O Level

Mathematics

Session: 1974 June
Type: Question paper
Code: 450

MATHEMATICS 450/1

ORDINARY LEVEL, SYLLABUS A

ARITHMETIC

(*One hour and a half*)

Mathematical tables and squared paper are provided.

All working must be clearly shown; it should be done on the same sheet as the rest of the answer.

SECTION I. [70 marks]

Answer **all** *the questions in this section.*

You must not use mathematical tables in working Questions 1–4.

1 (*a*) Simplify

$$\frac{(\frac{5}{8} \times 7\frac{1}{5}) - (\frac{3}{5} \times 2\frac{11}{12})}{4\frac{1}{8}}.$$

(5)

(*b*) In a certain village $\frac{3}{16}$ of the inhabitants are over 60 years old, $\frac{1}{8}$ are between 15 and 30 years old and $\frac{1}{4}$ are under 15. The remaining 35 people are between 30 and 60 years old. If there are 7 men in the village who are over 60 find the number of women over 60. (5)

2 (*a*) The difference between two numbers is 3·9. The larger of the two is 14·7. Find the other number and the product of the two numbers. (5)

(*b*) Find the greatest number of books, each costing £1·65, which can be bought for £75. If the money left over is spent on pamphlets costing 5p each, find the number of pamphlets bought. (5)

3 (*a*) A sheet of postage stamps consists of 20 rows of stamps, with 10 stamps in each row, these being surrounded by an edging 9 mm wide. The whole sheet is 49·8 cm long and 43·3 cm wide. Calculate the area of each stamp, neglecting perforations. (7)

(*b*) A man takes 80 equal steps to walk from his front gate to a bus-stop. His son takes 75 steps in the opposite direction to reach his school. Given that the distance between the school and the bus-stop is 117 metres and that the length of the son's step is two-thirds that of his father, calculate the length of the man's step as a decimal of a metre. Given also that the man takes 45 seconds to walk to the bus-stop, calculate his walking speed in kilometres per hour. (9)

4 (*a*) A window is in the form of a rectangle $ABCD$ with a semi-circular top on AD as diameter. Given that $BC = 28$ cm and that the area of the rectangle is three times the area of the semicircle, calculate

(i) the length of AB,

(ii) the perimeter of the window.

[Take π to be $3\frac{1}{7}$.] (8)

(*b*) A man invests a sum of money on which he receives interest at 8% per annum. His interest for the first year is £140. He pays tax at 40p in the pound on the interest. Find how much of the interest he has left and express it as a percentage of the sum invested. (8)

5 (*a*) Use tables to calculate, correct to three significant figures, the value of

$$\frac{342{\cdot}6 \times (5{\cdot}729)^2}{(62{\cdot}81)^3}.$$ (7)

(*b*) A steel bar is 30 cm long and its cross-section is a square of side 1 cm. Its mass is 239·4 g. A spherical ball-bearing of the same material has a mass of 6·798 g. Calculate the radius of the ball-bearing, giving your answer in centimetres correct to two places of decimals. (10)

[Take π to be 3·142. The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.]

Section II. [30 marks]

Answer **two** *questions in this section.*

6 The total charge made by a car-hire firm is made up of a fixed charge for each day for which the car is hired plus a charge for each kilometre travelled. The table gives the total cost of a day's hire when the given distances are travelled.

Distance (km)	250	400	750
Total cost (£)	6	7·80	12

Taking 2 cm to represent 100 km on one axis and to represent £2 on the other axis, draw a graph to show the total cost of a day's hire for distances from 0 to 750 km. Use your graph to find (4)

(i) the fixed daily charge, (3)

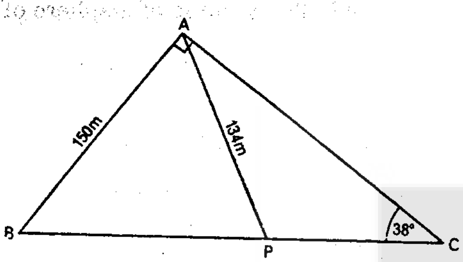
(ii) the total cost to a motorist who hires a car for two days and travels a total of 600 km. (5)

Another firm makes a fixed charge of £6 per day with no extra charge for the distance travelled. A motorist who hired a car for one day from the first firm finds that he could have saved £3 by hiring from the second firm. Find the distance he travelled. (3)

7 In a certain year the rateable value of a town was £3 200 000. Rates were collected at 80p for each pound of the rateable value. One eighth of the money so collected was allocated to road-works, the rest being divided between education and other services in the ratio 5:3. Find the amount allocated to education. (8)

In the following year the rateable value of the town was increased by 20% and the rates collected rose by $12\frac{1}{2}$%. Find the number of pence per pound of the new rateable value at which rates were charged. (7)

8



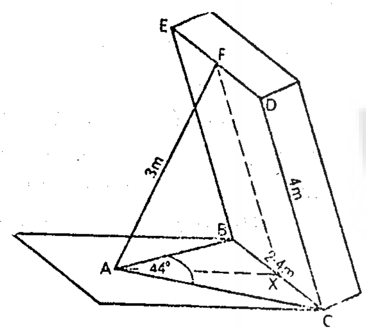
The diagram represents a triangular plot of ground ABC, situated at the junction of two straight roads BA and BC. Under a road-widening plan the section ABP is cut off. Given that $BA = 150$ m, $AP = 134$ m, $A\hat{C}B = 38°$ and $B\hat{A}C = 90°$, calculate

(i) AC, (3)

(ii) $A\hat{P}B$, (6)

(iii) the area of the remaining plot APC. (6)

9



The diagram represents the rectangular base $BCDE$ of a tipping trailer which has been raised from its horizontal position by the action of the extending bar AF. The horizontal bars AB and AC are equal. The points F and X are the midpoints of ED and BC respectively. Given that $DC = 4$ m, $BC = 2{\cdot}4$ m, $AF = 3$ m and $B\hat{A}C = 44°$, calculate

(i) AC, (4)

(ii) AX, (4)

(iii) the angle which $BCDE$ makes with the horizontal. (7)

MATHEMATICS 450/2

ORDINARY LEVEL, SYLLABUS A

GEOMETRY

(Two hours)

In questions involving calculations, no proofs are required but essential steps of the working must be shown.

SECTION I. [70 *marks*]

Answer **all** *the questions in this section.*

1 Without using set square or protractor, construct in a single diagram

(i) the triangle ABC such that $AB = 8$ cm, $A\hat{B}C = 60°$ and $BC = 6$ cm, (4)

(ii) the point D on the *opposite* side of AC to B such that $DA = DC$ and $BD = 9$ cm, (5)

(iii) the point P on AB such that PC bisects $B\hat{C}D$. (3)

2 (*a*) The sum of the interior angles of a convex polygon is five times the sum of the exterior angles. Calculate the number of sides of the polygon. (4)

(*b*) The diagonals AC and BD of the parallelogram $ABCD$ intersect at O. A line through O cuts AB and CD at P and Q respectively. Prove that

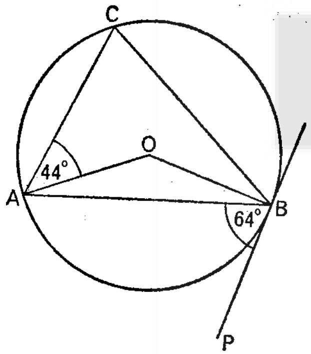
(i) the triangles AOP and COQ are congruent, (4)

(ii) the triangles ADP and CBQ are congruent. (6)

3 (*a*) A point R is 10 cm from the centre of a circle of radius 4 cm. A straight line RST cuts the circle at S and T. Given that $RS = 7$ cm, calculate ST. (4)

(*b*) In the acute-angled triangle XYZ, the point W is the foot of the perpendicular from X to YZ. Given that $XY = 7$ cm, $YZ = 10$ cm and $ZX = 9$ cm, calculate YW. (4)

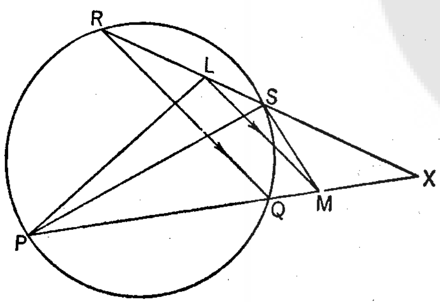
4 (*a*)



In the above figure, PB is the tangent to the circle at B. The centre of the circle is O. Given that $P\hat{B}A = 64°$ and $O\hat{A}C = 44°$, calculate $O\hat{A}B$ and $O\hat{B}C$. (6)

(*b*) Two circles of radii 9 cm and 4 cm touch externally. Calculate the length of the exterior common tangent. (4)

5



In the above figure, PQ and RS are chords of the circle which meet, when produced, at X. Points L and M, on RX and PX respectively, are such that LM is parallel to RQ. Prove that

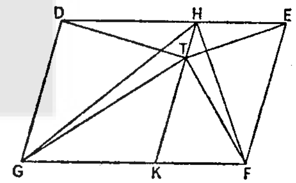
(i) $S\hat{L}M = S\hat{P}M$, (6)

(ii) $SLPM$ is a cyclic quadrilateral, (1)

(iii) the triangles XSM and XPL are similar, (3)

(iv) $PL.XM = SM.XL$. (2)

6 (*a*)



In the above figure, $DEFG$ is a parallelogram and the straight line HTK is parallel to DG. Prove that

Area $\triangle TDG$ + Area $\triangle TEF$ = Area $\triangle GHF$. (8)

(*b*) The point M is the mid-point of the chord AB of a circle, centre O, and S is a point on OM between O and M. Prove that $AS = SB$. (5)

Section II. [30 marks]

Answer **two** *questions in this section.*

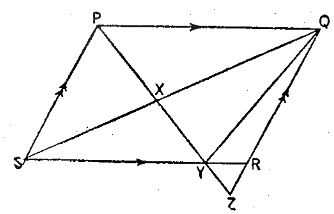
7 Prove that the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle. (6)

The point D is the mid-point of the side BC of triangle ABC. The bisector of $A\hat{D}B$ meets AB at X. The line through X parallel to BC meets AC at Y. Prove that

(i) YD bisects $A\hat{D}C$, (6)

(ii) XY is a diameter of the circle XDY. (3)

8



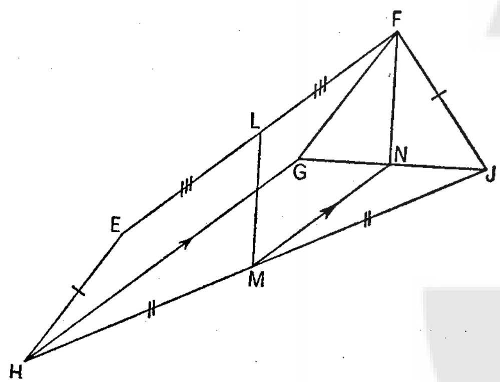
In the above figure, $PQRS$ is a parallelogram. A line through P cuts QS, RS, and QR produced at X, Y and Z. Prove that

(i) the triangles PXS and QXY are equal in area, (4)

(ii) $\dfrac{\text{Area } \triangle PXS}{\text{Area } \triangle QXZ} = \dfrac{XY}{XZ}$, (4)

(iii) $\dfrac{XY}{XZ} = \dfrac{PS^2}{QZ^2}$. (7)

9 Prove that the straight line drawn through the middle point of one side of a triangle parallel to another side bisects the third side. (6)



In the above figure, $EFGH$ is a parallelogram and $FJ = EH$. The mid-points of EF and HJ are L and M. The line through M parallel to HG meets GJ at N.

Prove that

(i) $G\hat{F}N = J\hat{F}N$, (6)

(ii) $FLMN$ is a parallelogram. (4)

10 Without using set square or protractor, construct in a single diagram

(i) a circle of radius 4 cm,

(ii) a triangle TPX where TP is the tangent to the circle at a point P, $TP = 3$ cm, $T\hat{P}X$ is obtuse and PX is a chord of the circle such that $PX = 7$ cm. (6)

(iii) the point Y on the circumference of the circle such that $P\hat{X}Y = 30°$ and $X\hat{Y}P = T\hat{P}X$, (4)

(iv) the point Z on YX produced such that the triangle PZY is equal in area to the quadrilateral $TXYP$. (5)

MATHEMATICS 450/3

ORDINARY LEVEL, SYLLABUS A

ALGEBRA

(One hour and a half)

Mathematical tables and squared paper are provided.

All working must be clearly shown; it should be done on the same sheet as the rest of the answer.

SECTION I. [70 marks]

Answer **all** *the questions in this section.*

1 (*a*) Given that $a = -2$, $b = 1$, find the value of

$$2-3(a+2b)^2 \quad \text{and of} \quad \frac{a^2+b^2}{(a-b)^2}.$$ (4)

(*b*) Express

$$2-\frac{5x}{x-1}+\frac{3x}{x+1}$$ (4)

as a single fraction in its simplest form.

2 (*a*) Solve the equation $0{\cdot}2(x-3)-0{\cdot}5(x-2) = 1$. (4)

(*b*) Solve the simultaneous equations

$$4x+7y-5 = 5x-4y-13 = x+2y.$$ (6)

3 Factorise completely

(i) $3+5k-2k^2$, (3)

(ii) $x(x-2)-y(2-x)$, (3)

(iii) $25-4(a+2)^2$. (3)

4 (*a*) Solve the equations

(i) $(2x+3)^2 = 9$, (3)

(ii) $(y+1)(y-1) = 3y+3$. (3)

(*b*) In an examination a boy gains p marks for each correct answer and loses q marks for each incorrect answer. He answers N questions altogether, of which x are found to be incorrectly answered, and the rest correctly answered. Find an expression for the total number of marks he obtains.

If $p = 3$, $q = 2$, $N = 25$, and his total mark is zero, calculate the value of x. **(5)**

5 (*a*) Solve the equation

$$x = \frac{1}{2x+3},$$ **(7)**

giving your answers correct to two places of decimals.

(*b*) Given that $(1-mk)t = m+k$, find an expression for m in terms of k and t. **(4)**

6 If A gives £5 to B then B will have four times as much money as A, but if, instead, B gives £9 to A then A will have twice as much money as B. Find how much money each had originally. **(7)**

7 (*a*) Using tables, calculate the value of the **cube root** of

$$\frac{uv}{u+v}$$

given that $u = 0{\cdot}234$ and $v = 5{\cdot}789$. Give your answer correct to three significant figures. **(7)**

(*b*) Given that $x = \frac{1}{2}$ and $y = 4$, calculate, without using any tables, the values of

(i) $x^{-y} + y^{-x}$, **(3)**

(ii) $\lg(10y) - \lg(8x)$. **(3)**

$$[\lg a = \log_{10} a]$$

SECTION II. [30 marks]

Answer **two** *questions in this section.*

8 (*a*) Solve the simultaneous equations

$$x - 2y = 3,$$

$$\frac{1}{x} - \frac{2}{y} = 4\tfrac{1}{2}.$$ **(8)**

(*b*) Given that the remainder is 12 when the expression

$ax^3 + 3x^2 - 11x - 6$ is divided by $x+2$, calculate the value of a. Using this value of a, find out whether $2x+1$ is, or is not, a factor of the expression, showing all necessary working. **(7)**

9 (*a*) The sag in a plank supported at its two ends is found to vary directly as the cube of its length and inversely as the square of its thickness. A plank 10 metres long and 2 cm thick is found to sag 2·5 cm. Calculate the sag in a similar plank 15 metres long and 3 cm thick. **(7)**

(*b*) The first and last terms of an A.P. are -10 and 410 respectively. The sum of all the terms of the progression is 7200. Calculate the number of terms, and the common difference, of the progression. **(8)**

10 Two concerts were held. Tickets for the first concert cost x pence each and the total receipts from the tickets sold were £450. For the second concert, when the price of each ticket was reduced by 5 pence, 300 more tickets were sold and the total receipts increased by £90. Obtain an equation for x. Hence find

(i) the price of a ticket for the first concert,

(ii) the number of tickets sold for the first concert. **(15)**

11 The following is an incomplete table of values for the graph of $y = +\sqrt{25 - 4x^2}$.

x	-2	$-1\frac{1}{2}$	-1	0	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$
y	3		4·58		4·58		3	

Calculate, and write down on your answer sheet, the missing values of y. **(8)**

Taking 2 cm as the unit for both x and y, draw the graph of $y = +\sqrt{25 - 4x^2}$ for values of x between -2 and $+2\frac{1}{2}$.

(i) From your graph estimate the range of positive values of x for which y lies between 3 and 4·5. **(3)**

(ii) By drawing another suitable graph on the same axes, estimate the value of the x-coordinate of the point P on the original graph such that the y-coordinate of P is twice the x-coordinate of P. **(4)**