CAMBRIDGE
UNIVERSITY PRESS & ASSESSMENT

A Level

Mathematics

Session: 1984 June
Type: Report
Code: 9200

ADVANCED LEVEL

MATHEMATICS

Mathematics (Syllabus A)

Paper 9200/1, 9208/1 Mathematics 1 and Pure Mathematics 1

The paper proved to be somewhat more demanding than the two previous June papers, but there was no evidence that candidates found difficulty in completing their quota of questions in Section B. The general standard of accuracy of working, both algebraic and numerical, showed some decline.

Section A

Q.1 This was very poorly done. Many candidates assumed that $(\frac{1}{2}x)^2 = \frac{1}{2}x^2$, and even more had no idea that $1/(2 + 3x) = \frac{1}{2}(1 + 3x/2)^{-1}$. Of those who

attempted long division, more than half divided in descending powers of x. No candidate sought to avoid the division by equating the numerator to $(2 + 3x)(a + bx + cx^2)$.

Q.2 Most candidates appeared to have no idea of the approach to this question, which involved only the summation of $1 + 2 + 3 + \ldots + n$.

Q.3 Many candidates seemed utterly unfamiliar with a modulus sign, despite this inequality being almost identical to one given in the Notes in the syllabus. With those who attempted to square, errors such as $(x + 3)^2 = x^2 + 9$ and $2^2 = 2$ abounded.

Q.4 This trigonometrical equation was well done.

Q.5 Most candidates knew how to manipulate complex numbers in algebraic form, but the standard of accuracy in both parts was deplorable.

Q.6 Generally well done, using the coordinates of the centres and the sum of the radii. Those who adopted the purely algebraic approach of solving the two equations were all unsuccessful as the non-existence of real intersections was only a necessary but not sufficient condition for the one circle to lie completely outside the other.

Q.7 There was much work of dubious integrity used in seeking to establish that the given point lay on the curve, negative signs being introduced on an as required basis. Many seemed blissfully unaware that the tangent to the curve at $(5, -8)$ must be an equation linear in x and y and not involving the parameter t.

Q.8 Those who observed that $\sin x \, dx = d(-\cos x)$, or who used an equivalent substitution, were generally successful. The existence of the answer had clearly prevented the appearance of many negative results! Those who attempted to use multiple angles were usually let down by their weakness in manipulation.

Q.9 Candidates from a number of centres had clearly never reached the last paragraph of the syllabus. Of the remainder, most solutions contained a correct relation between y and x, but there were many errors in expressing y in terms of x.

Section B

Most candidates confined themselves to any four from Questions 10, 11, 13, 14, 16 & 17.

Q.10 Many candidates did not understand the distinction between the words 'factor' and 'root'. The work of the first paragraph was often so full of algebraic inaccuracies that correct values for a and b were never found, thus making the determination of the third factor virtually impossible for them. Many division sums, faked to produce a remainder of zero, were exhibited; and the division was often unnecessarily performed in two stages, each with a linear divisor. In the last stage, false 'sandwiches' such as $-\frac{1}{2} > x > 2$ were not infrequently exhibited.

Q.11 The partial fractions and the determination of the negative nature of the gradient were usually well done. There were also some well-drawn graphs, but too many candidates made very 'heavy weather' of this exercise. It was really only necessary to observe that $y > 0$ for $x > 1$, that the gradient was always negative, and that, for single asymptotes, the curve approaches the asymptote from opposite sides at its extremities, to enable the graph to be drawn immediately. Most candidates were able to determine the linear graph required to establish the number of real roots of the final equation.

Q.12 There were few attempts at this question, most of which did not proceed beyond the second paragraph. Many of those attempting the third stage thought that the perpendicular from A to CV produced was the line through A perpendicular to AC. It should be emphasised that 'the exact value of $\cos ARB$' means exactly what it says – the use of a calculator to establish the approximate magnitude of the angle and hence the value of its cosine (to 6, 7, or more decimal places) is not a proper response to the question posed.

Q.13 On the whole, this question was well done. The identity was established – its application to the given equation observed – and solutions in the required interval obtained. As expected, the commonest error was to 'lose' the solution corresponding to the linear factor.

Q.14 At this level, candidates are expected to have a reasonable appreciation of what may be quoted and what is expected to be established. In this question, it was clearly unacceptable to say "$SP = PM$ – focus-directrix property of a parabola" since neither the focus nor the directrix had been mentioned in the question. Similarly, in the next part, it was unacceptable to quote the result "tangents at the ends of a focal chord meet at right angles on the directrix" in order to establish that $pq = -1$. Most candidates had the correct intentions in the last part but were often defeated by their lack of manipulative ability with the algebra.

Q.15 This question, rather surprisingly, was only infrequently attempted. Many found the integration of $(ax)^{1/2}$ quite beyond them – a check on the dimensions of the result would, in this case, have indicated the presence of any non-numerical error. Sketch graphs, which would have drawn attention to the symmetry of the figure were very infrequently drawn.

Q.16 The attempts were many but the marks gained were few. The first differentiation was generally correct. After this, the work tended to be naive in the extreme, with algebraic errors abounding – few were able to recognise the quadratic form as a perfect square. The failure to obtain the zeros of the first and second derivatives removed much of the guidance which it was hoped to give to the sketching of the graph. The non-negative nature of the first derivative was frequently at variance with the graph.

Q.17 This question proved to be a godsend to many candidates. Marks, however, were often lost in the second paragraph through failure to make clear the function to which the Newton-Raphson method was being applied.

Paper 9200/2

The paper gave a good spread of marks. Nevertheless there was a good number of candidates who gave seven completely correct answers. The proportion attempting the statistics section appears to have stabilised at about one third. The proportion attempting some topics from both sections is, perhaps, increasing, though still small.

Mechanics

Q.1 *(a)* Generally well done although many candidates are clearly used to using a polygon of forces for the resultant and resolution for equilibrium. As a result, the 'additional force' was often found after another page of working, instead of by reversing the resultant.

In *(b)* many candidates had little idea – indeed the vector notation was often not understood : $3\vec{AC} + 3\vec{BC}$ was taken to mean 3 along AC plus 3 along BC. There were many attempts at graphical solutions, often starting with $ABCD$ a square or a rectangle ($\vec{AB} = \vec{DC}$ etc.).

Q.2 The most common errors were indistinguishable. Either "the wind is blowing from due south" was misread as "the wind is blowing towards due south" or the

candidate endeavoured to "stop the wind" and got into a muddle with the reversed wind. Candidates must check that they have correctly read information (in this case, directions) from the paper. (A similar error in *Q.1* was to take N60°W to mean W60°N.) Trigonometric methods are still surprisingly popular, although the calculator has reduced their arithmetical terrors.

Q.3 The given answer helped many candidates and complete solutions were common. Some candidates needed pages of geometry concerning medians to deal with the triangle – $\frac{1}{3}M$ at each vertex is much easier. The most common error was to muddle the lengths involved in the second part.

Q.4 Part (i) was surprisingly badly done since once the acceleration has been written as $v\,dv/dx$ it is all calculus. Attempts to start with dv/dt or to use the constant acceleration formula were common. The maximum speed caused many problems. Very few argued: acceleration $= 0 \Rightarrow x = a \Rightarrow v^2 = 4k^2a^2$. Many differentiated the square root of the given expression for v^2 and got lost on the way. Another common "argument" was: acceleration $= 0 \Rightarrow a = 0 \Rightarrow v^2 = 4k^2x^2$.

Some candidates seemed to think that parts (i) and (ii) were related and the two expressions for v were equated. Of those who started correctly with $dx/dt = Ve^{\alpha t}$; few could integrate $e^{\alpha t}/(t + 1)$, $e^{\alpha t}/(\alpha t + 1)$ were popular candidates. Many failed to eliminate t to get $v = V + \alpha x$.

Q.5 This question was generally well done although often by roundabout methods. A few candidates confused themselves with $g = 10$ or $g = 9{\cdot}8$. Many candidates overlooked the request for U in terms of V.

Q.6, 7 The statics questions were, as usual, badly done. The difficulties lie in putting in the appropriate contact forces and in knowing on which body they act e.g. T along DC acting on AB at C. Another difficulty is with the trigonometry in a moment equation. The moment of the thrust, T, in CD, about A in *Q.6* caused problems (Ta, $Ta\sin\theta$ etc.) and the moment of W about D or A in *Q.7*. Candidates appear to be trained on "force X perpendicular to line of action" only and do not use the sum of the moments of the components $(T\sin\theta)(2a\cos\theta) + (T\cos\theta)(0)$ in *Q.6* and $(W\cos\theta)a \pm (W\sin\theta)\frac{1}{2}a$ in *Q.7* or, in *Q.7*, replace W by $\frac{1}{2}W$ at B and $\frac{1}{2}W$ at D.

Q.8 This was generally well done apart from those who ignored the 'k' in kW. (For some candidates, PkW = P000W!) Provided candidates equated power and not tractive force, the first part was generally correct. In the second part one or other of the forces was often omitted ($\lambda 10^2 = ma$ or $P/10 = ma$).

Q.9 The first part was generally correct, apart from the usual mistakes over impulse, for which, of course, only one mass should be considered. In the second part, many candidates failed to realise that A plays no part in the collision – it often appeared with different velocities on either side of the momentum equation.

Q.10 There were some cases of equal tensions in AP and PR. The most common error was $T_1\cos\theta + T_2\cos\theta = 3mg$, showing the usual confusion as to which force acts on which particle. A reasonable number of complete solutions was seen.

Statistics

Q.11 Most candidates correctly used midpoints for calculation of the mean, but many also used midpoints instead of the right handed points for the cumulative frequency distribution. It was clear that many candidates think that decimal places and significant figures are synonymous. With calculators, $\Sigma fx^2 - (\Sigma fx)^2/\Sigma f$ gave zero exactly due to loss of significant figures – few found it surprising to get zero variance from a spread of data. In order to find the variance correct to 3 significant figures it is necessary to change origin (or use an assumed mean).

Q.12 Many candidates interpreted the question to mean that "the next two engines" could include "the same engine, seen twice" – thereby turning the question into "with replacement" instead of "without". Solutions based on this were allowed substantial credit. Many candidates found $P(A \cap B)$ by multiplying $P(A)$ and $P(B)$ and then found $P(A/B)$ by dividing $P(A \cap B)$ by $P(B)$.

Q.13 Some candidates attempted to use the binomial distribution in (ii) and (iii), but many obtained the full 6 marks for the first part. The second part was rarely correct – many confused the proportion of pierced ears (with expected value 0·35) with the number of pierced ears per islander (with expected value 0·7). There were very few successful attempts to deal with the variances.

Q.14 Parts (i) and (iii) of this question were generally correct. In part (ii) it was quite common to see $P(X + Y = 0) = P(X = 0) + P(Y = 0)$. In the last part, those who considered all pairs (X, Y), such that $X < Y$ often treated the cases with $Y = 4$ to be negligible as well as those with $Y > 4$. Misconceived attempts to use the normal distribution in the last part were common.

Q.15 Once the probability of a 'hit' was seen to be $\frac{1}{3}$, the question was well done by most candidates although the continuity correction was often omitted in *(b)*.

Q.16 Few sensible attempts were seen – generally candidates assumed that the probability density function of Y was y^2 (from 0 to 9!). Very few started with an understanding of the distribution of X – either the p.d.f. or the cumulative distribution function. Amongst the rare attempts the most frequent was to assume (ii) and deduce (i) and (iii).

Q.17 Nearly all candidates failed to realise that e.g., P(Adam catches 11·30 bus) = P(Adam arrives before 11·30) – P(Adam arrives before 11·00). There were heroic attempts to obtain the given answer of 0·477 by using bogus continuity corrections but the fundamental misunderstanding scuppered the rest of the question. Attempts to use the sum of two normal variables were doomed. Very few realised that the second and third parts required $p_1p_1' + p_2p_2' + \ldots$.

Q.18 There were many confused attempts at this question. Some treated it as a test of the difference in the proportions $\frac{30}{100}$ and $\frac{27}{120}$. Others carried out a test using a variance of pq/n, with $p = 27/120$ instead of $3/10$. It is not enough to push the numbers into any old formula – the hypotheses have to be understood. Even when the correct formula was used, the hypotheses were often absurd – e.g. $H_0 : \frac{30}{100} = \frac{27}{120}$, *or* $\frac{30}{100}$ is not significantly different (at the 5% level) to $\frac{27}{120}$. Those who sought safety in words did not escape – e.g. H_0: 30% prefer butter, H_1: 30% do not prefer butter *or* 30% prefer margarine. Many candidates fail to realise that hypotheses are concerned with population parameters and not sample values. Similarly in the second part many failed to realise that the (null) hypothesis is still $p = 0{\cdot}3$. The continuity correction was frequently omitted (in both parts).

Q.19 This was generally much better than *Q.18*, although, in the first part, $1/\sqrt{40}$ was often omitted in the variance. In the second part the arithmetic was frequently correct, but, again, the hypotheses often did not refer to the mean length of the drives.

Q.20 This was frequently done well. The most frequent errors were failure to use graph paper and ridiculous scales, often with axes meeting at (0, 0) so that the data points were scattered in a minute region or a narrow strip.