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O Level

Mathematics

Session: 1957 June
Type: Question paper
Code: 19

ELEMENTARY MATHEMATICS 47

ORDINARY LEVEL, ALTERNATIVE A

ARITHMETIC

(Two hours)

Mathematical tables are provided.

Candidates are warned that misreading a question may lead to the loss of a number of marks.

All working must be clearly shown; it should be done on the same sheet as the rest of the answer.

SECTION I [76 marks]

Answer **all** *the questions in this section.*

You must not use Mathematical tables in working Questions 1–6.

1. (i) Simplify $(2\frac{1}{2}-1\frac{5}{8})\,(2\frac{1}{4}+1\frac{7}{8})\div 1\frac{3}{4}$.

(ii) Express 2 tons 2 cwt. 2 qr. as a fraction, in its lowest terms, of 10 tons.

(iii) An aeroplane flies non-stop for $2\frac{3}{4}$ hr. and travels 660 miles. Find its average speed in feet per second.

2. (i) Find the exact value of $3{\cdot}2\times 0{\cdot}25\times 0{\cdot}6125$.

(ii) Find the quantity, in litres, of ink required to fill 750 rectangular ink pots each 2·5 cm. long, 2·5 cm. wide and 2 cm. deep.

(iii) Taking 1 in. to be 2·540 cm., express 328·75 cm. in feet and inches, correct to the nearest inch.

3. (i) A load of wet sand weighs 7 cwt. 64 lb. When dry the sand weighed 6 cwt. 70 lb. Find the percentage of water in the wet sand.

(ii) The average weight of the 8 tallest boys in a class was 50·25 kg., and the average weight of the remaining 16 was 45·15 kg. Find the average weight of all the boys in the class.

4. (i) Working 8 hr. a day at the same average rate, 45 men could do a job in 12 days. If they only work $7\frac{1}{2}$ hr. a day and the job must be done in 9 days, find how many men should be employed.

(ii) A tennis court 78 ft. long and 36 ft. wide is to be surrounded by a rectangular fence of wire-netting 9 ft. high, erected 6 ft. away from each long side and 10 ft. away from each short side. If the wire-netting is sold only in complete rolls 10 yd. long and 4 ft. 6 in. wide, find the number of rolls which should be bought.

5. A geyser uses 75 cu. ft. of gas an hour to provide hot water, and is in operation, on the average, for $1\frac{1}{4}$ hr. each day. Gas costs 20·8 pence a therm, and a therm is the equivalent of 200 cu. ft. of gas. Find the cost of providing hot water from 1 October to 31 December inclusive.

An electric immersion heater would need to be in operation for $2\frac{1}{2}$ hr. daily at a cost of 3*d.* an hour to provide the same amount of hot water. Find the saving over the same period when electricity is used.

6. (i) On his fifteenth birthday a boy invested a gift of £80 at 5 per cent. per annum compound interest. On his eighteenth birthday he withdrew his £80, together with the accumulated interest, to help to pay for a £120 motor bicycle, his father paying the difference. Find, correct to the nearest shilling, how much his father paid.

(ii) A dealer bought a table for \$35 and sold it to a retailer at a profit of 20% on the cost price; the retailer sold it for \$56·70. Find the percentage profit made by the retailer on his outlay.

7. (i) Use logarithms to find the value of

$$24{\cdot}71 \times 0{\cdot}9426 \div 512{\cdot}0.$$

(ii) A solid cylindrical stone pillar is of diameter 1 ft. 9 in. and height 24 ft. Given that 1 cu. ft. of stone weighs 176·5 lb., find the weight of the pillar in tons, correct to three significant figures. [Take π to be 3·142.]

SECTION II. [24 marks]

Answer **two** *questions in this section.*

8. In 1955 the rateable value of a house was £51 and the rate was 20*s.* 8*d.* in the £. In 1956, the rateable value was re-assessed at £84 and the rate was 14*s.* 6*d.* in the £. Find the sum paid in rates in 1956, and express this as a percentage, correct to three significant figures, of the sum paid in 1955. Find also what rate, in shillings and pence, correct to the nearest penny, would cause no change in the sum paid after re-assessment.

9. A man invested £4200 in £1 shares at 56*s.* In the next three years he received dividend on the shares at the rate of 15, 12 and 8 per cent. per annum respectively, but in the fourth year no dividend was received. He paid income tax at 8*s.* in the £ in the first two years, and at 6*s.* in the £ in the third year. Find, for the four year period, his average annual net income from the shares.

10. Two points A and B are at the same horizontal level, B being 200 miles due east of A. An aeroplane flies at a constant height from A on a bearing 054° for 110 miles to P and then due east to Q, which is due north of B. Calculate (i) the distance PQ, (ii) the bearing of Q from A.

11. Two points A and O are on a horizontal straight road, O being due south of A. An observer at A sees T at ground level due east of him. Another observer at B, which is 125 ft. vertically above O, sees T at an angle of depression 12°. The bearing of T from O is 064°. Calculate, correct to the nearest foot, the distance of T from each observer.

ELEMENTARY MATHEMATICS 48

ORDINARY LEVEL, ALTERNATIVE A

GEOMETRY

(*Two hours and a half*)

Candidates are warned that misreading a question may lead to the loss of a number of marks.

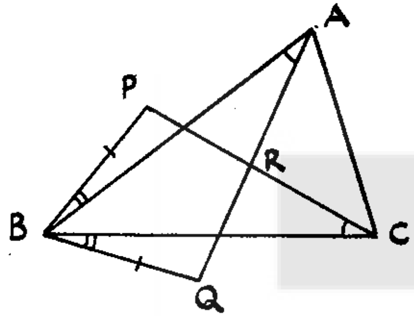
SECTION I [61 marks]

Answer **all** *the questions in this section.*

1. Without using set square or protractor, construct a quadrilateral $ABCD$ in which $AB = 2{\cdot}5$ in., $B\hat{A}D = 60°$, $AD = 2{\cdot}0$ in., $BC = DC = 2{\cdot}1$ in. Measure AC.

Construct also the circumcircle of the triangle BCD and measure its radius.

2.



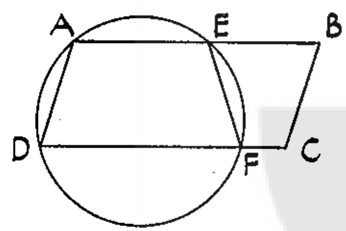
In the above figure, $BP = BQ$, $P\hat{B}A = C\hat{B}Q$, $B\hat{A}Q = B\hat{C}P$. Prove that

(i) the triangles PBC and QBA are congruent;

(ii) $RA = RC$.

3. (i) $PQRS$ is a quadrilateral and T is the mid-point of the diagonal QS. Prove that the area of the figure $PQRT$ is equal to one half of the area of the quadrilateral $PQRS$.

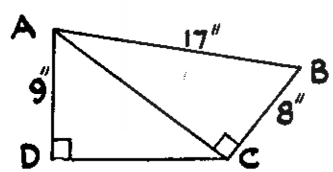
(ii)



In the above figure, $ABCD$ is a parallelogram. A circle through A and D cuts AB at E and DC at F. Prove that $F\hat{E}B = A\hat{B}C$.

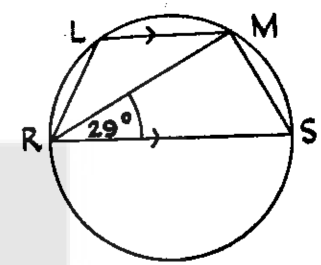
4. In this question, no proofs are required, but essential steps of the calculations must be shown.

(i)



In the above figure, $A\hat{C}B = A\hat{D}C = 90°$, $AB = 17$ in., $BC = 8$ in., $DA = 9$ in. Calculate the area of the quadrilateral $ABCD$.

(ii)



In the above figure, RS is a diameter, LM is parallel to RS, and $M\hat{R}S = 29°$. Calculate $R\hat{L}M$ and $L\hat{R}M$.

(iii) From a point, O, outside a circle, two straight lines OAB and OCD are drawn to cut the circle at A, B and C, D respectively. If $OA = 3$ in., $AB = 4$ in., and $OC = 2$ in., calculate CD.

5. If there are three or more parallel straight lines, and the intercepts made by them on any straight line that cuts them are equal, prove that the corresponding intercepts on any other straight line that cuts them are also equal.

[It is **not** necessary to assume any theorem on proportion or similarity, but if such a theorem is used it must be proved.]

6. If a straight line touch a circle, and from the point of contact a chord be drawn, prove that the acute angle which this chord makes with the tangent is equal to the angle in the alternate segment.

7. Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

SECTION II [39 marks]

Answer **three** *questions in this section.*

8. Draw two straight lines OX, OY, so that $X\hat{O}Y = 40°$. On OX mark two points A, B, so that $OA = 1$ in., $OB = 3$ in.

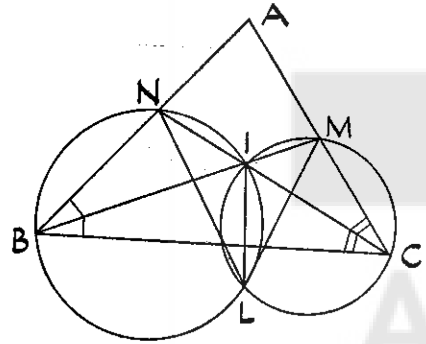
(i) Construct two points P, Q on OY, so that

$$A\hat{P}B = A\hat{Q}B = 55°.$$

Measure PQ.

(ii) Construct also a point R within the angle XOY, so that $A\hat{R}B=55°$ and the area of triangle $ARB=1{\cdot}7$ sq. in.

9.



In the above figure, the lines BIM, CIN bisect the angles ABC, ACB. The circles BNI, CMI intersect again at L. Prove that

(i) $N\hat{L}I=I\hat{B}C$;

(ii) $N\hat{I}M+N\hat{L}M=180°$.

10. ABC is a triangle inscribed in a circle. The tangent at A meets BC produced at T.

(i) Prove that the triangles ABT, CAT are similar.

(ii) Complete the statement

$$\frac{\triangle ABT}{\triangle CAT}=\frac{\quad}{CT}.$$

(iii) Prove that $\dfrac{BT}{CT}=\dfrac{AB^2}{AC^2}$.

11. In this question, no proofs are required, but each statement must be illustrated by a sketch.

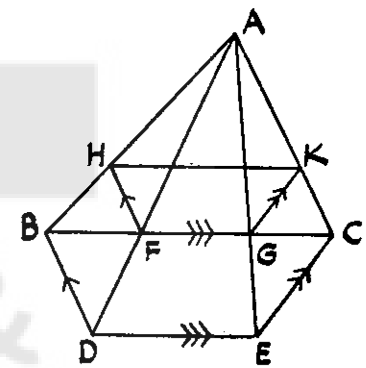
(i) P is a point equidistant from two fixed lines OA, OB and lying within the angle AOB. State the locus of P.

(ii) Q is the centre of a circle touching the fixed line XY at the fixed point C. State the locus of Q.

(iii) R is the mid-point of the side DE of a triangle DEF, of constant area. If E and F are fixed points, state the locus of R.

(iv) H is a fixed point outside a fixed circle, centre G, and S is the mid-point of a chord of this circle that passes through H. State the locus of S.

12.



In the above figure, the following pairs of lines are parallel:—DB, FH; GK, EC; BC, DE.

Prove that

(i) $\dfrac{HF}{BD}=\dfrac{KG}{CE}$;

(ii) HK is parallel to DE.

ELEMENTARY MATHEMATICS 49

ORDINARY LEVEL, ALTERNATIVE A

ALGEBRA

(Two hours)

Mathematical tables and squared paper are provided.

Candidates are warned that misreading a question may lead to the loss of a number of marks.

All working must be clearly shown; it should be done on the same sheet as the rest of the answer.

SECTION I [56 marks]

Answer **all** *the questions in this section.*

1. (i) Find the value of $(x+2)(x^2-2x+4)$, when $x=-1$.

(ii) Simplify

$$(3x-1)^2-(3x-2)(3x+1).$$

(iii) Reduce to a single fraction

$$\frac{4}{5a}+\frac{3}{2a}-\frac{1}{a}.$$

(iv) For what value of k is x^2-6x+k a perfect square?

2. (i) The diameter of a cycle wheel is d inches. Find, in its simplest form, an expression for the speed of the cycle in feet per second when the wheels are making n complete revolutions per minute. [Take π to be $3\frac{1}{7}$.]

(ii) Find the L.C.M. of a^3-4ab^2 and $a^2-4ab+4b^2$, leaving your answer in factors.

(iii) Solve the equation

$$\tfrac{3}{5}(2t-5)=1-\tfrac{2}{5}(t+1).$$

3. Solve the equation

$$\frac{x}{2x-1}-2=\frac{3}{x},$$

giving the roots correct to two decimal places.

4. (i) Simplify

$$\frac{x}{x+1}\div\frac{x^2-2x}{x^2-2x-3}.$$

(ii) An outward journey of $5x$ miles is covered at a speed of $(x+10)$ m.p.h. and the return journey at $(x-10)$ m.p.h. Find an expression for the total time in hours and reduce it to a single fraction.

5. (i) Make b the subject of the formula

$$P=\frac{ER}{R+b}.$$

(ii) Simplify

$$\sqrt{9a^6}\div\tfrac{1}{2}a^2.$$

(iii) Use logarithms to evaluate $7{\cdot}62^{\frac{1}{3}}$, correct to three significant figures.

6. The sum of the ages of two brothers Peter and John is 20 years 6 months. When Peter was half his present age he was 5 years older than John is now. Find how old Peter will be when John is 20 years of age.

7. (i) If V varies directly as the cube of r and $V=24{\cdot}8$ when $r=2$, find (a) the value of V when $r=3$, (b) an expression for r in terms of V.

(ii) Given that

$$\frac{a-b}{a+b}=\frac{3}{4},$$

find the value of $\dfrac{2a}{3b}$.

SECTION II [44 marks]

Answer **four** *questions only in this section.*

8. (i) Solve the simultaneous equations

$$2x-y=5,$$
$$xy+2=0.$$

(ii) The expression x^3+3x-7 can be written in the form

$$x(x-3)(x+1)+A(x-3)(x+1)+Bx+C,$$

where A, B, C are numbers. Find the values of A, B and C.

9. A rectangular plot of land whose length and breadth are in the ratio 5:2 was bought at 4*s.* per sq. yd. and fenced at a cost of 6*s.* 8*d.* per yd. The land cost £120 more than the fencing. Find the total cost of the land and the fencing.

10. Draw the graph of $y=x^2-\dfrac{4}{x+4}$ for values of x between -3 and $+2$ by plotting at least six points. Take 1 in. to represent one unit on each axis. From your graph find values of x for which

$$x^2=1+\frac{4}{x+4}.$$

11. The volume of the smaller part of a sphere, radius a, cut off by a plane at distance x from the centre of the sphere is

$\frac{\pi}{3}(2a^3-3a^2x+x^3)$.

Show that $(a-x)$ is a factor of this expression and complete the factorization. Find also the ratio of the volumes cut off by planes at distances $\frac{2a}{3}$ and $\frac{a}{2}$ from the centre.

12. (i) Given that $\log_{10} m = x+y$ and $\log_{10} n = x-y$, express in terms of x and y the values of $\log_{10} m^2n^2$ and $\log_{10} \frac{10m}{n}$.

(ii) If

$$(x-1)\log_{10} 4 = x \log_{10} 3,$$

use tables to evaluate x correct to three significant figures.

13. (i) The first term of an arithmetical progression is 16 and the sixth term is 83. Find the third term and the sum of the first 40 terms.

(ii) The sum of the first two terms of a geometrical progression is equal to six times the third term. Calculate the two possible values of the common ratio.

ELEMENTARY MATHEMATICS 50

ORDINARY LEVEL, ALTERNATIVE B

PAPER I

(Two hours and a half)

Answer **all** *the questions in Section I and any* **four** *in Section II.*

All working must be clearly shown; it should be done on the same sheet as the rest of the answer.

Mathematical tables and squared paper are provided.

Mathematical tables must not be used in Question 1.

SECTION I [52 marks]

1. (i) Divide 1·52388 by 0·612.

(ii) Find the total cost of 13 dozen plates if each plate costs 2*s*. 11*d*.

(iii) If the average length of my stride is 2 ft. 9 in., find the number of strides I take in walking a quarter of a mile.

2. (i) Factorize $1-4(x-y)^2$.

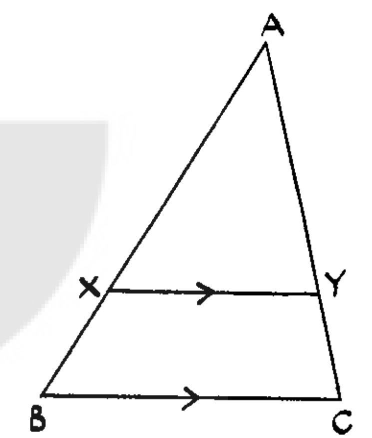
(ii) Simplify the expression

$$\frac{2}{x+1}-\frac{3}{2x+2}.$$

(iii) Given that $\frac{1}{x}+\frac{1}{y}=\frac{1}{4}$, express x in terms of y as simply as possible.

3. (i) A six-sided figure has five of its exterior angles equal. The sixth exterior angle is four times each of the others. Calculate the interior angles of the figure.

(ii)



In the figure, XY is parallel to BC. Given that $AX=3XB$, write down the values of (*a*) $XY:BC$, (*b*) the area of AXY: the area of ABC.

4. (i) In the triangle ABC, $AB=3$ in., $BC=4$ in. and the angle $B=40°$. Calculate the area of the triangle and the length of the third side.

(ii) Given that x is an angle between 0 and 180 degrees and that $\sin x = \frac{4}{5}$, calculate, without using tables, the possible values of $\cos x$.

5. AB is a chord of a circle of length 2 in. and subtends an angle of 35° at the circumference of the circle. Find (i) the radius of the circle, (ii) the ratio of the lengths of the arcs into which AB divides the circle.

SECTION II [48 marks]

Answer any **four** *questions in this section.*

6. (i) A sum of money invested at $3\frac{1}{2}\%$ Simple Interest amounts after 4 years to £513. Find the sum invested.

(ii) Solve the equation $5x^2 - 7x - 8 = 0$, giving your answers correct to two decimal places.

7. I smoked x packets of 20 cigarettes and y ounces of tobacco during the course of a year when a packet of 20 cigarettes cost 3*s*. 8*d*. and an ounce of tobacco cost 4*s*. 1*d*. My total expenditure was £25. 10*s*. After the budget, the cost of a packet of cigarettes was increased by 2*d*. and the cost of an ounce of tobacco was increased by 3*d*. I calculated that my annual expenditure on tobacco and cigarettes would increase by £1. 8*s*. 4*d*. Find x and y.

8. Prove that the angle in a semicircle is a right angle.

Two unequal circles touch externally at P. The common tangent at P meets at X another common tangent, which touches the circles at S and T. Prove

(i) X is the mid-point of ST;

(ii) the angle SPT is a right angle.

9. A and B are two points on level ground on opposite sides of a vertical flagstaff and in line with its foot. The angles of elevation of the top of the flagstaff from A and B are 42° and 68° respectively. Given that $AB = 20$ ft., calculate the height of the flagstaff.

10. A pilot wishes to make good a track of 045°. If his air speed is 300 knots and the wind is blowing at 60 knots from 125°, find, by drawing or by calculation, the course which should be steered and the ground speed of the aircraft.

11. Two places both in latitude 50° N. differ in longitude by 20°. Calculate, in statute miles,

(i) the distance of either place from the north pole;

(ii) the distance between the places measured along their parallel of latitude.

[Take the earth to be a sphere of radius 3960 statute miles and π to be 3·142.]

ELEMENTARY MATHEMATICS **51**

ORDINARY LEVEL, ALTERNATIVE B

PAPER II

(Two hours and a half)

Answer **all** *the questions in Section I and any* **four** *in Section II.*

All working must be clearly shown; it should be done on the same sheet as the rest of the answer.

Mathematical tables and squared paper are provided.

Mathematical tables must not be used in Question 1.

SECTION I [52 marks]

1. (i) Express 17*s*. $7\frac{1}{2}$*d*. as a decimal of one pound.

(ii) Simplify $15\frac{1}{2} \times 2\frac{2}{5} \div 6\frac{1}{5}$.

(iii) Express 3 cwt. 7 lb. as a percentage of 17 cwt. 14 lb. Give your answer to three significant figures.

2. (i) Factorize $ab - ac - db + dc$.

(ii) The cost of potatoes is p pence per x pounds. Express the price in shillings per cwt.

(iii) Find the number which, when added to $(9x^2-30x)$, will make this expression a perfect square.

3. P is a point inside the square $ABCD$ of side 2 in. such that $PC=PD=CD$. Prove that $PA=PB$.

Calculate the angle APB and the areas of the triangles PCD, PBC, and PAB.

4. A man in a boat pulls himself towards the vertical wall of a harbour by means of a rope fastened to a point on the wall 15 ft. above the level of his hands. Calculate the length of rope which he hauls in while moving from a position 30 ft. away to one 10 ft. away from the wall. Calculate also the change in the angle which the rope, assumed straight, makes with the vertical.

5. Without using set square or protractor, construct a triangle ABC in which $AB=5$ in., $B\hat{A}C=45°$, $A\hat{B}C=60°$. Construct also the perpendicular bisector of AB and the perpendicular to BC at C. If these perpendiculars meet at P, measure PA.

SECTION II [48 marks]

Answer any **four** *questions in this section.*

6. Two girls, Ann and Beryl, with one bicycle between them, wished to go a journey of 14 miles, starting together. Ann cycled x miles at 8 m.p.h., left her bicycle and walked the rest of the way at 3 m.p.h. Beryl walked the x miles at 4 m.p.h. and then cycled the rest of the way at 9 m.p.h. If Beryl arrived 20 min. ahead of Ann, find x.

7. Calculate the weight in pounds of a mile of cylindrical steel wire if the diameter of the cross-section is 0·096 in., and a cubic foot of steel weighs 495 lb. [Take π to be 3·142.]

Find the length of the same weight of wire whose diameter is twice that of the first wire.

8. Two lines PAB, PCD are drawn from an external point P to cut a given circle, centre O, at AB, CD respectively. Prove that the triangles PAC, PDB are similar.

If $PA=4$ cm., $AB=5$ cm., $PC=3$ cm., calculate the length of CD.

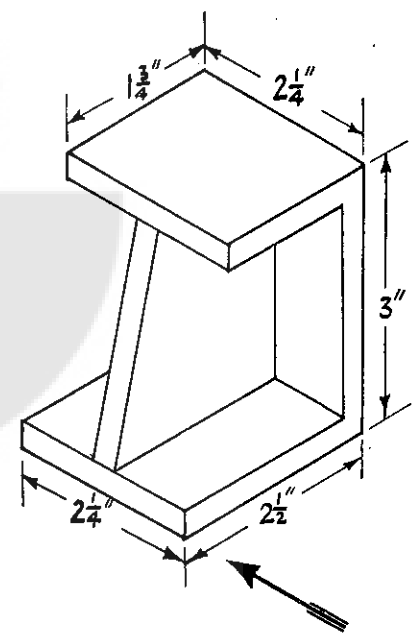
If the point P remains fixed while the line PCD varies, state, without proof, the locus of the mid-point of CD. Illustrate your answer by a rough sketch.

9. (i) Solve the equation $9y+\dfrac{4}{y-1}=46$.

(ii) If $(x-1)^{\frac{2}{3}}=\frac{1}{9}$, find x without using tables.

10. A pyramid $VABCD$ stands on a horizontal rectangular base $ABCD$, in which $AB=4$ cm., $BC=6$ cm. V is vertically above the intersection, O, of the diagonals AC, BD, and $VA=5$ cm. Find, by drawing or by calculation, the angles made with the horizontal by (i) the edge VA, (ii) the plane VBC.

11.



The figure represents a symmetrical metal casting with horizontal rectangular base and top. All parts are $\frac{3}{8}$ in. thick. Draw, full size, the plan and an elevation in the direction shown.