CAMBRIDGE
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# A Level

## Mathematics

Session: 1984
Type: Syllabus
Code: 9200

Subject Syllabus
**SS9(HCO)**
**1984**
Mathematics

**For All Centres**

GENERAL CERTIFICATE OF EDUCATION
SCHOOL CERTIFICATE
HIGHER SCHOOL CERTIFICATE

EXAMINATION SYLLABUSES FOR

# 1984

# MATHEMATICAL SUBJECTS

UNIVERSITY OF CAMBRIDGE
LOCAL EXAMINATIONS SYNDICATE
INTERNATIONAL EXAMINATIONS

2/82/9-5M/F&J

*June 1982*

# MATHEMATICS (SYLLABUS A) (9200)
# FURTHER MATHEMATICS (SYLLABUS A) (9219)
# PURE MATHEMATICS (9208)

## ADVANCED LEVEL AND HIGHER SCHOOL CERTIFICATE (PRINCIPAL SUBJECTS)

### Introduction

In revising the 'traditional' Advanced level and H.S.C. Mathematics syllabuses, consideration was given to (i) the declining entry for the subjects Pure Mathematics and Applied Mathematics and the continuing small entry for Statistics, (ii) the popularity of the two subjects Mathematics (Syllabus B) and Further Mathematics, (iii) the intention of the Mathematics Committee that the 'modern' and 'traditional' Advanced level and H.S.C. subjects should be brought together at some time in the future. The result of these considerations is that 'traditional' syllabuses in Mathematics and Further Mathematics have been drawn up. In addition, it was noted that there is a demand for a subject Pure Mathematics and provision is made for this.

It was decided to weight the pure mathematics and the applied mathematics elements of the examination equally and to offer candidates a choice of mechanics and/or statistics in the applied sections of the syllabuses.

The forms of the question papers testing the pure mathematics sections of the syllabus will be somewhat different from the Advanced level and H.S.C. mathematica question papers previously set by the Syndicate in that they will each contain a compulsory section of short questions. It is felt that certain topics in pure mathematic can adequately be examined by short questions.

Questions set in Further Mathematics will be of the same standard of difficulty (a far as possible) as those set in Mathematics, i.e. questions in the examination fo Further Mathematics will be set on the further topics in the syllabus. Where the topi in the Mathematics syllabus are to be taken to a greater depth for Furth Mathematics, this is indicated in the syllabus.

Notes are included with the syllabuses to clarify certain sections and to act as a gui to teachers.

# MATHEMATICS (SYLLABUS A) (9200)

(*May not be taken with* **9202, 9208, 9220, 8180,** *Additional Mathematics or any Mathematical subject at Ordinary level.*)

### Scheme of Papers

The examination will consist of two three-hour papers as follows:

Paper 1 (Pure Mathematics) will contain two sections A and B. Section A (52 marks) will consist of about ten short questions and Section B (48 marks) will consist of eight longer questions. Candidates will be required to answer **all** the questions in Section A and any **four** questions in Section B.

Paper 2 (Applied Mathematics) will contain two sections A and B containing ten Mechanics questions and ten Statistics questions respectively. Candidates will be required to answer **any seven** questions.

A Special paper (3 hours) will be set for G.C.E. Home Centres only. This paper will contain five questions set on each of the three sections of the syllabus and candidates will be required to answer **any seven** questions.

### Detailed Syllabus

PURE MATHEMATICS

It will be assumed that candidates are familiar with the basic principles of algebraic manipulation (including the laws of indices and logarithms) and with mensurational trigonometry in the plane (including the use of the sine and cosine formulae for triangles).

| SYLLABUS | NOTES |
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| **Algebra** | |
| 1. Polynomial functions; the remainder theorem and the factor theorem. | Questions will mainly be concerned with the use of the factor theorem in factorising polynomials and in solving polynomial equations. |
| The theory of quadratic functions and equations; graphical representation. | Completion of square should be known. Candidates should be able to deal with the variation in sign of $ax^2+bx+c$ as $x$ varies. The relations between the coefficients of the equation $ax^2+bx+c=0$ and the symmetric functions of the roots are required. |
| Solution of equations in one unknown involving surds, indices and/or logarithms. | Knowledge of the routine process for surd equations of squaring and the necessity of checking solutions at the end is required. |
| Solution of simultaneous equations. | Equations will be linear (in two or three unknowns) or linear and quadratic (not more than one quadratic). |
| Locating the roots of an equation by simple graphical and numerical methods. | |
| Arithmetic series; finite and infinite geometric series. | The terms 'arithmetic mean' and 'geometric mean' should be understood. |
| Simple algebraic inequalities; the use of the modulus sign. | Questions may be set on inequalities such as $(2x-1)(x-1)(2x+3) \geqslant 0$, $\frac{x}{x+2} < \frac{2}{x-1}$, $\left\lvert\frac{x}{x+1}\right\rvert < 1$. |



| SYLLABUS | NOTES |
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| 5. Simple problems on arrangements and selections. | The terms 'permutation' and 'combination' should be understood. |
| 6. Binomial theorem for a postive integral index; use of the series for $(1+x)^n$ when $n$ is not a positive integer. | Applications to approximations may be set. Knowledge of the range of convergence of infinite binomial expansions is required. Questions on greatest terms or relationships between binomial coefficients will not be set. The notation $\binom{n}{r}$ may be used in questions. |
| 7. Simple partial fractions. | Denominators will be no more complicated than $(ax+b)(cx+d)(ex+f)$ or $(ax+b)(cx+d)^2$ or $(ax+b)(x^2+c^2)$. |
| 8. Complex numbers: algebraic form. | Knowledge of real and imaginary parts and modulus (but *not* argument) is expected. Use of the principle $a+ib=c+id \Leftrightarrow a=c$ and $b=d$ will be required. |
| Sum, product and quotient of two complex numbers; conjugate complex $z^*$ of $z$, and the result $zz^*=\lvert z\rvert^2$. | Applications to quadratic equations with real coefficients. |
| **Trigonometry** | |
| 9. Circular measure. Small angles. | Includes application to arc length and sector area. Use of $\sin x \approx x$, $\tan x \approx x$ and $\cos x \approx 1-\frac{1}{2}x^2$ ($x$ in radians). |
| 10. The six trigonometric ratios for angles of any magnitude. | Idea of periodicity is required. |
| Graphs of simple trigonometric functions. | |
| 11. Use of the formulae for $\sin(A\pm B)$, $\cos(A\pm B)$, $\tan(A\pm B)$. | No proofs will be required. Applications to multiple angles and simple identities will be set. |
| Use of the formulae for $\sin X\pm\sin Y$, $\cos X\pm\cos Y$. | Applications to identities such as $\sin 2x+\sin 4x+\sin 6x+\sin 8x \equiv 4 \sin 5x \cos x \cos 2x$ but *not* identities such as $\sin A+\sin B+\sin C \equiv 4\cos\frac{1}{2}A \cos\frac{1}{2}B \cos\frac{1}{2}C$ when $A+B+C=\pi$. |
| 12. The use of $a\cos\theta+b\sin\theta \equiv R\cos(\theta-\alpha)$. | The range of values and period of $a\cos\theta+b\sin\theta$ are expected. |
| Solution of simple trigonometric equations in a given interval. | Solution of equations reducing to $\sin\theta=k$, $a\sin^2\theta+b\sin\theta+c=0$ or similar is included. Solution of $a\cos\theta+b\sin\theta=c$ is included. The use of $\sin\theta=\frac{2t}{1+t^2}$, $\cos\theta=\frac{1-t^2}{1+t^2}$ may be required. |

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| | Questions requiring graphical solutions may be set. |
| 13. Easy three-dimensional problems. | |

**Coordinate geometry**

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| 14. Elementary two-dimensional rectangular cartesian coordinate geometry. | Distances, angles, area of a triangle. |
| The linear equation; perpendicular distance from a point to a line. | |
| 15. Elementary treatment of the circle. | |
| Elementary treatment of the parabola $y^2 = 4ax$ and the rectangular hyperbola $xy = c^2$. | Knowledge of parametric form is required and methods for obtaining equations of chords, tangents and normals. |
| 16. Easy locus problems. | |
| 17. Simple curve sketching. | E.g. $y = x(1-x)$, $y^2 = x(1-x)$, $y = \frac{1}{1-x}$. Asymptotes parallel to the axes may be required. Features particularly required in a sketch will be specified in the question. |

**Differential and integral calculus**

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| 18. The idea of a derived function. Second and higher derivatives. | An intuitive concept of limit is expected. The derivatives of the following should be known: $x^n$, $\sin x$, $\cos x$, $\tan x$, $\cot x$, $e^x$, $\ln x$. |
| Differentiation of sums, products, quotients and composite functions. | |
| Differentiation when a relationship is defined parametrically or implicitly. | Only the first derivative is required. |
| Application of differentiation to rates of change, velocity, acceleration, maxima and minima, inflexions, sketch graphs, small increments (one independent variable), tangents and normals (including curves defined parametrically). | |
| 19. Newton-Raphson method for approximate solution of equations. | No proof is required but knowledge of the graphical meaning of the method, and its limitations, is expected. |



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| 20. Integration as the inverse of differentiation; standard forms. | The following standard forms should be known (constants of integration omitted): $\int x^n\,dx = \frac{x^{n+1}}{n+1}$ $(n \neq -1)$, $\int \frac{1}{x}\,dx = \ln\lvert x\rvert$, $\int e^x\,dx = e^x$, $\int \sin x\,dx = -\cos x$, $\int \cos x\,dx = \sin x$, $\int \sec^2 x\,dx = \tan x$, $\int \operatorname{cosec}^2 x\,dx = -\cot x$. Recognition of the form $\frac{f'(x)}{f(x)}$ is expected. |
| Integration by substitution. | In some cases, a substitution may be suggested. |
| Integration by use of partial fractions. | |
| The definite integral; its evaluation and its applications. | Applications will include evaluation of plane areas and volumes of solids of revolution. |
| 21. Formation of differential equations in a stated context. | No particular knowledge of physical, chemical, biological, economic laws, etc. is expected, but mathematical formulation of stated laws may be required. |
| The solution of first order differential equations with separable variables. | |

MECHANICS

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| 22. Scalar and vector quantities; knowledge that displacement, velocity, acceleration, force, momentum are vector quantities. | An approach by vector algebra is not expected and questions will not be set requiring the use of the unit vectors **i**, **j**, **k**. |
| Addition and subtraction of vectors; the multiplication of a vector by a scalar. | Knowledge of the ratio theorem $\lambda\overrightarrow{OP}+\mu\overrightarrow{OQ} = (\lambda+\mu)\overrightarrow{OR}$ will not be expected. |
| 23. Relative velocity. | Emphasis will be placed on the ability to produce correct velocity triangles. |
| 24. Composition and resolution of coplanar forces; moments; couples. Equilibrium of a particle and of a rigid body under coplanar forces. Friction. | An experimental basis is sufficient; proofs of fundamental theorems are not required. Problems set will in general require clear force diagrams and their interpretation by means of equations obtained by resolving and/or taking moments. Alternatively, graphical solutions may be permitted. Knowledge that three forces in equilibrium are concurrent or parallel will be assumed, but problems which involve complicated trigonometry will not be set. Lami's theorem will not be demanded explicitly. |
| 25. Determination of centres of mass of composite or truncated bodies. | The necessary formulae for the centres of mass of standard bodies will be given. The use of calculus will not be required. |

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| 26. Kinematics of a particle moving in a straight line, including graphical treatment. | |
| 27. Newton's laws of motion; mass and force. | |
| 28. Dynamics of a particle moving in a straight line under forces expressed as functions of time, velocity or distance. | Candidates will be expected to know the forms $\frac{d^2x}{dt^2}, \frac{dv}{dt}, v\frac{dv}{dx}$ for acceleration. Problems set will require only the solution of first order differential equations with separable variables. |
| 29. Interaction principle; simple problems on connected bodies. | Either statical or dynamical problems may be set, and may involve a single fixed smooth light pulley. |
| 30. Motion of a projectile, excluding the range on an inclined plane. | Simple problems involving the cartesian parametric equations (parameter $t$) or the $(x, y)$ equation of the path of a projectile may be set. Knowledge of the properties of a parabola is not required. |
| 31. Impulse and momentum; conservation of momentum; coefficient of restitution. | Questions will be confined to motion in a straight line. |
| Energy, work, power. | Candidates will be expected to know, and to be able to apply in simple cases, the principle 'Gain/loss of kinetic energy is equal to the work done by/against the external applied forces.' |
| 32. Uniform circular motion; angular velocity. | Proof of 'acceleration $= a\omega^2$' is not required. |

STATISTICS

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| 33. Tabulation and representation of statistical data; mean ($\mu$), median and mode; cumulative frequency graph (ogive); percentiles; variance $\left(\frac{1}{n}\Sigma(x-\mu)^2\right)$ and standard deviation; use of an assumed mean or a scale factor. | Knowledge of the various types of diagrams for continuous or discrete variables, either grouped or ungrouped, is expected. |
| 34. Theoretical and empirical interpretations of probability; the basic probability laws including $P(A \cup B) = P(A)+P(B) - P(A \cap B)$; | Problems will involve simple applications of the basic laws, but not Bayes' theorem in general. Tree diagrams, Venn diagrams and/or Karnaugh maps may be useful. |



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| mutually exclusive events; conditional probability; $P(A \cap B) = P(A).P(B\|A) = P(B).P(A\|B)$; independent events. | |
| 35. Probability distributions for discrete random variables: expectation; variance; (cumulative) distribution function; uniform (rectangular), binomial and Poisson distributions; their means and variances (without proofs); the use of the Poisson as an approximation to the binomial distribution (without proof). | Simple general examples may be set in whi the distribution has to be found. Knowled and use of $E(aX+b) = aE(X)+b$ and $Var(aX+b) = a^2 Var(X)$ will be expected. |
| 36. Probability distributions for continuous random variables: probability density function, f($x$); expectation, variance; cumulative distribution function, F($x$); simple general examples; uniform (rectangular) distribution; the normal distribution excluding detailed analytical treatment; use of tables; the normal distribution as an approximation to the binomial distribution (without proof). | Problems in which f($x$) or F($x$) may be giv or may have to be found. Interpretation their graphs. $f(x) = F'(x)$. Calculations means, medians, modes and expected values simple functions of $X$. No detailed knowlec of bivariate distributions will be expected, I the following results (without proof) should known: $E(aX \pm bY) = aE(X) \pm bE(Y)$ and $Var(aX \pm bY) = a^2 Var(X) + b^2 Var(Y)$ for dependent variates $X$, $Y$; if $X$, $Y$ are indep dent normal variates then $aX \pm bY$ is a norr variate. |
| 37. General ideas of sampling methods and distributions: distribution of sample mean and of the difference of two sample means; the central limit theorem; distribution of a sample proportion; the expected value of $\Sigma(x-\bar{x})^2$ (without proof). | The use of random numbers in obtaining unbiased sample. Detailed knowledge of p ticular sampling methods will not be expect but candidates may be asked to comment answer simple probability questions or method of sampling given in a question. |
| 38. Estimation of population parameters from a sample; unbiasedness; standard error and confidence limits for mean and for proportion from large samples or from samples from normal distributions of given variance. | Candidates will be expected to know that unbiased estimate of the population varia from a sample of size $n$ is $\frac{1}{n-1}\Sigma(x-\bar{x})^2$. |
| 39. Hypothesis testing: null and alternative hypotheses; test of significance of the sample | Samples will be large, or else be drawn fi normal populations with known variance. $t$-distribution will not be required. |

mean, the difference of two means or of two proportions; one-tailed and two-tailed tests; Type I and Type II errors in simple cases where the alternative hypothesis is given.

40. Bivariate samples: scatter diagrams; graphical treatment of linear regression and correlation; calculation of rank correlation coefficient (either Spearman's or Kendall's).

Calculation of product moment coefficient of correlation or of the equation of regression lines is not required, but questions may require the fitting of a regression line by eye, or the deduction from a scatter diagram as to whether $r$ is positive, negative, large or small. Knowledge that the regression lines each pass through the mean of the array will be assumed.