

tariff scores by taking more qualifications is not indicative of higher ability levels (as demonstrated by Figures 1 and 2). For instance, students achieving 5 A\* grades at A level (700 UCAS points) are probably not much more able than those achieving 4 A\* grades (560 points).

This suggests that the current tariff measure, based on total points score could be improved by taking account of this in some way.

Finally, it is worth considering to what extent admissions tutors (particularly those with many years' experience) are aware of some of these issues and account for them when making offers to students. They may, for instance, take some account of the number of qualifications contributing to a student's UCAS tariff score, or they may value points scores gained from some qualifications more than scores

gained from other qualifications. This should go some way to making up for any lack of equivalence between UCAS tariff scores for different qualifications.

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## Post-16 Mathematics qualifications: Differences between GCE A level, International A level, Cambridge Pre-U and Scottish examination questions

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### Introduction

This article describes the application of a taxonomy in order to compare and contrast the mathematical skills required to answer examination questions from four different post-16 Mathematics qualifications taken by students both in the UK and overseas: A levels and Advanced Subsidiary (AS) levels, International A and AS levels, Cambridge Pre-U, and Scottish Highers and Advanced Highers. Though the precise content and structure of the different qualifications differ slightly, they are all qualifications which should provide students with a sound basis for university study in Mathematics. All UK universities accept these qualifications as prerequisites for their Mathematics courses. It is therefore of interest to establish whether the questions asked in the assessments of these qualifications require the same kinds of mathematical skills. If there are notable differences among the qualifications, this could suggest that there might be corresponding differences in how well prepared students are for studying Mathematics at university.

In recent years the number of UK schools offering alternative qualifications to General Certificate of Education (GCE) A level has increased. This perhaps may be attributable to head teachers' diminishing confidence in the A level system, with 67 per cent of those surveyed by the Office of Qualifications and Examination Regulation (Ofqual) in 2014 reporting that constant changes to the A level system were of concern. Furthermore, 12 per cent of head teachers surveyed said that they thought that international qualifications such as the International Baccalaureate (IB) and the Cambridge Pre-U were more challenging than A levels. A levels have been criticised for being "oblique

at measuring academic ability" (de Waal & Cowen, 2007, p.8), with mathematicians in Higher Education (HE) claiming that it is easy for A level Mathematics students to "learn the exam" rather than the subject" (Higton et al., 2012, p.58).

Furthermore, concerns are regularly voiced by educational researchers and university admissions and teaching staff regarding the preparedness of new undergraduate mathematicians. For example:

- a restructure of the modular system in A level Mathematics in 2006 resulted in complaints that there was diminishing content (Bassett, Cawston, Thraves, & Truss, 2009; Porkess, 2003, 2006) and that the newer examinations were easier (Qualifications and Curriculum Authority, 2007);
- the modular system of examinations has been criticised for failing to test students' synoptic understanding of Mathematics (Hodgson & Spours, 2004; Quinney, 2008; Wilde, Wrighton, Hayward, Johnson, & Skerrett, 2006);
- some have commented that the A level does not prepare students well for undergraduate Mathematics (Smith, 2004);
- the Engineering and Physical Sciences Research Council (EPSRC) has claimed that "mathematical A-levels are not as rigorous as they used to be." (EPSRC, 2004, p.17);
- the value of the top grade has been questioned, as some stakeholders have claimed that it can be "...achieved through high levels of accuracy rather than extended mathematical reasoning." (Smith, Mitchell, & Grant, 2012, p. 30); and
- claims have been made that standards are falling in the A level, that higher grades are becoming easier to obtain (Coe, 2011; Lawson, 1997).

A rigorous analysis of the types of skills required to answer Mathematics examination questions in GCE A level and its equivalents should therefore be an important source of evidence for these various debates. In particular, it can help shed light on the validity of the assessments (e.g., Shaw, Crisp & Johnson, 2012) in the sense of whether the skills actually assessed match those that are claimed to be assessed; and it can help the various 'users' of these different qualifications (e.g., students, teachers, university admissions tutors) to understand any differences among them.

This article first describes the different Mathematics qualifications that were analysed, then describes the taxonomy used to classify the skills, before presenting and discussing the findings.

## Mathematics qualifications

The following post-16 Mathematics qualifications were analysed in this research:

**GCE A levels:** This is the most common qualification taken by students aged 16–19 in England, Wales and Northern Ireland. A level Mathematics was the most popular subject in 2014, constituting 10.4 per cent of all A levels examined (Joint Council for Qualifications, 2014).

Most students take three A levels (Gill, 2014), choosing from a wide variety of subjects. They may stop after one year and earn an AS level by taking examinations in the units which were taught in the first year of the A level. In Mathematics, Applied units are available in three topics: Mechanics, Statistics and Discrete Mathematics<sup>1</sup> and students may choose to study a narrow or broad range of these topics (see Figure 1).

The qualifications are currently offered by five different awarding bodies, which are all accredited by Ofqual, the regulator of qualifications and assessments in England. Mathematics and Further Mathematics<sup>2</sup> each consist of six equally-weighted units (three of which constitute the AS level) which are individually examined. Further Mathematics may only be studied in addition to Mathematics, with the units in Further Mathematics building upon the knowledge of earlier units taken as part of A level Mathematics.

**International AS and A levels:** These are a very popular qualification all around the world and an increasing number of schools in the UK are beginning to offer them.

Cambridge International Examinations (Cambridge) offers A levels in Mathematics and Further Mathematics and the way in which students choose and take the International A level is much the same as with the GCE A level: students most often take three subjects from a wide variety of their choosing, the A level is studied over a two-year period, and AS levels are available. However, Cambridge A levels are assessed linearly, unlike the modular assessment in GCE A levels, and the number and content of the units is not the same (see Figure 1).

**Cambridge Pre-U's:** This is a relatively recent qualification, which currently has a small number of candidates, although it is continuing to grow.

Launched in 2008, the Cambridge Pre-U is a post-compulsory qualification which is aimed at those students wishing to go on to tertiary study (see University of Cambridge International Examinations, 2012).

The Cambridge Pre-U Mathematics is divided into three components, each with one two-hour examination of equal weighting.

Though its uptake is small, it is recognised by UK universities and an increasing number of institutions across the globe. Steinberg and Hyder (2011) describe the Cambridge Pre-U as being among the best international qualifications, with some arguing that it is more demanding than GCE A levels, partly because of its linear (as opposed to modular) assessment structure (University and Colleges Admissions Service [UCAS], 2008a). It has also been found to act as a good predictor of degree outcome (Gill & Vidal Rodeiro, 2014).

**Scottish Highers and Advanced Highers:** This is the most common qualification for students in Scotland. Students aged 16–19 study for Highers and, sometimes, Advanced Highers, typically studying four or five subjects for Highers over the course of one year. In 2012, 86 per cent of students doing five or more Highers took Mathematics as one of their subjects, with nearly 5,000 doing Advanced Higher Mathematics, and over 18,000 doing Higher Mathematics (Nuffield Foundation, 2013, p.5).

Higher Mathematics, which only covers topics in Pure Mathematics, consists of three compulsory progressive<sup>3</sup> units which are assessed by the means of two terminal examinations. Advanced Highers in Mathematics and Applied Mathematics are available, and are each assessed by the means of one terminal examination based on three compulsory progressive units (Scottish Qualifications Authority, 2010).

Advanced Highers are considered to be equivalent in standard to the first year of undergraduate study in that subject at Scottish universities, where typical degree programmes take a year longer to complete than in the rest of the UK. It is possible for students with Advanced Highers (or A levels) to skip the first year of undergraduate study if they wish. Consequently, the Highers are generally viewed by universities as approximately equivalent to AS level, and Advanced Highers to A level (The Association of Graduate Careers Advisory Services [AGCAS] Scotland, 2008; Munro, 1998; UCAS, 2008b).

Though all of the four aforementioned qualifications are accepted by universities as prerequisites to study undergraduate Mathematics, they do not all necessarily follow the same structure or examine exactly the same content. Figure 1 shows how the different qualifications have different compulsory and optional elements, and how the total number of examined elements differs.

## MATH Taxonomy

A number of taxonomies are available for analysis and classification of Mathematics questions according to a set of criteria. *Bloom's Taxonomy of Educational Objectives* (Bloom, Engelhart, Furst, Hill, & Krathwohl, 1956) is perhaps the most famous example of this. However, whilst Bloom describes levels of learning and is not subject-specific, the *MATH Taxonomy* (Smith et al., 1996) is a modification of it for the context of undergraduate Mathematics, and can be used to describe the skills required to complete a task. It classifies skills according to three broad groups (A, B, C), which have two or three subgroups (see Table 1). It makes no claims to describe the level of difficulty of a question; that is, a Group A question might be more difficult than a Group C question.

1. Referred to in some specifications as Decision Mathematics.

2. Only the awarding body AQA offers A level Statistics, with Oxford, Cambridge and RSA (OCR) Examinations offering an AS level in Statistics.

3. That is, each unit builds upon the content of previous units.

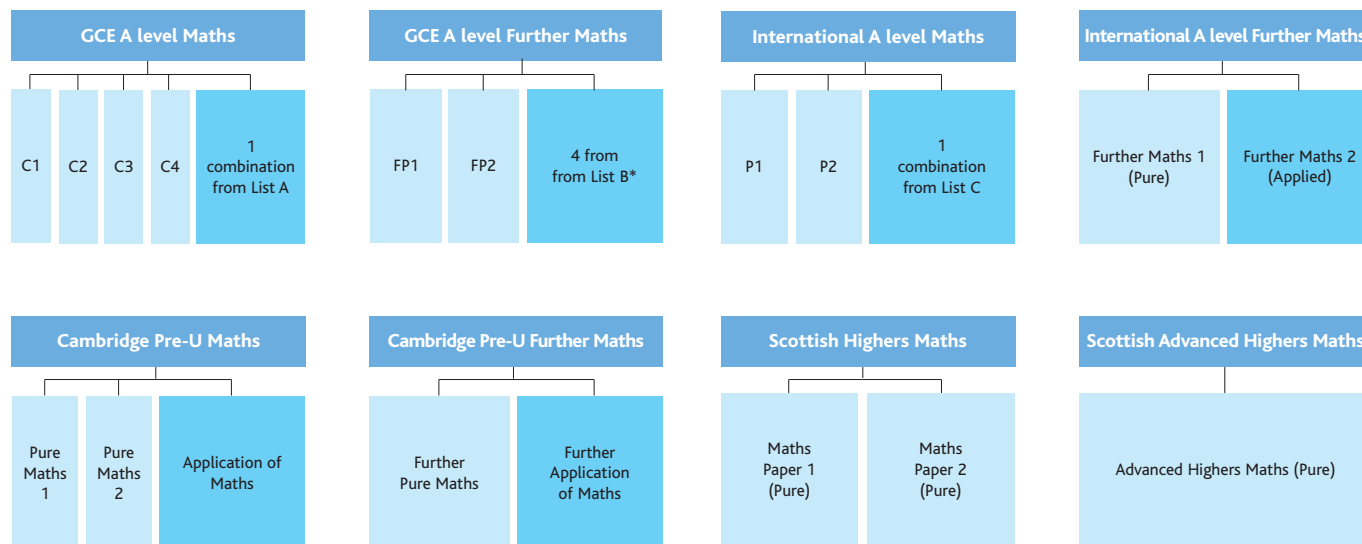


Figure 1: Different structures of post-16 Mathematics qualifications

\* Students may not take a unit in Further Mathematics that they have already taken in Mathematics.

List A M1+M2, D1+D2, S1+S2, S1+D1, M1+S1, M1+D1

List B FP3-4, M1-5 (where available), S1-4, D1-2

List C M1+S1, M1+M2, S1+S2

Where 'C' is Core Pure Mathematics, 'P' is Pure Mathematics, 'M' is Mechanics, 'S' is Statistics, 'D' is Discrete Mathematics and 'FP' is Further Pure Mathematics.

Table 1: Groups of mathematical skills according to the MATH Taxonomy

Group	Subgroup	Outline
A	Factual knowledge and fact systems (FK&FS)	Factual recall and routine procedures
	Comprehension (Comp)	
	Routine use of procedures (RUOP)	
B	Information transfer (IT)	Using existing mathematical knowledge and techniques in new ways
	Application in new situations (AINS)	
C	Justifying and interpreting (J&I)	Application of conceptual knowledge to construct mathematical arguments
	Implications, conjectures and comparisons (IC&C)	
	Evaluation	

Group C skills have been found to be associated with students who have deeper understandings of the material (Malabar & Pountney, 2002), and associated with university-level Mathematics (Barnett, 1990; Pountney, Leinbach, & Etchells, 2002).

Existing work suggests that GCE A level Mathematics examinations rely heavily on Group A tasks (Darlington, 2013a, 2014; Etchells & Monaghan, 1994), as do undergraduate Mathematics examinations (Ball, Smith, Wood, Coupland, & Crawford, 1998; Darlington, 2013a, 2013b, 2014, 2015; Smith et al., 1996). This is by no means a phenomenon confined to the UK. For example, work by Crawford (1983, 1986) and Crawford, Gordon, Nicholas, and Prosser (1993) found that new undergraduate mathematicians in Australia had very little prior experience of Group C tasks.

Examples of A level Mathematics and Further Mathematics, undergraduate Mathematics and university entrance examination questions associated with each group and subgroup in the *MATH Taxonomy* may be found in Darlington (2013a, 2013b, 2014, 2015). Table 2 gives examples of questions from GCE A level, International A level, Cambridge Pre-U and Scottish Highers and Advanced Highers papers in Mathematics and Further Mathematics which would fit into each category and subcategory.

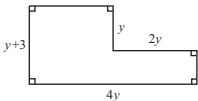
### Sample

Question papers from A level Mathematics and Further Mathematics were analysed so as to give an indication of the nature of the skills required at the two different levels. Analysis was conducted at the subquestion level. For maximum contrast, the introductory Pure Mathematics unit was analysed from the Mathematics qualifications, along with the most 'advanced' Pure Mathematics unit from Further Mathematics (see Table 3). All analyses were conducted on the five<sup>4</sup> most recent publicly-available question papers from all awarding bodies, where applicable, at the time of analysis.

Pure Mathematics units were selected rather than Applied Mathematics units because: (1) the different qualifications had different emphases on Applied Mathematics, and as such the content was not comparable; and (2) Applied Mathematics units were not available for some of the qualifications (see Figure 1). Furthermore, it is highly likely that the majority of marks in Applied Mathematics units would be skewed towards Group B due to its focus (see Table 1).

4. Only four Cambridge Pre-U papers were available at the time of analysis.

Table 2: Examples of questions in each MATH Taxonomy subgroup

Category	GCE A level <sup>5</sup>	International A level	Cambridge Pre-U	Scottish Highers
<b>GROUP A</b>				
FK&FS	Sketch the curve $y = \frac{1}{x}$			Write down the derivative of $\sin^{-1}x$ .
Comp	The curve $y = -\sqrt{x}$ is stretched by a scale factor of 2 parallel to the x-axis.  State the equation of the curve after it has been stretched.	The function $f$ is defined by $f: x \mapsto 2x^2 - 12x + 13$ for $0 \leq x \leq A$ , where $A$ is constant.  State the value of $A$ for which the graph of $y = f(x)$ has a line of symmetry.	Let $I_n = \int_0^{\alpha} \tanh^2 \theta \, d\theta$ for $n \geq 0$ , where $\alpha > 0$ . Given that $\alpha = \frac{1}{2} \ln 3$ , evaluate $I_0$ .	A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 3$ and $u_0 = 1$ .  What is the value of $u_2$ ?
RUOP	Express $\sqrt{18} - \sqrt{2}$ in simplified surd form.	Find the first 3 terms in the expansion of $(2x - \frac{3}{x})^5$ in descending powers of $x$ .	Find the equation of the line passing through the points $(-2, 5)$ and $(4, -7)$ . Give your answers in the form $y = mx + c$ .	For what value of $\lambda$ is $\begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ -1 & \lambda & 6 \end{pmatrix}$ singular?
<b>GROUP B</b>				
IT	Sketch the curve $y = 9x^2 + 18x - 7$ , giving the coordinates of all intercepts with the axes.	Sketch the curve $y = (x - 2)^2$ .	Sketch, on a single diagram, the graphs of $y = e^{\frac{1}{5}x}$ and $y = x$ and state the number of roots of the equation $e^{\frac{1}{5}x} = x$ .	Describe the loci in the complex plane given by $ z + i  = 1$
AINS	A rectangular tile of length $4y$ cm and width $(y + 3)$ cm has a rectangle of length $2y$ cm and width $y$ cm removed from one corner as shown in the diagram.    Given that the perimeter of this tile is between 20 cm and 54 cm, determine the set of possible values of $y$ .	A television quiz show takes place every day. On day 1 the prize money is \$1000. If this is not won the prize money is increased for day 2. The prize money is increased in a similar way every day until it is won. The television company considered the following two different models for increasing the prize money.  Model 1: Increase the prize money by \$1000 each day. Model 2: Increase the prize money by 10% each day.  On each day that the prize money is not won the television company makes a donation to charity. The amount donated is 5% of the value of the prize on that day. After 40 days the prize money has still not been won. Calculate the total amount donated to charity (i) if Model 1 is used (ii) if Model 2 is used	The curve $C$ has Cartesian equation $x^2 - xy + y^2 = 72$ .  Find the exact area of the region of the plane in the first quadrant bounded by $C$ , the $x$ -axis and the line $y = x$ .  Deduce the total area of the region of the plane which lies inside $C$ and within the first quadrant.	The radius of a cylindrical column of liquid is decreasing at the rate of $0.02 \text{ ms}^{-1}$ , while the height is increasing at a rate of $0.01 \text{ ms}^{-1}$ .  Find the rate of change of the volume when the radius is 0.6 metres and the height is 2 metres.
<b>GROUP C</b>				
J&I	The variables $x$ and $y$ satisfy the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}$  Explain briefly why there is no particular integral of either of the forms $y = ke^{3x}$ or $y = kxe^{3x}$ .	The function $g$ is defined by $g: x \mapsto 2x^2 - 12x + 13$ for $x \geq 4$ . Explain why $g$ has an inverse.	Let $f(x) = x^2$ and $g(x) = 7x - 2$ for all real values of $x$ .  Give a reason why $f$ has no inverse function.	Prove by induction that, for all positive integers $n$ , $\sum_{r=1}^n (4r^3 + 3r^2 + r) = n(n+1)^3$
IC&C	$w$ denotes the complex number $\cos \frac{2}{5}\pi + i \sin \frac{2}{5}\pi$  Write down a polynomial equation of degree 5 which is satisfied by $w$ .	For the series $\sum_{n=1}^N \frac{4n+9}{(n+2)(n+3)(2n+3)(2n+5)}$ find the sum to infinity.	The cubic equation $x^3 + x^2 + 7x - 1 = 0$ has roots $\alpha, \beta, \gamma$ .  State what can be deduced about the nature of these roots.	Let $n$ be a natural number.  For each of the following statements, decide whether it is true or false. If true, give a proof; if false, give a counterexample. A If $n$ is a multiple of 9 then so is $n^2$ B If $n^2$ is a multiple of 9 then so is $n$ .

Note: No examples of Group C's 'Evaluation' questions could be found in any of the papers analysed.

5. All GCE A level examples taken from OCR question papers.

**Table 3: Question papers analysed**

Qualification	Introductory Pure Mathematics Question Paper	'Advanced' Pure Mathematics Question Paper
GCE A level	Core Pure 1	Further Pure 3
Cambridge International A level	Pure Mathematics 1	Further Pure Mathematics 1
Cambridge Pre-U	Pure Mathematics 1 Paper 1	Further Pure Mathematics
Scottish Higher/Advanced Higher <sup>6</sup>	Higher Mathematics Paper 1	Advanced Higher Mathematics

Note: See Figure 1, which illustrates the selection of these particular examinations.

GCE A level question papers from the different awarding bodies were treated together as one group because all awarding bodies are regulated by Ofqual, and a number of studies have found that there are no differences between the awarding bodies' papers in terms of difficulty. For example, Taverner (1996) compared students' A level Information Systems (ALIS) scores<sup>7</sup> with their A level results in order to see whether there were any differences between awarding bodies, and found no significant differences.

Six examples of analysis for each qualification were checked by a Mathematics education specialist, and there were no disagreements in the classifications of skills for those questions.

## Results

For each question paper, the proportions of marks awarded for Group A, B and C tasks were calculated, and averaged across the qualification to enable comparisons to be made.

The results of this analysis are given in two sections: (1) for qualifications equivalent to GCE A level Mathematics C1, and (2) for qualifications equivalent to GCE A level Further Mathematics FP3.

Data for GCE qualifications and the associated analysis are taken from Darlington (2015).

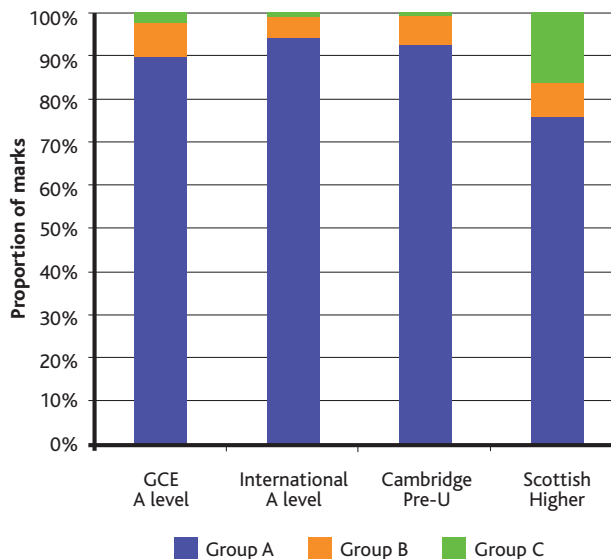
### A level Mathematics equivalent

For all of the qualifications analysed, the majority of the marks awarded in the question papers were for Group A skills (see Figure 2), of which 88.5% were routine uses of procedures.

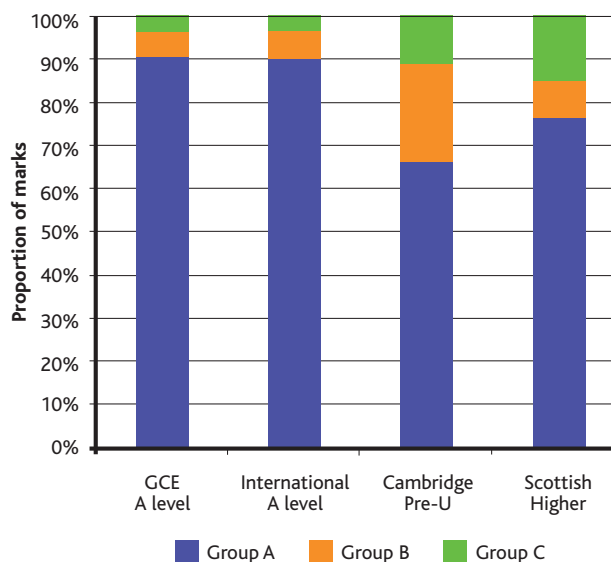
Figure 2 shows that, with 75.43% of marks, Scottish Highers put less focus on Group A skills than all of the other qualifications. The International A level had the highest proportion of marks awarded for Group A skills (94.13%). Scottish Highers awarded substantially more marks for Group C skills (16.57%) than GCE (2.25%) and International (0.52%) A levels and the Cambridge Pre-U (0.31%). It is unsurprising that there were relatively few marks awarded for Group B skills in all of the qualifications (ranging from 5.33% of the marks in the International A level to 8.47% in the GCE A level) because Pure Mathematics examinations were analysed, whilst Group B skills are more associated with Applied Mathematics.

6. Introductory question paper from Higher Mathematics, and Advanced paper from Advanced Higher Mathematics.

7. ALIS tests are run by the Centre for Evaluating and Monitoring (CEM) at the University of Durham. The scores act as performance indicators for post-16 students, using data from GCSE grades and CEM's baseline tests.



**Figure 2: Contrasts in question type composition in qualifications equivalent to A level Mathematics C1**



**Figure 3: Contrasts in question type composition in qualifications equivalent to A level Further Mathematics FP3**

### A level Further Mathematics equivalent

As with qualifications equivalent to GCE A level Mathematics, the majority of marks in question papers equivalent to GCE A level Further Mathematics FP3 were for Group A skills (see Figure 3).

Both the GCE (90.6%) and the International A levels (89.45%) awarded more marks for Group A skills than Scottish Advanced Highers (76.6%) and the Cambridge Pre-U (66.04%). The converse could be said for Group C skills, where the A levels awarded fewer marks than the other qualifications. The Cambridge Pre-U awarded significantly more marks for Group B skills (23.34%) than the other qualifications.

## Discussion and Conclusion

Analyses conducted for the purpose of this article revealed that the majority of marks awarded in examinations at both the C1-equivalent and FP3-equivalent level were for Group A skills. That is, the majority of



questions required students to demonstrate an ability to answer questions which could be prepared for by doing drill-style practice, something perhaps aided by the apparent frequency of the topics and similar types of questions posed year-to-year (see Darlington, 2013a). This was the same for all qualifications, though it was more extreme in some instances than others. Specifically, the concentration of Group A skills appeared to be higher in both Mathematics A levels than Scottish Highers, and higher in A level Further Mathematics than both Scottish Advanced Highers and the Cambridge Pre-U.

It should be noted that the GCE A level C1 examinations can be taken after just one school term of learning, whereas the Cambridge A level and Scottish Highers are taken at the end of one year of study, and the Cambridge Pre-U after two. Therefore, it *might* be possible that the non-GCE A level students may be better-practiced with certain techniques and therefore more freely-able to use this Mathematics when eventually assessed later on in their study of the qualification. Consequently, there may be more scope for assessment to assess a wider range of skills.

Whilst this research might suggest that A levels, Cambridge Pre-U's and international qualifications may not place an emphasis on students demonstrating certain Group C skills, this does not necessarily have to be interpreted as a criticism. Not only do many students do these qualifications out of interest or as a service subject for Science or Social Science degrees, but developing and marking Group B and Group C tasks can be time-consuming and challenging for examiners and teachers (Leinch, Pountney, & Etchells, 2002). However, one could question the validity of the assessment objectives (AOs) of A level Mathematics, as current guidelines describe AO2 as:

*Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.* (Ofqual, 2011, p.12)

As AO2 is supposed to constitute at least 30 per cent of the overall marks for the qualification, and shares a similar meaning to the definition of Group C skills (see Table 1), this brings into question whether A level Mathematics effectively examines that particular AO. However, it should be noted that this analysis using the *MATH Taxonomy* was conducted on a subquestion level, whereas AOs are categorised on a mark-by-mark basis. Hence, a question classified here as Group C may, in reality, reward students for a number of AOs depending on the mark allocation. Therefore, comparisons between AOs and the groups in the *MATH Taxonomy* can only be crude – AOs refer to what is assessed and what is rewarded by the mark scheme, whereas the *MATH Taxonomy* refers to the skills required to answer the questions.

The differences in the mathematical skills assessed between these qualifications should be read with caution. Whilst 30 GCE A level question papers were analysed, only four or five question papers from the other qualifications were subjected to the same analysis due to limited availability. Furthermore, Scottish universities have different teaching structures to those in the rest of the UK in accordance with the different secondary school examinations there, meaning that Highers do not serve exactly the same purpose for universities in Scotland as A levels do for English universities. The reader should not necessarily interpret the data as meaning that any of these qualifications are 'better' or 'worse' than the others, but recognise that some differences do appear to exist between them.

However, this is perhaps an opportunity to recognise the value of Mathematics admissions tests and extension papers such as the *Sixth Term Examination Papers* and the *Advanced Extension Award*, as described by Darlington (2015), rather than any apparent shortcoming of post-16 qualifications. However, although problems with access to prepare for and take these assessments mean that these should not be seen as a 'solution' to the gap between the skills assessed at A level and university (Darlington 2014, 2015).

GCE A level Mathematics and Further Mathematics are currently undergoing revisions which will reportedly see them involve more problem solving in examinations, as well as restructures to units, syllabuses and content. Quite what impact this will have in terms of the skills required to answer examination questions remains to be seen; however, the research here suggests that GCE A level Mathematics and Further Mathematics questions are not vastly different to some of the alternatives available in both the UK and overseas.

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