The position of statistics in the Key Stage 4 curriculum and assessments in England: a discussion paper

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1 The importance of statistics

We believe that it is vital that young people have a good understanding of statistics to help in their work, further study and more generally to understand the use of statistics by the media, government and other agencies to help make informed judgements. It is a crucial life skill.

Statistics is a key application of numeracy in its wider sense – termed "mathematical literacy" by Celia Hoyles. However, young people will only develop literacy regarding statistics skills when it is studied using realistic situations and realistic data.

> Alongside the modelling side of mathematical literacy, there is also the need to know how to calculate and estimate and to have a feel for numbers, percentages and proportions. Mathematical literacy is, however, much more than a set of simple and disconnected skills; and it goes well beyond a command of number or basic numeracy. **It is anchored in real data (often in the form of data output from spreadsheet models), and set in the local and global context of the work** (emphasis added). It involves an appreciation of the thresholds and constraints of a model (such as the limitations on factory output, the costs of machinery, or throughput of production lines**), flexibility in understanding different representations of the model (the columns of a spreadsheet; charts or graphs** (emphasis added), or, less commonly, symbolic forms), and being able to modify the model to improve the simulation of workplace practices and outcomes.
>
> (Hoyles et al, 2002, p. 12)

The report *Mathematical Skills in the Workplace* (Hoyles et al, 2002) identifies a range, which is by no means exhaustive, of mathematical skills required in certain sectors in the UK through interviews with a range of employers. The statistical skills identified by each type of employer are listed below (p. 15-17).

- Electronic Engineering and Optoelectronics
  "Statistics of normally-distributed data"
  "Communication of data and mathematical information"
- Financial Services
  "Ability to read, interpret and transform data from charts and spreadsheets"
  "Confidence in identifying, appreciating and using concepts of risk and probability"
  "Ability to use approximations, estimates and formal probabilities to model likely events"
- Food Processing
  "The ability to read, interpret, transform and communicate data, in the form of charts, graphs and numbers (in monitoring and improving operational efficiency of a production line)"
- Health Care
  "Communication of data (charts, numbers)"
  "Statistics of normal distributions (for establishing 'normal ranges' for new … equipment)"

"2 and 3-dimensional representations of data"

- Packaging

  "Measurement (statistical sampling)"

  "Statistics of normally-distributed data"

  "Communication of data (to improve operational efficiencies)"
- Pharmaceuticals

  "Statistics of normally-distributed data"

  "Data trending (correlation between chemical samples, and in the time variation of individual samples)"

  "Communicating mathematical and statistical information"
- Tourism

  "Identifying trends statistically"

  "Appreciating and using (informal) concepts of risk and probability"

In a focus group of employers run by OCR, data handling was highlighted as the third most important mathematical skill for the workplace, after problem solving and mental number skills. The importance of statistical skills has also been noted in the ACME report *Mathematical Needs* (2011). The report states that "off-the-shelf and purpose-designed computer software packages are creating ever more data sets, statistics and graphs" (p. 2). Later they identify "…the present lack of statistics, mathematical modelling and problem solving skills" (p. 4). ACME is reporting concerns regarding about the level of young people's statistical, and other, skills in the UK at the same time as the importance of those skills is increasing.

As well as the use of data in the workplace, data are used to help make sense of the world in a range of disciplines, including biology, chemistry, physics, psychology, geography, economics, medicine, engineering, environmental science, sociology and more. ACME notes that "the quantitative demands of almost all university courses are increasing; even subjects like history … now recognize the importance of statistics" (2011, p. 1). Study of statistics is optional at A Level and in fact some students studying A Level Mathematics do not select any statistics modules. It is hence possible that, even among strong students of A Level Mathematics, some will have insufficient grounding in statistics to take a degree in one of the above subjects. If new proposals for post-16 courses come to fruition, enabling everyone to continue with mathematics post-16, then a good grounding in statistics before age 16 is likely to enhance progression to these courses.

As well as its use in higher education and the workplace in the UK, statistics provides young people with tools of analysis to interpret and understand information they will come across every day. Michael Gove asked the Royal Society, "… what about statistics? There are a vast array of issues that people are confronted with in daily life – from health scares to claims about the effect of drugs to financial news – which require statistical understanding" (2011). Statistics plays an increasingly important role in the modern world and it is right that it is part of the core of the National Curriculum.

Statistics is crucial for everyone, irrespective of what they will be doing after the age of 16. This is why we believe the subject needs to remain compulsory. However, in order for it to be taught as fully as possible in schools we might want to consider whether it should be recognised as a subject in its own right.

2 The current position

Traditionally and at present, the vast majority of statistics content has been placed within the curriculum for mathematics. The statistics content taught at KS4 is therefore generally delivered within mathematics lessons, and the assessment of statistics has had to fit within the criteria for GCSE Mathematics.

The main advantages of the current system are:

- All students are taught the same topics in statistics: there is universal provision.
- All students complete an assessment (GCSE Mathematics) which includes questions concerning statistics: there is a universal form of assessment.

However, the disadvantages of statistics being included in the assessment for GCSE Mathematics and the national curriculum for mathematics are:

- Statistics is assessed through routine closed questions.
- The questions concerning statistics appear at different points within a GCSE Mathematics paper, depending on the apparent difficulty of the statistical techniques being tested.
- As a result, it is not possible to develop a sense of coherence between the statistics questions on a GCSE Mathematics paper – they exist as 'islands' within the mathematics questions.
- Statistics can become marginalised when it forms a minority part of the mathematics curriculum.
- Its nature, apparently mathematical but actually very distinct from mathematics, may be lost.

OCR's focus group of teachers noted these effects, with one teacher saying:

> I don't think anything is gained by doing a lot of work on drawing charts and things like that [as in GCSE Mathematics]. The big difficulty is how you can make people learn statistics, and understand it, and interpret it and know what makes sense and what doesn't when questions of that sort don't get set on the [mathematics] exams. (January 2013)

Roger Porkess (2012), in *The Future of Statistics in our Schools and Colleges*, highlights the importance of what he terms the "statistics cycle", a process with four distinct stages. However, the restrictions of format and space of the GCSE Mathematics examinations mean that the vast majority of statistics questions assess only data presentation. The important skills of problem analysis, data collection and data analysis, that form the other three stages of the cycle, cannot be assessed to any great extent.

As an applied subject, statistics needs a broad range of contexts. Indeed, unlike mathematics, statistics is meaningless without context. Clearly, there is a range of other subjects taught in schools which naturally give rise to these contexts. A problem with the current delivery of statistics entirely within mathematics lessons is that these contexts are seldom explored (Porkess, 2012).

There is also currently a slight mismatch between what is taught within the statistics element of the curriculum and the statistics which could be used in other subjects at GCSE level. Geography, for example, could make extensive use of Spearman’s rank correlation coefficient, but correlation is not covered in the current programme of study beyond informally looking at scatter graphs. Sampling methods can also be more realistically covered in other subjects than the narrow approach currently used in mathematics, and can include physical sampling methods such as transects, used in ecology and geography for example.

Making better links between statistics and other subjects is beneficial for both, but there also needs to be a coherent approach to statistics as its own subject, and mathematics departments will need to continue to play a leading role, ensuring quality of delivery across the school.

The GCSE in statistics is one way for the subject to have its own assessment space and coherent curriculum. However, only a minority of students are entered for dedicated GCSE Statistics specifications: 50,620 took the qualification in June 2012, compared to 675,789 for GCSE Mathematics (JCQ, 2012). We would suggest that a coherent study of statistics should be the norm for students, and not the exception.

3 The future of statistics

Porkess (2012) proposes several recommendations to improve delivery, assessment and attainment in this vital subject. The first two recommendations made in the report are about recognising the importance of statistics in society and statistics education, with which we wholly concur.

Recommendations 3 to 5 concern provision. We agree with the recommendation that all students should be given a good education in statistics to equip them with at least the basic knowledge and skills required to make progression in further education or the workplace and to help them better understand information. We also agree with the recommendation that staff in schools and colleges are aware of and emphasise to students the importance of statistics to them in the next stages of their lives. We agree that effective delivery will require teachers who are themselves confident with statistics and that mathematics teachers should, within each school, play a leading role regarding statistics.

Porkess' sixth and seventh recommendations are:

> Recommendation 6: Under present conditions, statistics is best placed in the mathematics curriculum.
>
> Recommendation 7: To ensure that sufficient account is taken of the importance of statistics, and of its special requirements, it should be represented separately from mathematics, but alongside it, when policy decisions are being made.
>
> (Porkess, 2012, p. 2)

Taken together, recommendations 6 and 7 appear contradictory – statistics cannot be both "in the mathematics curriculum" and "represented separately from mathematics but alongside it". However, we do understand the concern expressed regarding the position of statistics.

The main argument in favour of retaining statistics within the mathematics curriculum is that it goes a long way to ensuring coverage, as already noted. But, as we have also seen, and as Porkess agrees, there are problems with the current approach.

The key issues, then, are to ensure that **all students are taught a coherent body of statistics**, and that it **is validly and meaningfully assessed**, in the same way as for any other subject.

One way to ensure that the subject is awarded the status it deserves, and its unique facilitating nature is recognised, would be to list statistics as a full and **separate core** (mandatory) **subject** in the National Curriculum for England at KS3 and KS4. Basic probability theory should remain within the mathematics curriculum (even if it is also included in statistics).

Porkess' proposals regarding the assessment of statistics are as follows.

> Recommendation 11: The assessment techniques used should ensure that, at every level, students carry out work covering all the processes required to use statistics to solve problems and make decisions.
>
> Recommendation 12: The assessment of statistics within mathematics should be informed by good practice in other subjects.
>
> (Porkess, 2012, p. 3)

We agree that the way statistics is assessed within GCSE Mathematics is often unsatisfactory, and evidence from our focus groups supports this view. Many of the questions involve drawing particular graphs, performing standard algorithms or writing a very short comparison based on central tendency or spread. Porkess recommends reforming the assessment of GCSE Mathematics to improve the assessment of statistics. These recommendations are, of course, based on the previous recommendations to retain the statistics content within the mathematics content of the National Curriculum.

Creating a new type of assessment for statistics within GCSE Mathematics is likely to be highly problematic. Introducing a new type of assessment for statistics, such as coursework or a pre-released scenario for a statistics paper, would require an additional assessment component. Although these are indeed good ways of assessing students' application of their statistical knowledge, they do not sit well within the models currently used for GCSE Mathematics in England.

Such assessments would require more space than can be made available within the single GCSE in mathematics, in order that they are done in a valid, meaningful way. Such inclusion would, we feel, also compromise the GCSE assessment of the rest of mathematics, firstly in terms of space and secondly in terms of its impact on a student's grade in GCSE Mathematics. The different nature of the statistics component could mean that a student does much worse (or better) in this than in the mathematics component(s) and it could make the difference of a grade in either direction to their overall result for mathematics.

We believe it would be clearer for attainment in statistics to be reported separately from mathematics and, as for the curriculum, we believe that separating the assessment of statistics from mathematics would be beneficial.

Statistics and mathematics focus on different things, in different ways. Both mathematicians and statisticians recognise the difference between the subjects. The American statisticians Cobb and Moore (1997) identify the following differences between the two:

- Statistics is the study of variability and this gives it a particular set of subject content which distinguishes it from mathematics.
- Statistics only have meaning in context, whereas underlying mathematical relationships are obscured by contexts.
- Mathematical axioms and theorems deal in truth, whereas statistical methods allow inferences to be made when dealing with variability.

(Cobb and Moore, 1997)

Cobb and Moore go on to say that “statistics should be taught as statistics … not a subfield of mathematics” (p. 814). They point out that statistics makes great use of mathematics but, like other subjects such as economics or physics, it has its own space, concepts and language to explore and use. Fundamentally, the teaching of statistics should concern “the basics of statistical thinking … the need for data, the importance of data production, the omnipresence of variability, the quantification and explanation of variability” (p. 815).

Both statisticians and mathematicians recognise the differences between the subjects. The mathematician Tony Gardiner, in his draft mathematics curriculum (2012), also notes how statistical numbers such as ‘the height of a UK adult male in 2010’ are not as concrete or singular as mathematical numbers, and require sampling and inferences rather than calculation (2012, p. 43–4). Gardiner proposes the removal of statistics from his mathematics curriculum, for similar reasons to Cobb and Moore. Gardiner feels that statistics has become ‘inflated’ within mathematics and “areas which are essential prerequisites for beginners have suffered a loss of time and emphasis as a result” (p. 45).

4 A possible way forward

We suggest that statistics be given the status of a subject in its own right, rather than a subset of mathematics. Removing the assessment of statistics from GCSE Mathematics should be beneficial to both subjects. It will allow more assessment space in mathematics to cover algebra, for example, more fully. Statistics will no longer have to fit within the assessment of GCSE Mathematics, which is designed to suit the assessment of mathematics, rather than statistics. It will be able to have an assessment model which best fits the nature of the subject and allows much more of the statistics cycle to be included in the assessment.

This approach was mentioned by OCR's focus group of teachers. Some teachers felt that in order to better prepare students for A Level Mathematics, it would be useful to take some of the statistics content out and replace it with more algebra and (geometric) proof at KS4. A few teachers suggested developing a double award with one part focusing on statistics and the other on mathematics. It was also a recommendation of Adrian Smith's maths inquiry, *Making Mathematics Count* (2004):

> The Inquiry strongly believes that knowledge of Statistics and Data Handling is fundamentally important for all students and would wish to see these topics continue to be given due emphasis and timetable allocation. However, we believe it would be timely – in the context of a radical re-think of future 14–19 mathematics pathways within the general structure that may emerge following the 14–19 Working Group review – to reconsider the current positioning of Handling Data within the GCSE mathematics timetable, it occupies some 25 per cent of the timetable allocation. Many respondents believe the current mathematics curriculum at Key Stage 4 to be overloaded. We have no doubt that much of the concern expressed to us about the perceived decline of fluency with core mathematical operations reflects the pressure on the mathematics timetable that has resulted from the inclusion of this significant element of Handling Data.
>
> (Smith, 2004, p.85-6).

The argument in both cases is not concerned with making improvements to the assessment of statistics; it is to facilitate improvements to the assessment of core mathematical skills by increasing the available assessment space for them. There are, therefore, benefits to both subjects in bringing about this separation.

In our national context – with HE and employers demanding practical statistical skills - the focus of a statistics curriculum and qualification should be on promoting understanding of the use of statistics and the whole statistical process. Tony Gardiner points out that the descriptive statistical methods currently taught in English schools "often cannot be understood at the level they are used" and that "serious mathematical analysis of statistical problems remains well out of reach"' (2012, p. 45). We take this to mean an underlying mathematical understanding, rather than an understanding of what the techniques *do*, or what the *results mean*. For example, secondary school students may struggle to explain how and why the Spearman's rank correlation coefficient works in mathematical terms, but they may well appreciate the concept in practical terms – that it indicates the strength of association between two variables based on paired rankings (or words to that effect). They should be able to understand when it is an appropriate calculation to carry out, and when it is not. In terms of practical use of statistics, this is enough. Cobb and Moore point out that a desire to explain statistics in mathematical (probabilistic) terms is tempting for some teachers, to avoid teaching "statistics-as-magic".

> The danger of statistics-as-magic is real. But the proper defense is not a retreat to a mathematical presentation that is inadequate to the subject and often incomprehensible to students. Mathematical understanding is not the only kind of understanding. It is not even the most helpful kind in most disciplines that employ mathematics, where understanding of the target phenomena and core concepts of the discipline take precedence. We should attempt to present an intellectual framework that makes sense of the collection of tools that statisticians use and encourages their flexible application to solve problems.
>
> (Cobb and Moore, p. 815)

We propose, therefore, that the new GCSE in statistics should have a 'short course' (half a GCSE) option. The short course GCSE should entirely cover the national curriculum for statistics at KS4 – the body of knowledge and skills which students should be taught by the end of KS4. Part of the assessment should draw strongly on pre-released case study material, to enable students to familiarise themselves with the data relating to a specific scenario before being asked to complete an external examination. A practical, utilitarian approach to statistics will make the subject more useful and appealing to English students, and it will be an easy decision for teachers to offer the short course to students, since the schools will be mandated to teach the content it contains.

The full course would include some more content and a further assessment for students who wish to study the subject in greater detail, for any number of reasons.

5 Recommendations

- Separate the statistics content from the mathematics content in the National Curriculum
- List the body of knowledge and skills for statistics as a core (mandatory) subject in its own right
  - This will increase the status of statistics
- Remove the requirement for GCSE Mathematics to assess the programme of study for statistics
  - This will facilitate improvements to GCSE mathematics assessments
- Create a new short course GCSE in statistics
  - This will be required to cover the programme of study for statistics
  - As such it is expected that the vast majority of students will take this qualification
  - It will also use the most appropriate assessment methodologies for the subject, such as pre-released case studies, or internal assessment
  - A full course option should remain available for students who wish to take it
- Make better links between the content of statistics and its use in other subjects
  - This could help to improve motivation and students' ability to use and apply statistics in a range of subjects

6 References

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