced courses are included, NSW and Alberta have very little statistical content at all.

It is not clear why there is such a disparity between the statistical content of the different qualifications. A possible explanation may lie in the intended audience of these qualifications. The advanced Mathematics qualifications investigated in this study are primarily intended for students progressing to Mathematics or Physical Sciences courses at university. Consequently, this leads to a strong emphasis on areas of Pure Mathematics such as calculus rather than Statistics in order to better prepare students for the demands of their university courses.

This explanation however would still leave the majority of students progressing to Mathematics and Physical Science degrees with very little experience of Statistics. An alternative explanation may be that Statistics is taught to a higher level earlier in the school curriculum in these jurisdictions, and therefore there is a reduced demand for Statistics at this level. A whole-scale analysis of General Certificate of Secondary Education (GCSE) level Mathematics qualifications is beyond the scope of this article but may prove to be an interesting area for future research.

Conclusion

This article has shown that there are substantial differences between the applied content of A level Mathematics and Further Mathematics and their international counterparts. Significant differences in both Mechanics and Statistics content were identified, which raises the question of how the content of Mathematics qualifications at this level is decided. There appears to be very little international consensus in how much Statistics or Mechanics a student should be exposed to before beginning university or entering the workplace. Additionally, the UK is unique amongst the jurisdictions investigated for incorporating its Mechanics content into Mathematics courses, rather than Physics.

However, it is difficult to fully appreciate the differences in international Mathematics qualifications without investigating their preceding qualifications (e.g., GCSE level courses) as well as analysing the Pure Mathematics content. This study also does not investigate the cultural contexts in which these qualifications are taken and how they are used by students and other stakeholders. Rather, this article acts as an exploratory basis from which to consider how pre-university Mathematics is taught and assessed across the world.

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The roles of teaching styles and curriculum in Mathematics achievement: Analysis of TIMSS 2011

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Introduction

In these times of rapid change fuelled by technological advances, the demand for improved mathematical knowledge is growing worldwide. Mathematical skills such as problem solving and inference are increasingly becoming part of both university access and labour market requirements. In some countries such as the United Kingdom, the importance of Mathematics has been recognised by policy-makers for individual progression, as well as for the economy and society at large (Oates, 2010; Department for Education [DfE], 2010). Evidence from the UK as well as comparative studies recognises Mathematics as a key subject (see, among others, Andrews, 2014). In particular, a wealth of research based on international benchmarking surveys, such as the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA), have investigated the connection between achievement in specific subjects and contextual factors (such as student and family background, school inputs and institutional background)[1]. However, much less is known about how aspects of a Mathematics curriculum are associated with Mathematics achievement.

This article provides empirical evidence about the link between Mathematics achievement, curriculum, teaching methods and resources used in the classroom. More specifically, this research explores common teaching styles and topics taught across countries with respect to their Mathematics achievement.[2] In order to do so, we make use of the fifth TIMSS survey, which provides a rich set of information regarding aspects of the curriculum (e.g., the emphasis on problem solving and interpreting data sets), resources used by teachers in the classroom (e.g., calculators and textbooks) and teaching styles (e.g., how often students are asked to take written tests, to work out problems individually rather than with teachers' guidance), along with measures of achievement in Mathematics gathered in 2011. Although TIMSS is administered to students and their teachers in both Grades 4 and 8 (Years 5 and 9 respectively, within England), analysis in this research is restricted to the Grade 8 students (aged 14). When analysing data aggregated at jurisdictional level, this allows us to explore relationships in the Mathematics achievement of 15 year-olds as measured by PISA 2012.

There are two main elements which connect with achievement that are investigated in this article. The first is the relationship between Mathematics performance and the prevalence of different teaching methods (also referred to as teaching styles or instructional practices) within different countries. It should be noted that in undertaking this analysis we are not assuming that countries will necessarily limit themselves to one dominant teaching style. Indeed, even at the level of the individual classroom we are aware that the same teacher will use a variety of styles and methods of teaching within the same lesson (Boaler 2008, Benbow & Faulkner 2008). This is likely to be even truer at the level of whole nations where different teachers within different schools are likely to have a different emphasis within their own style of teaching. The second element is the role of the curriculum in Mathematics performance measured by international assessment tests, such as TIMSS and PISA. In exploring the Mathematics curriculum, it is important to consider that it is possible that teachers do not teach the topics that the National Curriculum requires (Andrews, 2011). To overcome this issue, this research looks into the curriculum actually adopted by teachers and measured by means of their responses to a set of items which summarise the Mathematics topics they have taught in their classrooms up to Grade 8.

The underlying idea of the research in this article is to use international data from TIMSS to inform the debate about which methods of teaching Mathematics, which resources used in the classroom, and which topics taught may be most effective in terms of achievement in Mathematics measured by international tests. Since the 1990s, these tests have depicted East Asian students outperforming their Western counterparts (for the most recent results, see Mullis, Martin, Foy & Arora, 2012: Exhibit 1.1; Organisation for Economic Co-operation and Development [OECD], 2013: Figure I.2.14). This has led to the desire among policy-makers and education practitioners in countries such as the United States and the UK to emulate practices in use in high-performing jurisdictions, such as Singapore. As argued by Usiskin (2012), at both policy-level and school-level, the curriculum and the textbooks used have been identified as the main determinant of Singapore's Mathematics performance without any justification. Our aim is to check whether Mathematics achievement is linked to the curriculum and the use of textbooks as well as other resources and particular teaching styles.

In carrying out this analysis it is important to consider that a country's characteristics (such as geographical and economic conditions, and aspects of the education system) can have a strong influence on students' Mathematics achievement (Andrews, 2012; Usiskin, 2012). Therefore, drawing conclusions from such comparisons may be misleading (Gill & Benton, 2013). To help avoid this problem, as far as possible, the characteristics of students in each country that may affect achievement are allowed for in the analysis presented.

The results from the analysis should not be taken as a suggestion that the particular teaching methods, and the specific topics taught within high-performing jurisdictions, should be adopted elsewhere. Rather, we accept that establishing the causal factors behind attainment levels of different countries is problematic and that, at best, these studies can be used as a mirror to reflect upon teaching practices within our own country rather than a blueprint (Clarke, 2004; Elliott 2013). However, we feel that such reflections are best based upon detailed quantitative analysis of the type provided, rather than a simple 'eyeballing' of international league tables of achievement.

1. For a broad review of the contextual factors affecting achievement using data from international tests, see Hanushek and Woessmann (2011).
2. Results presented in this article are drawn from Zanini and Benton (2015) which also includes a more in-depth case study set within the UK.

Data

Our analysis makes use of data from TIMSS 2011[3]. The data focuses upon students in Grade 8 and their teachers. Data from a total of 42 countries participating in the TIMSS Mathematics assessments were included within the analysis. Four countries[4] which used the Grade 8 TIMSS instruments with students in grades other than Grade 8 were excluded. Non-country, benchmarking participants (such as individual states within Canada) were also excluded.

In some previous analyses of PISA (see Gill & Benton, 2013) the focus has deliberately been on countries identified as being sufficiently similar to each other. However, in this research we decided to take a different route as part of our aim was to attempt to group countries with regard to their curriculum and use of different teaching methods. For this element of analysis, the diversity of countries included was an advantage as cultural diversity may lead to differences in curriculum and teaching practice which may in turn be identifiable. On the other hand, including the entire, diverse array of countries within our analysis decreases the confidence with which we may be able to generalise any findings to the unique context within a single country.

Our analysis examines Mathematics achievement within different countries in two different ways. Firstly, achievement is defined using the overall Mathematics achievement scale as measured by TIMSS. This provides a measurement of students' achievement in relation to the kind of Mathematics curriculum that is generally taught internationally (Wu, 2009). It was also of interest to examine how the methods of teaching and the curricula within different countries might relate to students' abilities to apply Mathematics more broadly beyond the specific topics they have been taught. For this reason we also analysed the relationship between identified patterns of teaching from TIMSS, and Mathematics achievement as measured by PISA 2012 (published in OECD, 2013). PISA "does not just ascertain whether students can reproduce knowledge; it also examines how well students can extrapolate from what they have learned and apply that knowledge in unfamiliar settings." (OECD, 2013). As such, this provides a distinct concept of Mathematics achievement which was of interest within our research. Data on achievement in PISA 2012 was available for 29 of the 42 countries with relevant data for TIMSS 2011.

As we describe in the next section, in examining the relationship between Mathematics achievement and aspects of teaching, it was important to at least attempt to control for the impact of other background variables. This supplementary information was almost exclusively drawn from the school and student surveys collected as part of TIMSS 2011 itself. However, as pointed out by the OECD (2013):

> *The relative prosperity of some countries allows them to spend more on education, while other countries find themselves constrained by a lower national income. It is therefore important to keep the national income of countries in mind when comparing the performance of education systems across countries.* (OECD, 2013, p.24)

For this reason we also included data on the per capita Gross Domestic Product (GDP) of countries in 2011 in our analysis.[5]

3. This data is freely available for download from http://timss.bc.edu/timss2011/international-database.html. More technical details on questionnaires preparation, sampling and data collection are described in Martin and Mullis (2012).

4. Botswana, Honduras, South Africa and Yemen.

5. Additional data sources were used to retrieve GDP information for specific countries (see Zanini & Benton, 2015: Section 2).

Methods

Using latent class analysis to segment countries into groups

One approach to analysing the links between teachers' responses to questionnaire items and the achievement of students would be to consider each questionnaire item separately. However, assuming we wish to distinguish the effect of one teaching method from the effect of another, such an approach would immediately run into problems due to the number of questions we might wish to analyse coming close to, or possibly exceeding, the number of countries available for analysis. In order to estimate linear regression coefficients and to disentangle the effect of one teaching method from the effect of another, the number of variables considered must be less than the number of observations included in the analysis.

Given the above consideration, our analysis needed to condense the information contained in the questionnaire, avoid treating individual items as if they give a raw quantity of the way instructional time is used, and ensure that we examine the pattern of countries' responses across all items together. For this reason, our analysis focuses upon identifying groups of countries with similar patterns of answers to the questions of interest. This part of the analysis was achieved using *latent class analysis*.

In its theoretical formulation, latent class analysis attempts to explain the relationships between various measured variables in terms of respondents belonging to one of a number of discrete latent (or unmeasured) classes (or groups). A typical latent class analysis would assume that all of a respondent's questionnaire answers are independent of one another once we know the grouping they fit in to. In our own situation we are less concerned with these theoretical underpinnings than with using the technique as a convenient way to segment countries into groups with similar patterns of responses across various questionnaire items. Thus we are not assuming that any of the theoretical assumptions given as examples above are actually true. Instead, we use the associated software for the sake of convenience and can verify whether the method has been effective by examining whether the derived groupings of countries actually display relatively similar behaviour in terms of the questions of interest.

Analysis, was completed using *Mplus Version 7* (Muthén & Muthén, 2012). This software provides a convenient feature whereby data is analysed at teacher level and groupings are produced at country level. Separate latent class analyses were undertaken to produce groupings of countries in terms of:

1. How frequently students were required to do various tasks during Mathematics lessons and the importance of various types of teaching materials for instruction.
2. When and whether various Mathematics topics were taught to Grade 8 students.

The second of these analyses focuses on what is taught in different countries whilst the first focuses on some aspects of how it is taught. The groupings of countries derived for each of these two research areas are described further in the Results section.

Using meta-regression to account for the impact of background variables on achievement

Once country groupings were produced, we examined how these groupings related to students' achievement. In order to explore this it is necessary to account for the influence of background characteristics.

One possible method was to account for each country's GDP per capita in 2011. However, initial analysis revealed a correlation of only 0.45 between GDP per capita and achievement in TIMSS 2011, rising to only 0.60 once the country of Qatar was removed.

For this reason we instead attempted to aggregate the information collected within the background section of the TIMSS student questionnaire to derive a single number for each country capturing the majority of the important information from the background questionnaire. This was achieved using the following process. Firstly the full international student-level data set was restricted to those students with listwise complete data on the following background questions:

- Gender
- How often they speak the language of the test at home
- How many books they have in their home
- Whether they have a computer at home
- Whether they have a study table/desk at home
- Whether they have books of their own (not school books) at home
- Whether they have their own room at home
- Whether they have an internet connection at home
- Highest level of education completed by their mother ('Don't know' was a valid response)
- Highest level of education completed by their father ('Don't know' was a valid response)
- Whether their mother was born in the country they are living in
- Whether their father was born in the country they are living in
- When they moved to the country they are living in (Being born there was a valid response).

A student level regression was then performed using Mathematics achievement data upon each of the above background characteristics. Using the results of this regression, for each student in the full data set (i.e., including those with listwise *incomplete* data) their response to each of the above questions was replaced with the corresponding regression coefficient. For each of the questions above, the average level of these effects was calculated for each single country. For each country in turn, these average effects were added up across questions to generate an overall, aggregated measure of background characteristics[6]. This measure was linearly rescaled so that the country with the highest measure (Korea) was assigned 100 and the country with the lowest measure (Ghana) was assigned 0.[7] Unsurprisingly, given the way it was derived, this measure was found to have a very high correlation with achievement in TIMSS 2011 Mathematics (correlation=0.80). More encouragingly, this measure was also found to correlate very highly with countries' Mathematics achievement in PISA 2012, which was not used to help create the scale (correlation=0.72). As a result, it is clear that this measure provides a useful mechanism to help control for the differences between countries.

6. The questionnaire in Israel did not include the question about whether students had a room of their own. For this reason, Israel was assigned the average effect (across countries) for this question.

7. The aggregated background measure assigned to each country is provided in Zanini and Benton (2015) Table 2.

8. For further details on meta-regression see Benton (2014). The meta-regression analyses were completed in R using the package metafor (Viechtbauer, 2010).

9. Further analysis aimed at validating the grouping revealed that the simple five class segmentation of countries captures nearly half of the overall variation in scores and that this was hardly improved by increasing the number of classes to six (see Zanini & Benton, 2015: Section 3.3).

Once the groupings of countries had been derived, it was possible to analyse the relationship between these groupings and Mathematics achievement whilst taking account of the aggregated background measure and GDP per capita using meta-regression. In essence meta-regressions are simply country-level regressions of achievement on country groupings, the background measure and GDP per capita. However, in contrast to standard linear regression, meta-regression allows us to account for the fact that the outcome (in this case Mathematics achievement) is measured with error. As such, the technique allows for the fact that some of the variation between countries will be purely due to measurement error and that the magnitude of this error may vary between countries. For our analysis, the technique also allowed for the possibility that there may be variation between countries that is neither due to measurement error nor explicable in terms of the covariates included within the regressions.[8]

Results

Teaching styles: grouping and meta-regressions

To begin with, our analysis examined countries in terms of the teaching styles used in lessons and the resources used to support learning; that is, questions 19 and 20 from the TIMSS 2011 Mathematics teacher questionnaire. The first question asked teachers to report how often ('every or almost every lesson', 'about half the lessons', 'some lessons', and 'never') they ask students to do specific activities. The second question asked teachers whether they use a list of resources (textbooks, workbooks/worksheets, concrete objects that help, or computer software) and, if so, if they use it as a 'basis for instruction' or as a 'supplement' (see Appendix).

As we have described, the analysis was undertaken using latent class analysis with the aim of identifying a small number of country groupings where, within each group, the extent of teachers' reported use of various strategies and resources was relatively similar. The latent class analysis identified five groups of countries. The choice of the number of classes was driven partly by statistical indices such as the Bayesian Information Criterion (BIC), (see Nylund, Asparouhov & Muthén, 2007), and partly by a desire to ensure that each group contained a reasonable number of countries.[9]

A summary of the groupings, the countries they contain, an overall interpretation of the response patterns and a suggested label for each grouping are shown in Table 1. Briefly this suggests that the five groupings are:

1. *Simplified*. A set of mainly Nordic countries where teachers are the least likely to report using many of the techniques in every, or almost every, lesson.
2. *Learn, repeat, and check*. A set of East Asian and former Soviet countries with high frequencies of teacher demonstration and independent or routine work, but lower frequencies for explaining answers or relating to daily lives.
3. *Routine independence*. A set of English speaking countries with fairly low frequencies of teacher demonstrations and more frequent use of independent or group work.
4. *Restrained diversity*. A set of mainly European countries with a high frequency use of a range of techniques, but with a relatively high reliance on textbooks as the basis for instruction, and fairly infrequent use of written tests and quizzes.

5. *Test-centric diversity*. A diverse group of countries although many of them (10 out of 17) are Middle Eastern. Teachers in these countries report high frequency use of many different techniques, particularly tests and quizzes, indicating that a variety of methods may be used within the classroom.

Table 1 shows that many countries that share similar contextual (i.e., geographical, historical, societal and/or cultural) factors are grouped together. It is both reassuring and disappointing that the groupings of countries match relatively well with simple contextual descriptions of the countries. It is reassuring in that it is to be expected that countries with similar geographical, historical, societal and/or cultural backgrounds might be expected to share similar styles of teaching. However, this also means that the groupings in terms of teaching style are also strongly interrelated with other (possibly unmeasured) contextual factors. This means that any attempt to examine the impact of the different teaching styles over and above these other factors is problematic.

The performances in TIMSS 2011 and PISA 2012 of each country

Table 1: Descriptions of country groupings by teaching styles

Grouping	*Countries included*	*Notes*	*Label*
1	*Mainly Nordic:* Finland, Norway, Sweden, Slovenia	Teachers in these countries are least likely to indicate using any particular method 'every or almost every lesson'. This indicates that students are less likely to experience multiple teaching methods in every lesson. Lower frequency of written tests or quizzes being used than in other countries. These countries are also the most likely to use textbooks as the basis for instruction.	*Simplified*
2	*East Asian and Former Soviet:* Chinese Taipei, Hong Kong, Japan, Kazakhstan, Korea, Malaysia, Russian Federation, Ukraine	High frequency of listening to the teacher and watching demonstrations of problem solving. Also high frequency of independent working and applying procedures to solve routine problems. Lower frequencies than other countries for explaining answers, relating to learning to daily lives or finding own solutions to complex problems. Testing is used fairly frequently; possibly to check progress.	*Learn, repeat, and check*
3	*English Speaking:* Australia, New Zealand, Singapore, United States, England	Along with Group 1, these countries have the least frequent amount of listening to the teacher explain how to solve problems. Memorisation of rules, procedures and facts is also relatively low. High frequency of independent/group work. The lesser use of textbooks as the basis for instruction in these countries indicates that a variety of resources may be being used. High frequency of time addressing routine problems and comparatively little spent on complex problem solving. Written assessments and quizzes are used relatively infrequently.	*Routine independence*
4	*Mainly other European:* Armenia, Georgia, Hungary, Israel, Italy, Jordan, Lithuania, Macedonia	Similar to the final category but less extreme. However, a greater reliance on textbooks and less use of assessment in the form of quizzes and tests.	*Restrained diversity*
5	*Scattered geographical regions:* Bahrain, Chile, Palestinian National Authority, Ghana, Indonesia, Iran, Lebanon, Morocco, Oman, Qatar, Romania, Saudi Arabia, Syrian Arab Emirates, Tunisia, Turkey	For many techniques teachers were the most likely to indicate that they will incorporate these techniques into 'every or almost every lesson' indicating an intention from teachers to incorporate a range of techniques and resources in most lessons. It includes more than a third of teachers stating that they will use written tests or quizzes in 'every or almost every lesson' (and two-thirds in at least half of lessons). This group of countries are the second least likely to indicate that textbooks are used as the basis of instruction indicating that a range of materials are being used.	*Test-centric diversity*

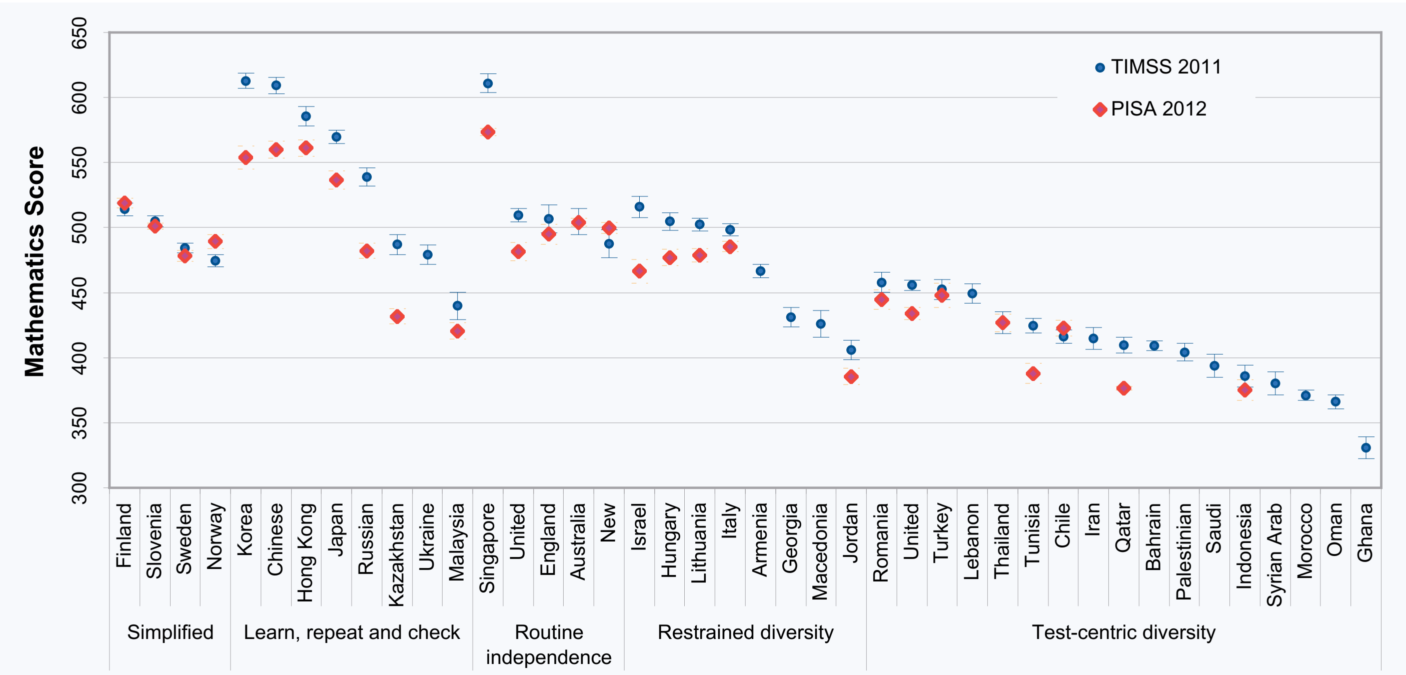

Figure 1: TIMSS 2011 Grade 8 and PISA 2012 Mathematics performance by countries within each teaching style group

within each grouping are shown in Figure 1. As can be seen, there is little obvious difference in the average performance of the different groups of countries with the exception of the generally lower performance of countries in the *Test-centric diversity* teaching category. However, this may itself be explained by the contextual nature of the countries within this grouping rather than the effect of this particular teaching style.

A more interesting picture emerges when we examine the difference between performance in PISA and performance in TIMSS. In order to explore this, performance in PISA is plotted against performance in TIMSS in Figure 2. Countries in each of the different groupings are identified separately and a regression line showing countries' expected performance in PISA 2012 given their performance in TIMSS 2011 is included. As can be seen, countries in the *Learn, repeat, and check* category tend to perform worse on PISA than might be expected from their performance in TIMSS. In contrast, countries in the *Simplified* category tend to perform better in PISA than might be expected from their TIMSS results. Such results might be explicable in terms of a *Learn, repeat, and check* approach being helpful in terms of learning the Mathematics associated with a particular curriculum but less helpful in enabling such knowledge to be applied in new situations. In contrast a *Simplified* approach may potentially allow teachers to provide more depth in their instruction so that students can understand how a particular piece of Mathematics may be applied in numerous situations.

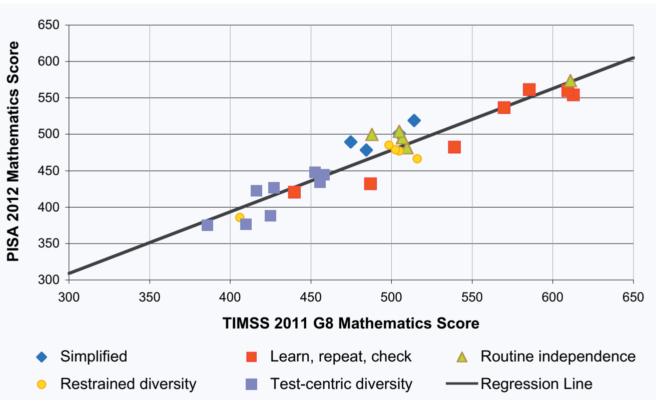

Figure 2: PISA performance against TIMSS performance for countries with different reported approaches to teaching Mathematics

The results in the figures do not account for the influence of other background factors upon results. In order to do this, performance in TIMSS is plotted against the newly derived aggregated background measure provided in Figure 3. Countries with different approaches to teaching are identified separately. A regression line is added to this graph showing the expected level of achievement in TIMSS for countries with different levels of the aggregated background measure. This graph suggests that countries with a *Learn, repeat, and check* approach to teaching tend to over-perform relative to their background characteristics (seven out of eight countries are above the line). In contrast countries with a *Simplified* or *Restrained diversity* approach to teaching tend to under-perform (four out of four and eight out of eight below the line respectively – albeit only slightly). Having said this, given the cultural differences between the groups of countries noted earlier, it is difficult to be confident that these results are due to the impact of teaching styles.

Performance in PISA is plotted against the aggregated background measure in Figure 4. When Mathematics attainment is quantified using this measure, differences between the groups of countries are far less evident. Countries with a *Restrained diversity* approach are still universally below the line of expected performance, but only slightly so. All other groups of countries are spread both above and below the line. This suggests that particular teaching styles may favour performance in TIMSS over performance in PISA (as seen in Figure 2), although once again this finding may be confounded by other cultural or societal factors.

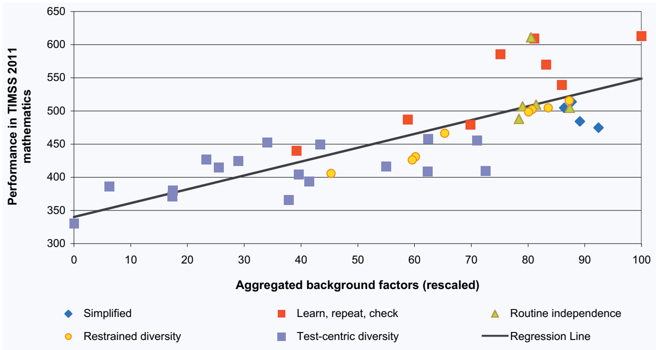

Figure 3: Performance in TIMSS by countries' aggregated background measures with different reported approaches to teaching Mathematics

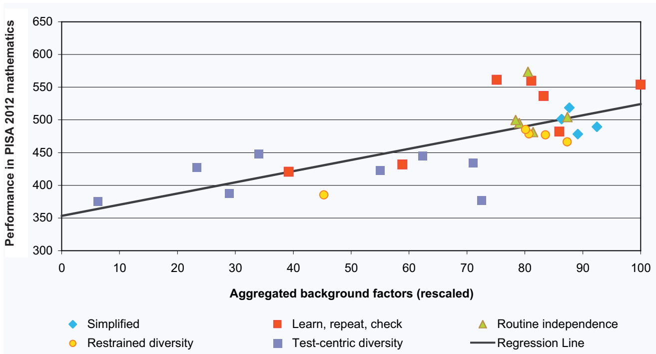

Figure 4: Performance in PISA by countries' aggregated background measures with different reported approaches to teaching Mathematics

Temporarily setting aside concerns over the potential confounding influence of other unmeasured variables, we further explore the statistical significance of the relationships suggested in Figures 2, 3 and 4 using meta-regression. Findings of meta-regression are shown in Table 2. The first set of results confirms that countries with a *Learn, repeat, and check* style of teaching tend to significantly outperform similar countries with other styles in TIMSS 2011. Specifically, this style of teaching is associated with countries achieving an additional 69 points on average compared to similar countries with a test-centric diverse approach. Other teaching styles do not appear to be associated with significantly greater performance, although countries within the *Routine independence* group come close.

The second set of results in Table 2 shows that the above results are not repeated within PISA 2012. Although countries within the *Learn, repeat, and check* group tend to outperform other similar countries within the diverse group, the extent of the difference is smaller and is no longer statistically significant. It should also be noted that the difference between this group and the *Simplified* group is substantially smaller than for the analysis of outcomes from TIMSS. Furthermore, the achievement of the *Learn, repeat and check* group is now slightly behind the achievement of similar countries in the *Routine independence* group, which may be largely attributed to the influence of Singapore on the latter group.

The meta-regression of performance in PISA whilst accounting for performance in TIMSS (and other background variables) confirms that

Table 2: **Results of meta-regression examining the relationships between Mathematics performance and teaching style**

	Performance in TIMSS 2011 (1)			*Performance in PISA 2012 (2)*			*Performance in PISA 2012 (3)*		
	Coeff.	*S.E.*	*P-val.*	*Coeff.*	*S.E.*	*P-val.*	*Coeff.*	*S.E.*	*P-val.*
Intercept	346.1	15.0	<.0001	356.0	22.4	<.0001	36.4	33.2	0.2850
Aggregated background measure	1.8	0.4	0.0002	1.5	0.5	0.0072	-0.1	0.3	0.8184
Per capita GDP	-0.2	0.4	0.6280	-0.4	0.5	0.3800	-0.1	0.2	0.7309
Country group (Ref.: Test-centric diversity):									
Simplified	1.9	25.2	0.9419	24.7	28.7	0.3996	28.5	12.2	0.0295
Learn, repeat & check	68.9	20.1	0.0016	47.1	23.8	0.0607	-12.8	11.8	0.2899
Routine independence	45.2	22.6	0.0535	51.0	25.6	0.0586	15.9	11.4	0.1795
Restrained diversity	2.6	20.4	0.9005	-3.8	26.4	0.8874	-4.0	11.3	0.7270
TIMMS 2011 score	-	-	-	-	-	-	0.9	0.1	<.0001

countries with a *Simplified* teaching style significantly over-performed in PISA 2012 relative to their performance in TIMSS 2011. Specifically they tended to achieve almost 30 points higher than would be expected in PISA given their performance in TIMSS and the background characteristics of their students. None of the other groups displayed any significant difference in their PISA performance once performance in TIMSS was accounted for.

Taken together, the results above suggest that particular styles of teaching may be more beneficial for the particular focus of TIMSS and that others may be more beneficial in terms of PISA. Specifically the results support the idea that a *Learn, repeat, and check* style may be helpful in boosting performance against curriculum-related Mathematics questions but that this advantage may not translate to the ability of students to apply Mathematics to real life situations. In contrast the *Simplified* approach appears to be disadvantageous (compared to certain other teaching styles) when assessed using a TIMSS style assessment, but this disadvantage disappears if students are assessed on their ability to apply their knowledge.

Although interesting, the results above must be treated with caution. As we have mentioned, the groupings by teaching style are closely related to particular geographical areas. As such the possibility that there may be other confounding cultural or societal factors influencing the results cannot be understated. For this reason, the results we present here should be seen as interesting results for reflection rather than definitive proof that particular styles of teaching can boost achievement in one or other of the international tests.

Curriculum: grouping and meta-regressions

Having examined teaching styles and resources employed by teachers in their lessons, we now turn our attention to the curriculum actually taught across countries as reported by teachers in question 23 of the TIMSS 2011 Mathematics questionnaire[10]. In the questionnaire, topics were grouped into four different domains: 'Number', 'Algebra', 'Geometry' and 'Data and Chance'. For each topic the possible answers were 'Mostly taught before this year', 'Mostly taught this year' or 'Not yet taught' (see Appendix). From an inspection of the data (see Zanini & Benton, 2015: Table 8) it emerges that most of the topics under the 'Number' domain seemed to be taught before Grade 8. Also, it is quite clear that some topics such as simultaneous equations and those linked to geometrical representations of shapes are considered quite advanced at international level, as they had not yet been taught by most of the teachers.

In order to identify a small number of country groupings where the topics taught by teachers were relatively similar across countries within each group, a latent class analysis was performed. Following exactly the same procedure employed for teaching styles, the latent class analysis on the curriculum identified five groups of countries. The choice of the number of classes was driven partly by the BIC statistical index and partly by the need to ensure that each group contained a reasonable number of countries. As the 'five-group solution' proved to fit the data better than any 'one to four classes' model and that increasing to six or seven classes led to very small groups (of which one constituted of one country), the five-class segmentation was adopted (for details see Zanini & Benton, 2015: Table 9).

From the careful inspection of the results of the grouping analysis it is possible to derive that the five groups are:

1. *Number and Algebra (N&A):* the smallest group, (only comprising of Chinese Taipei and Ukraine), where the Mathematics curriculum up to the year of interview was mainly focused on 'Number' and 'Algebra';
2. *Delayed introduction 1 (DI1):* a group of countries characterised for having delayed the introduction of most topics relative to other countries, even those fundamental to a Mathematics curriculum;
3. *Non-algebraic focus (NAF):* teachers in these countries were the least likely to report having already taught topics in 'Algebra' which are usually considered basics of the Mathematics curriculum;
4. *Delayed introduction 2 (DI2):* as for Group 2, it seems that most of the Mathematics curriculum was delayed relative to countries in other groups;
5. *Geometry and Data (G&D):* when compared to other countries, the Mathematics curriculum in these countries seems to be more focussed on 'Geometry' and 'Data and Chance', of which some topics can be quite advanced for Grade 8 students.

A more detailed summary of the response patterns and the list of countries in each grouping are shown in Table 3. In contrast to the results shown for teaching styles, groupings of countries by mathematical curriculum do not match with any simple ways of describing them. Even when countries sharing similar cultural backgrounds or from the same geographical area are in the same group, they are also mixed up with

10. Kazakhstan and Russian Federation are not considered in the analysis in this section because of the high number of missing values in the teachers' responses to the items in the TIMSS questionnaire related to the curriculum.

Table 3: Descriptions of country groupings by curriculum

Grouping	*Countries included*	*Notes*	*Label*
1	Chinese Taipei, Ukraine	In these countries teachers reported that 'Number' and 'Algebra' topics have been taught 'mostly before this year'. Teachers in these countries were among the least likely to report teaching topics in the other domains 'before this year' and among the most likely to indicate having 'not yet taught' these topics. This clearly suggests that for countries in this group the Mathematics curriculum up to the year of interview were mainly focused on Number' and 'Algebra'.	*Number and Algebra (N&A)*
2	Palestinian National Authority, Lebanon, Morocco, Norway, Sweden, Syrian Arab Republic, Tunisia	Teachers in these countries were among those least likely to have taught a number of topics under different domains 'mostly before this year'. When compared to other countries, not many Mathematics topics were introduced before Grade 8.	*Delayed introduction 1 (DI1)*
3	Finland, Georgia, Indonesia, Iran, Italy, Lithuania, Slovenia	Teachers in these countries were among the most likely to indicate having taught topics in the 'Number' and (partially) 'Geometry' domains 'before this year', but the least likely to report having already taught 'Algebra' topics, most of which are considered basics of a Mathematics curriculum.	*Non-algebraic focus (NAF)*
4	Australia, Chile, Ghana, Malaysia, New Zealand, Qatar, Saudi Arabia, Thailand, United Arab Emirates, England	All Mathematics topics were less likely to be reported to be taught by teachers 'mostly before this year'. This is particularly true in the domain of 'Number'. Along with Group 2, it seems that most of the introduction of the Mathematics curriculum was delayed with respect to countries in other groups.	*Delayed introduction 2 (DI2)*
5	Bahrain, Armenia, Hong Kong, Hungary, Israel, Japan, Jordan, Korea, Oman, Romania, Singapore, Turkey, Macedonia, United States	Teachers in these countries were the most likely to indicate having 'taught before this year' most of the topics related to the domains of 'Geometry' and 'Data and Chance'.	*Geometry and Data (G&D)*

countries with very different contextual factors. For example, it can be seen that in Group 2, Norway and Sweden are grouped with Morocco and Tunisia among others, but not with Finland which is grouped in Group 3 with Italy and Iran. This can be interpreted as an indication that the Mathematics curricula may be less connected to societal and contextual factors than teaching styles.

Figure 5 shows the performances in TIMSS 2011 and PISA 2012 respectively of each country within each grouping. It is straightforward to see that no striking difference in the average performance of the different groups of countries arises. However, it should be noted that the highest performance in both TIMSS and PISA tests was achieved by four countries (Korea, Singapore, Hong Kong and Japan) in Group 5 (i.e., those with the label 'Geometry and Data'), with only one exception, Chinese Taipei. In interpreting this evidence it has to be considered that the societal and contextual factors within these countries can be a confounding factor of the impact of the curriculum on performance. Similarly, no clear patterns arise from the analysis of the relationship between performance in PISA and performance in TIMSS, indicating that a grouping's performance in

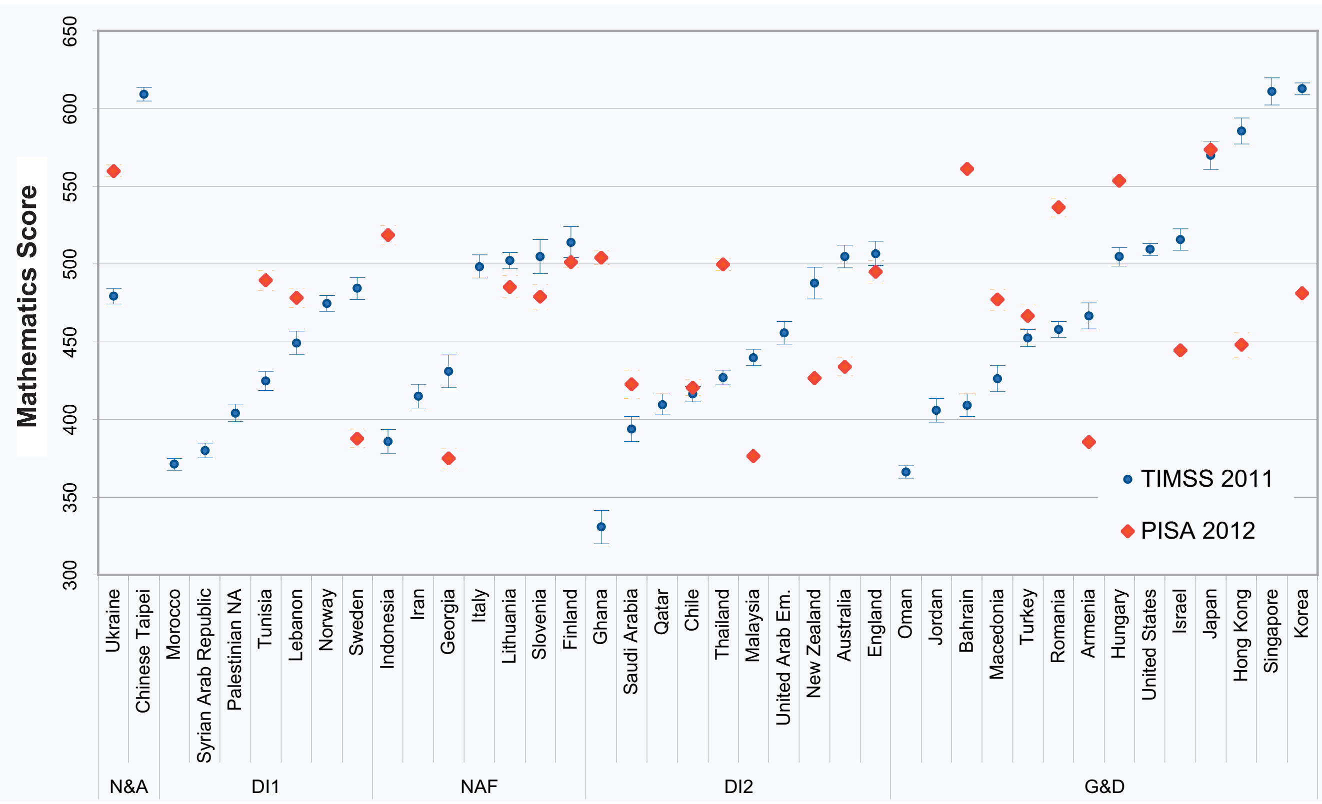

Figure 5: TIMSS 2011 Grade 8 and PISA 2012 Mathematics performance by countries within each curriculum group

Table 4: Results of meta-regression examining the relationships between Mathematics performance and curriculum

	Performance in TIMSS 2011 (1)			*Performance in PISA 2012 (2)*			*Performance in PISA 2012 (3)*		
	Coeff.	*S.E.*	*P-val.*	*Coeff.*	*S.E.*	*P-val.*	*Coeff.*	*S.E.*	*P-val.*
Intercept	366.13	23.13	<.0001	374.29	28.77	<.0001	6.86	40.35	0.87
Aggregated background measure	1.79	0.39	<.0001	1.68	0.46	0.00	0.15	0.26	0.55
Per capita GDP	0.00	0.00	0.75	0.00	0.00	0.76	0.00	0.00	0.73
Country group (ref.: Geometry and Data)									
Number and Algebra	39.59	32.01	0.22	55.65	41.86	0.20	-12.49	19.29	0.52
Delayed introduction 1	-26.28	20.71	0.21	-34.10	26.97	0.22	16.60	12.70	0.21
Non-algebraic focus	-13.92	19.57	0.48	-12.91	21.79	0.56	17.49	9.83	0.09
Delayed introduction 2	-32.06	19.50	0.11	-27.28	20.69	0.20	17.46	10.04	0.10
TIMMS 2011 score	-	-	-	-	-	-	0.91	0.10	<.0001

PISA 2012 was generally in line with their performance in TIMSS 2011.

These results suggest that the curriculum may be related to TIMSS and PISA performance in Mathematics. More specifically, it seems that delaying the introduction of topics like those related to 'Number' and Algebra' is negatively correlated to TIMSS performance. However, this finding may be confounded by other cultural and societal factors.

Setting aside the potential confounding influence of the cultural differences between countries, in order to further investigate the statistical significance of differences after accounting for the influence of the aggregated background factors, a meta-regression analysis was undertaken. More specifically, three different models were estimated and the results are shown in Table 4.

The first set of results (column 1) relate to the TIMSS 2011 performance. This confirms that, although the estimate of the coefficients of countries in both *Delayed introduction* groups and those in the *Non-algebraic focus* class are negative (which means that these countries tend to be out-performed by similar countries with different curricula), the differences are not statistically significant. It should be noted, however, that the magnitude of the coefficient associated to the *Delayed introduction 2* group is -32 (p-value = 0.11), suggesting that students in these countries tend to achieve 32 points lower than those in countries within the *Geometry and Data* group.

The results relating to PISA 2012 performance (column 2) are not particularly different from those relating to TIMSS 2011. The coefficients associated with the groupings are not significant, suggesting that curriculum differences across countries do not affect PISA Mathematics performance. The last set of results (column 3) indicates that Mathematics performance in PISA is not significantly affected by the groupings, which confirms the above results. However, it is worth mentioning that, for countries in the *Delayed introduction 2* group, the association with PISA performance is positive relative to achievement in TIMSS (column 3), whilst the association with TIMSS performance as a whole is negative (see column 1) - albeit the coefficients are not significant in both cases.

To summarise, these results provide no strong evidence that different curricula may be more beneficial for achievement in either PISA or TIMSS.

Conclusions and discussion

In this article we have investigated the relationship between Mathematics performance and the prevalence of different teaching styles, resources used in the classrooms, and curriculum using TIMSS 2011 Grade 8 (and partly PISA 2012) data. In undertaking this analysis, we relied on teachers' responses about the activities they asked their students to do, the resources they employed and the actual topics they taught.

The country-level analysis highlighted that countries were grouped differently by teaching styles than by curriculum. More specifically, grouping by teaching styles matched with contextual descriptions of the countries which include geographical, historical, societal and cultural factors. This suggests that, within countries with a similar background, teachers tended to share the same methods of teaching. Conversely, countries within the same groupings by topics taught did not share a common contextual description. This is also an indication that the Mathematics curriculum may be less influenced by countries' contextual characteristics than by teaching styles. On the other hand, the results of the meta-regression analysis suggested that teaching styles can be more connected to students' Mathematics performance on TIMSS and PISA than curriculum. In particular, our findings indicate that some teaching methods may be more beneficial in terms of PISA rather than TIMSS results and vice versa. Using the labels proposed in Table 1, a *Learn, repeat, and check* style appeared to be helpful in improving achievement measured by means of curriculum-related questions but not to apply Mathematics to real-life. In contrast, a simplified style appeared to be relatively disadvantageous in terms of TIMSS achievement, but positively associated to performance in PISA.

In interpreting these findings, it is important to stress that, since there may be other unobservable background factors affecting students' achievement, it is not possible to infer causal relationships from the results we have shown in this article. However, our research shows how data from international benchmarking studies can be exploited to provide empirical evidence about the link between teaching styles, curriculum, and Mathematics achievement. Although we cannot draw conclusions about which specific teaching practices and topics in the curriculum lead to better results in Mathematics achievement, our findings, based on detailed quantitative analysis, can be used to reflect upon Mathematics teaching styles and curriculum and their role in providing a more effective Mathematics education aimed at preparing students for their future lives and careers.

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APPENDIX

The reported questions 19, 20 and 23 from the TIMSS 2011 Teacher Questionnaire Mathematics Grade 8 (International Association for the Evaluation of Educational Achievement (IEA) (2011) are presented below:

QUESTION 19: When you teach mathematics to this class, how often do you usually ask students to do the following?

	Every or almost every lesson	*About half the lessons*	*Some lessons*	*Never*
a) Listen to me explain how to solve problems	O	O	O	O
b) Memorize rules, procedures, and facts	O	O	O	O
c) Work problems (individually or with peers) with my guidance	O	O	O	O
d) Work problems together with the whole class with direct guidance from me	O	O	O	O
e) Work problems (individually or with peers) while I am occupied by other tasks	O	O	O	O
f) Apply facts, concepts, and procedures to solve routine problems	O	O	O	O
g) Explain their answers	O	O	O	O
h) Relate what they are learning in mathematics to their daily lives	O	O	O	O
i) Decide on their own procedures for solving complex problems	O	O	O	O
j) Work on problems for which there is no immediately obvious method or solution	O	O	O	O
k) Take a written test or quiz	O	O	O	O

QUESTION 20: When you teach mathematics to this class, how often do you usually ask students to do the following?

	Basis for instruction	*Supplement*	*Not used*
a) Textbooks	O	O	O
b) Workbooks or worksheets	O	O	O
c) Concrete objects or materials that help students understand quantities or procedures	O	O	O
d) Computer software for mathematics instruction	O	O	O

QUESTION 23: The following list includes the main topics addressed by TIMSS mathematics test. Choose the response that best describes when the students in this class have been taught each topic

Domain	*Topic*	*Before this year*	*This year*	*Not yet*
Number	a) Computing, estimating, or approximating with whole numbers	O	O	O
	b) Concepts of fractions and computing with fractions	O	O	O
	c) Concepts of decimals and computing with decimals	O	O	O
	d) Representing, comparing, ordering, and computing with integers	O	O	O
	e) Problem solving involving per cents and proportions	O	O	O
Algebra	a) Numeric, algebraic, and geometric patterns or sequences (extension, missing terms, generalization of patterns)	O	O	O
	b) Simplifying and evaluating algebraic expressions	O	O	O
	c) Simple linear equations and inequalities	O	O	O
	d) Simultaneous (two variables) equations	O	O	O
	e) Representation of functions as ordered pairs, tables, graphs, words, or equations	O	O	O
Geometry	a) Geometric properties of angles and geometric shapes (triangles, quadrilaterals, and other common polygons)	O	O	O
	b) Congruent figures and similar triangles	O	O	O
	c) Relationship between three-dimensional shapes and their two-dimensional representations	O	O	O
	d) Using appropriate measurement formulas for perimeters, circumferences, areas, surface areas, and volumes	O	O	O
	e) Points on the Cartesian plane	O	O	O
	f) Translation, reflection, and rotation	O	O	O
Data and Chance	a) Reading and displaying data using tables, pictographs, bar graphs, pie charts, and line graphs	O	O	O
	b) Interpreting data sets (e.g., draw conclusions, make predictions, and estimate values between and beyond given data points)	O	O	O
	c) Judging, predicting, and determining the chances of possible outcomes	O	O	O

Research News

Karen Barden Research Division

Conferences and seminars

Association for Language Learning Annual Conference

The annual Language Word conference took place in Newcastle in March under the theme of *ALL Connected*, emphasising the importance of joining together across educational sectors to achieve the best in language learning. Frances Wilson, OCR, presented a paper entitled *Not dumbing down but stimulating up: reading resources for the reformed GCSEs languages classroom* based on a study completed in the Research Division. The paper was co-authored with Katherine Smith, OCR.

Southeast Asian Ministers of Education Organization (SEAMEO) Regional Language Centre (RELC)

The 50th RELC International Conference was held in Singapore in March. The main theme was *Transcending Boundaries in Language Learning: Language Arts and ELT Across the Curriculum*.

Stuart Shaw, Cambridge International Examinations, presented a paper co-authored with his colleagues Helen Imam and Sarah Hughes entitled *Language Rich: Insights from Multilingual Schools*.

British Society for Research into Learning Mathematics (BSRLM)

The BSRLM is a major forum for sharing research in Mathematics education in the UK. At the conference held in June, Jessica Munro, Research Division, presented a paper entitled *A comparison of the Applied Mathematics content in international qualification*. *A paper on Students' perceptions of A level Further Mathematics as preparation for undergraduate Mathematics* was presented by Ellie Darlington, Research Division.

Journal of Vocational Education and Training (JVET)

The JVET 11th International Conference was held in July at Worcester College, Oxford, with the theme of *Researching Vocational Education and Training*. Jackie Greatorex presented a paper co-authored with her Research Division colleagues Joanne Ireland, Prerna Carroll and Sylvia Vitello on *Linking instructional verbs from assessment criteria to mode of assessment*. Martin Johnson, Research Division, presented a paper on *What types of feedback support (assessors') professional learning?*

Publications

The following articles have been published since Issue 19 of *Research Matters*:

Benton, T. and Elliott, G. (2015). The reliability of setting grade boundaries using comparative judgement. *Research Papers in Education*. Advance online publication available at: http://dx.doi.org/10.1080/02671522.2015.1027723

Bramley, T. (2015). Book review: Rasch Measurement in the Social Sciences and Quality of Life Research. *Europe's Journal of Psychology*, *11*(1), 169–171. Available online at: http://ejop.psychopen.eu/issue/view/58

Child, S., Theakston, A. and Pika, S. (2014). How do modelled gestures influence preschool children's spontaneous gesture production? Social vs semantic influence. *Gesture*, *14*(1), 1–25. Available online at: http://www.jbe-platform.com/content/journals/10.1075/gest.14.1.01chi

Child, S., Johnson, M., Mehta, S. and Charles, A. (2015). Finding the common ground: Teachers' and employers' representations of English in an assessment context. *English in Education*, *49*(2), 150–166. Available online at: http://onlinelibrary.wiley.com/enhanced/doi/10.1111/eie.12066/

Crisp, V. (2015). Exploring the difficulty of mathematics examination questions for weaker readers. *Educational Studies*, *41*(3), 276–292. Available online at: http://dx.doi.org/10.1080/03055698.2014.992863

Darlington, E. (2015). What benefits could extension papers and admissions tests have for university mathematics applicants? *Teaching Mathematics and its Applications*. Advance online publication available at: http://teamat.oxfordjournals.org/content/early/2015/03/17/teamat.hrv003.full.pdf+html

Darlington, E. and Dunn, K. (2015). Motivations for case study selection in GCSE geography. *Teaching Geography*, *40*(1), 20–21. Available online at: http://www.geography.org.uk/Journals/Journals.asp?articleID=1277

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Newton, P.E and Shaw, S.D. (2015). Disagreement over the best way to use the word 'validity' and options for reaching consensus. *Assessment in Education: Principles, Policy & Practice*. Advance online publication available at: http://dx.doi.org/10.1080/0969594X.2015.1037241

Further information on all journal papers and book chapters can be found on our website: http://www.cambridgeassessment.org.uk/our-research/all-published-resources/journal-papers-and-book-chapters/

Reports of research carried out by the Research Division for Cambridge Assessment and our exam boards, or externally funded research carried out for third parties, including the regulators in the UK and many ministries overseas, are also available from our website at: http://www.cambridgeassessment.org.uk/our-research/all-published-resources/research-reports/

Statistical Reports

Tim Gill Research Division

The on-going *Statistics Reports* Series provides statistical summaries of various aspects of the English examination system, such as trends in pupil uptake and attainment, qualifications choice, subject combinations and subject provision at school. These reports, mainly produced using national-level examination data, are available in both PDF and Excel format on our website:
http://www.cambridgeassessment.org.uk/our-research/all-published-resources/statistical-reports/

The most recent additions to this series are:

- *Statistics Report Series No.76: A Level Uptake and Results by Gender, 2004-2013*
- *Statistics Report Series No.77: GCSE Uptake and Results by Gender, 2004-2013*
- *Statistics Report Series No.78: A Level Uptake and Results, by School Type 2004-2013*
- *Statistics Report Series No.79: GCSE Uptake and Results, by School Type 2004-2013*
- *Statistics Report Series No.80: Uptake of GCSE subjects 2014*
- *Statistics Report Series No.81: Provision of GCSE subjects 2014*
- *Statistics Report Series No.82: Uptake of GCE A level subjects 2014*
- *Statistics Report Series No.83: Provision of GCE A level subjects 2014*
- *Statistics Report Series No.84: Uptake of level 2 qualifications in English schools 2014*
- *Statistics Report Series No.85: Provision of level 2 qualifications in English schools 2014*
- *Statistics Report Series No.86: Uptake of level 3 qualifications in English schools 2014*
- *Statistics Report Series No.87: Provision of level 3 qualifications in English schools 2014*
- *Statistics Report Series No.88: Age distribution of GCSE candidates in England in 2013.*

10th Anniversary cover feature answers

Karen Barden Research Division

This 10th Anniversary issue coincides with the publication month of the very first issue of *Research Matters* which was published in September 2005. We have now published 20 'standard' issues to date and three 'Special Issues'. Were you able to spot the issue numbers and publication dates from our commemorative cover feature? Here are the answers:

From top left to right as illustrated:

Special Issue 2: Comparability, October 2011
Issue 18, Summer 2014
Issue 2, June 2006
Issue 6, June 2008
Issue 8, June 2009

Issue 13, January 2012
Issue 15, January 2013
Issue 12, June 2011
Issue 14, June 2012
Issue 1, September 2005

Issue 4, June 2007
Issue 19, Winter 2015
Issue 9, January 2010
Issue 10, June 2010
Issue 7, January 2009

Issue 3, January 2007
Issue 11, January 2011
Issue 5, January 2008
Issue 17, January 2014
Issue 16, June 2013.

If you wish to check your answers in full to see all of the covers of the last 10 years in full size, all previous issues of *Research Matters* are available from our website: www.cambridgeassessment.org.uk/our-research/all-published-resources/research-matters/

A limited number of back issues of *Research Matters* are available on request. Please email: researchprogrammes@cambridgeassessment.org.uk stating the issue number and/or copy of the Special Issue you are interested in.

Requests from readers wishing to be added to our mailing list to receive regular copies of all future issues of *Research Matters* are also welcome at the above email address.

Many thanks to all our contributors and reviewers over the last 10 years, and to all our readers for the ongoing support and feedback. Special thanks also to our designer, George Hammond, for his expert contribution to the production of *Research Matters* from Day 1, and for his help with the 10th Anniversary cover feature design and answers. Here's to the next 10 years!

All details are correct at the time of going
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