CAMBRIDGE
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A Level

Mathematics

Session: 1994 June
Type: Mark scheme
Code: 9205

MS9 (UK)

University of Cambridge
Local Examinations Syndicate

GCE Examinations June 1994

MARKING SCHEME
for
MATHEMATICS

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GCE ADVANCED LEVEL EXAMINATIONS
REVISED MARKING SCHEME JUNE 1994

1 *EITHER*: State or imply $2(-\frac{1}{2})^3 + a(-\frac{1}{2})^2 + 16(-\frac{1}{2}) + 6 = 0$ — M1
Obtain given answer $a = 9$ — A1

OR: Evaluate $2(-\frac{1}{2})^3 + 9(-\frac{1}{2})^2 + 16(-\frac{1}{2}) + 6$ — M1
Show correctly that this comes to zero — A1 — **2**

Carry out division by $(2x + 1)$ far enough to obtain 3-term quotient, or equivalent (e.g. factorise by inspection) — M1
Obtain correct quotient $x^2 + 4x + 6$ and no remainder — A1
Write as $(x + 2)^2 + k$, or differentiate, equate to zero and solve for x, or solve $x^2 + 4x + 6 = 0$, or sketch U-shaped quadratic graph, or consider $b^2 - 4ac$ — M1
Demonstrate given result correctly — A1 — **4**

[Not much need be said for the last A1, but *something* is needed; e.g. they could just state without further explanation that $(x + 2)^2 + 2$ is always positive. Algebraic (or other) details need to be correct for the mark to be given.]

[Candidates who appear to omit the first part and start on the second part will forfeit the first 2 marks for showing $a = 9$ unless they *state explicitly* that they've shown it by the fact that $(2x + 1)$ divided exactly; if they say this they can get B2 as a special case.]

[The question can be done back-to-front, with the first part appearing as a result of working to find the quadratic factor, though perhaps not many will try this. Stating $(2x + 1)(px^2 + qx + r)$ and finding numerical values for p and r: M1; using coefficient of x to find q: M1, $q = 4$: A1; deducing $a = 9$: A1. The last two marks for the question are then as normal.]

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2 *EITHER*: State any series of positive integer powers of x beginning $1 + \frac{1}{2}x$ — B1
Show correct method for either binomial coefficient $\dfrac{\frac{1}{4}(\frac{1}{4} - 1)}{2}$ and/or $\dfrac{\frac{1}{4}(\frac{1}{4} - 1)(\frac{1}{4} - 2)}{3!}$ — M1
Obtain $-\frac{3}{8}x^2$ correctly — A1
Obtain $+\frac{7}{16}x^3$ correctly — A1

[There could be numerical errors in the unsimplified coefficients with the M1 being earned; e.g. $\frac{1}{4} - 1$ mentally calculated as $-\frac{1}{4}$. The details of the $(2x)^2$ and $(2x)^3$ don't matter for the M mark, except that it should be the correct integer powers of x involved.]

OR: Differentiate and substitute $x = 0$ at least once — M1
Obtain first two terms $1 + \frac{1}{2}x$ — A1
Obtain $-\frac{3}{8}x^2$ correctly — A1
Obtain $+\frac{7}{16}x^3$ correctly — A1 — **4**

[Single uncancelled fractions OK for A1, A1, or exact decimals.]

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3 *EITHER*: State or imply $x \log 2 = y \log 3$ (any base, or none at this stage) — M1
Obtain simplified equation in one unknown, e.g. $x \log 2 = (1 - x)\log 3$ — A1
Carry out correct processes to solve a linear equation — M1
Obtain given answer $x = \dfrac{\ln 3}{\ln 6}$ — A1

OR: Substitute e.g. $y = 1 - x$ and use rules of indices to get $2^x = \dfrac{3}{3^x}$ — M1
Simplify to $(2 \times 3)^x = 3$ — A1
Obtain $x \ln 6 = \ln 3$ or $x = \log_6 3$ — M1
Obtain given answer $x = \dfrac{\ln 3}{\ln 6}$ — A1 — **4**

[No credit for numerical verification of given answer by calculator. For exact verification, they might find y from $x + y = 1$, getting M1 A1 for $y = \dfrac{\ln 2}{\ln 6}$, and then M1 A1 for checking this (via $x \ln 2 = y \ln 3$ presumably) in the other equation. Other possibilities seem to involve an M1 A1 for log/index rules as in one of the regular methods, then M1 A1 for checking the given x rather than finding it.]

[Writing $2^x - 3^y = 0$ followed immediately by $x \ln 2 - y \ln 3 = 0$ without any explanation is M0 (and therefore A0) for use of log rules; however they can then recover and get the next M1 A1 if the solution is completed.]

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4 State or imply arc length $= 2r$ — B1
Calculate sector area via value of θ found from $s = r\theta$, or via $\pi r^2 \times \dfrac{2r}{2\pi r}$ — M1
Obtain answer r^2 — A1 — **3**

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5 Use $7^2 = 2^2 + b^2 - 4b\cos 30°$ and exact $\cos 30°$ to show given result correctly — B1

Make recognisable attempt at quadratic formula or completing the square, or carry out any complete 'otherwise' method for b — M1

Show exact working at least as far as $\frac{1}{2}(2\sqrt{3} \pm \sqrt{192})$ (or equivalent result given by other methods) *and* state the single correct answer for b (though maybe not in exact form) — A1 — **3**

[It's no good picking out the correct one in the next part; the uniqueness and the exactness (not necessarily fully simplified) must both be demonstrated (but not justified) in this part to get the A1]

Use sine rule with B, 30°, 7 and previous answer for b, or any other complete method — M1

Obtain given result — A1 — **2**

[Allow the final A1 if the given answer is reached, even if some decimal working is involved in this part. Special case: b not evaluated earlier, but $\frac{b}{\sin B} = \frac{7}{\sin 30°}$ stated: allow the M1.]

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6 [No penalty for minor rounding errors; e.g. answers for angles (rounding to) within $\pm 0{\cdot}1°$ of correct values are OK.]

State or imply $R = \sqrt{34}$ (which is 5·83...) — B1

State or imply $\alpha = \arctan\frac{5}{3}$, or equivalent, (which is 59·0...°) — B1 — **2**

EITHER: Evaluate $\arccos\left(\frac{2}{\text{their } R}\right)$ — M1

Obtain value 10·9 and/or −129·0 or any other single correct value — A1

Use correct general formula $360n \pm$ something before subtracting α — M1

Obtain answer $360n + 10{\cdot}9$ and $360n - 129{\cdot}0$ or equivalent — A1

OR: Use correct $\tan\frac{1}{2}\theta$ substitutions and obtain quadratic (it's $5t^2 + 10t - 1 = 0$) — M1

Find any one correct value for θ, e.g. 10·9 — A1

Use correct general process, i.e. $2(\arctan(t) + 180n)$ — M1

Obtain answer $360n + 10{\cdot}9$ and $360n - 129{\cdot}0$ or equivalent — A1

OR: Carry out some mad squaring method and reduce to a soluble equation in one trig function (e.g. $34s^2 + 20s - 5 = 0$) — M1

Find any one correct value for θ, e.g. 10·9 — A1

Use relevant correct process for a general form of solution — M1

Obtain answer $360n + 10{\cdot}9$ and $360n - 129{\cdot}0$ or equivalent (and no extras) — A1 — **4**

[N.B. Answer $360n \pm 69{\cdot}9 - 59{\cdot}0$ is fully acceptable; if this is seen, ignore any subsequent wrong simplification. However, the common wrong answer 360 ± 10.9 does *not* of itself imply the correct version and will not normally score the last two marks.]

[No penalty for degree/radian muddles; values corresponding to 69·9 and 59·0 are 1·22 and 1·03; acceptable accuracy ±0.01.]

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7 State $\dot{x} = 1 + e^t$ and/or $\dot{y} = 1 - e^{-t}$ — B1

State $\frac{dy}{dx} = \frac{1 - e^{-t}}{1 + e^t}$ — B1✓

Equate gradient (or $\dot{y}$) to zero — M1

Obtain $t = 0$, or correct explicit unsimplified expression. e.g. $t = -\ln 1$ — A1

State coordinates (1, 1) fully simplified — A1✓ — **5**

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8 Use factor formula in expression of the form $\frac{y_2 - y_1}{x_2 - x_1}$ — M1

Show given result correctly — A1 — **2**

[No explanation really required for the A1; they can just write down the answer following a correct factor formula statement.]

State $(\phi - \theta)$ is small and use $\sin x \approx x$ — B1

Identify $\frac{1}{2}(\phi + \theta)$ as being (approximately) θ — B1 — **2**

[For the first B1, it's no good if they talk about *zero* angles, or try to say that $\frac{0}{0} = 1$. The second B1 is independent of first; no explanation required for it.]

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9	State $B = -1$ and/or $C = 1$	B1	
	Carry out any complete method for finding A	M1	
	Obtain $A = -1$ correctly	A1	3
	[Obtaining the false identity $1 \equiv Ax^2(x-1) + Bx(x-1) + Cx^3$ is M0.]		
	State terms $-\ln x$ and $+\ln(x-1)$	B1✓	
	State term $+x^{-1}$	B1✓	2
	[Follow through on non-zero values of A, B, C only. Special case: if no values for A, B, C were found, the last two B marks can be earned if $A\ln x + C\ln(x-1)$ and $-Bx^{-1}$ are stated.]		
10	(i) State answer 15 504	B1	1
	(ii) *EITHER*: State expression involving $\binom{10}{2} \times \binom{10}{3}$	M1	
	Obtain answer 10 800 correctly	A1	
	OR: State expression involving both $\binom{10}{4} \times \binom{10}{1}$ and $\binom{10}{5}$, and subtract from the previous answer	M1	
	Obtain answer 10 800 correctly	A1	2
11	(i) Expand LHS completely, or divide RHS by $(k+1)^2$ (in 1 or 2 steps)	M1	
	Demonstrate the identity correctly	A1	2
	[The marks for (i) may be earned in (ii), but only if *exactly* equivalent work is fully carried out in the course of doing the induction.]		
	(ii) Check $1^3 = 1^2(2 \times 1^2 - 1)$	B1	
	Consider $k^2(2k^2-1) + (2k+1)^3$ or equivalent	M1	
	Obtain $2k^4 + 8k^3 + 11k^2 + 6k + 1$ from the above expression, and complete	A1	3
	[M1 requires $S_k + T_{k+1}$ attempt; not allowed to count as valid attempts at T_{k+1} are e.g. $(k+1)$, $(k+1)^3$, $(2k-1)^3$.]		
12	(a) State equation $\frac{a(1-r^8)}{1-r} = \frac{1}{2} \times \frac{a}{1-r}$ or $\frac{a(1-r^8)}{1-r} = \frac{ar^8}{1-r}$	B1	
	Eliminate a to obtain equation in r only	M1	
	Obtain answer (rounding to) 0·917 correctly	A1	3
	Use $ar^{16} = 10$ with numerical r to find a	M1	
	Obtain answer $a = 40$ (no penalty for using decimal working)	A1	2
	[N.B. Using S_{17} instead of T_{17} is not to be counted as MR.]		
	(b) Equate (reasonable attempt at) sum to n terms of an AP with $d = 10$ to 10 000	M1	
	Obtain $a = \frac{10\,000}{n} - 5(n-1)$ or equivalent	A1	
	Deduce given result $\frac{10\,000}{n} + 5(n-1)$ for nth term	A1	3
	State $\frac{10\,000}{n} + 5(n-1) < 500$ and multiply through by n	M1	
	Simplify correctly to given form	A1	
	Use (recognisable attempt at) quadratic formula, or e.g. trial and error	M1	
	State answer 73	A1	4
	[If no working shown for value of n: correct answer 73 stated gets M1 A1; integer answer 74 or decimal answer 73·96 stated gets M1 A0; any other answer gets M0]		

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13 **(a)**	**(i)** State $f(x) = \ln(1+x)$ or imply this e.g. by 1st quadrant graph starting at O, concave down	B1	
	Sketch correct graph, i.e. quads 1 and 3, through O, (implied) vertical asymptote on left	B1	2
	[The second B1 implies the first B1 in this case.]		
	(ii) Identify $g^{-1}(x)$ as e^x	B1	
	Identify $h^{-1}(x)$ as $x-1$	B1	2
	(iii) Identify $g^{-1}h^{-1}(x)$ as e^{x-1}	B1✓	1
	(iv) Sketch an appropriate exponential shape for the graph	B1✓	
	Show curve correctly located, e.g. through $(1,\ 1)$ or $(0,\ e^{-1})$	B1	2
	[The only expressions we follow through on are ones they could 'reasonably' have found for **(iii)**; i.e. $e^{\pm 1 \pm x}$ or $\pm 1 \pm e^{\pm x}$]		
(b)	Sketch the (relevant part of the) parabola	M1	
	Use correct graph to explain the one-one property	A1	
	Use quadratic formula to solve $x^2 - 4x - y = 0$ for x, or equivalent	M1	
	Use known point, e.g. $x = 0$, $y = 0$, to select correct sign	M1	
	Obtain answer $q^{-1}(x) = 2 - \sqrt{(4+x)}$ or equivalent	A1	5
	[First M1 to be given generously, but the next A1 will be hard to earn. They must say (somehow) that each y-value corresponds to just one x-value and their sketch must support this assertion.]		
14	[No penalties for misuse of vector notation, so long as the meaning is clear.]		
(i)	Carry out all calculations with coordinates needed for $\overrightarrow{PM}$	M1	
	Obtain the given equation correctly	A1	2
	[Don't worry if the details they show are a bit sketchy, so long as it's clear they know that it's $\overrightarrow{PM}$ that's the direction vector.]		
(ii)	Carry out all calculations with coordinates needed for $\overrightarrow{QN}$	M1	
	State answer $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} -3 \\ -1 \\ h \end{pmatrix}$ or equivalent	A1	2
	[If no working is shown, the M1 would be implied by an answer with e.g. one sign error.]		
(iii)	Equate at least two components from the equations found	M1	
	Show that $t = s = \frac{1}{2}$ works for all three components	A1	
	Derive given position for X, i.e. $(\frac{1}{2},\ -\frac{1}{2},\ \frac{1}{2}h)$ from t or s	A1	3
	[If they use the same letter for the parameter in both lines, they'll get the 'right' answer; however, max M1 out of 3 in this case. Note that this part can be done otherwise, e.g. by verification or by noting that X is the mid-point of each of PM, QN. However, any method must deal with all three components if full marks are to be earned.]		
(iv)	Show correct processes for the calculation of any scalar product	M1	
	Equate the scalar product of the two relevant vectors to zero	M1	
	Obtain $h = \sqrt{2}$ correctly	A1	
	Carry out the correct processes to evaluate $\cos^{-1}\left(\frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}\right)$ for the two relevant vectors	M1	
	Obtain answer 70·5° or 109·5°	A1	5
	[It appears to me not obvious, at the start of **(iv)**, that VB is inclined at 45°; they need to say e.g. that OX produced goes to the mid-point of VB or that OX is parallel to DV to justify this, I think. Hence using this 'fact' to find $h = \sqrt{2}$ gets M0 M0 A0; however, the last 2 marks remain available. Thinking that OX is the x-axis leads to such obvious impossibilities so quickly that I think we can (and should) do nothing about it.]		

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15	Attempt product rule to differentiate $\cos(x+\alpha)\cos^2 x$ and equate to zero	M1	
	Obtain $-\sin(x+\alpha)\cos^2 x - 2\cos(x+\alpha)\cos x \sin x = 0$	A1	
	Identify the solution $\cos x = 0$	A1	
	Divide through simplified equation by $\cos(x+\alpha)\cos x$	M1	
	Obtain $\tan(x+\alpha) + 2\tan x = 0$ correctly (ignore any other 'possibilities' that they turn up)	A1	5
	[If they expand $\cos(x+\alpha)$ first, they might still get the first 3 marks, but the last 2 will almost certainly be out of reach. The equation for the first A1 is $-3\cos^2 x \sin x \cos\alpha - \cos^3 x \sin\alpha + 2\cos x \sin^2 x \sin\alpha = 0$.]		
	(i) Use $\tan(A+B)$ formula to express equation in terms of $\tan x$ only	M1	
	Obtain $(2\sqrt{2})t^2 - 3t - \sqrt{2} = 0$ or equally simplified exact equivalent	A1	
	Use quadratic formula or equivalent	M1	
	Obtain exact $\sqrt{2}$ and $-\frac{1}{4}\sqrt{2}$ or exact equivalents	A1	4
	[Ignore subsequent working once the exact values are seen.]		
	(ii) Use Pythagoras or equivalent to calculate $\sin x$ and/or $\cos x$	M1	
	Use $\cos(A+B)$ or $\cos 2A$ formula with relevant numerical values to calculate y	M1	
	Show given answer $-\frac{1}{9}$ correctly	A1	3
	[If they use approximate methods (i.e. calculator evaluation of α, etc) please give max M1 out of 3 for the complete calculation of y]		
16	Obtain $\frac{2}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}}$ for the integral	B1	
	Use limits 1 and 4 to find the area under the curve	M1	
	Subtract from the area of the rectangle (or subtract the other way round!)	M1	
	Obtain answer $\frac{1}{3}$ or equivalent	A1	4
	State equation of the form $y = \sqrt{x} + \dfrac{2}{\sqrt{x}} + k$	B1	
	State correct equation (i.e. $k = -3$)	B1	2
	State or imply required volume is $\pi\int_1^4 y^2\,dx$ using their transformed y	B1✓	
	Show correct processes for squaring a trinomial y	M1	
	Obtain given integrand correctly	A1	
	Integrate at least 3 of the given terms correctly	M1	
	Obtain $\frac{1}{2}x^2 - 4x^{\frac{3}{2}} + 13x - 24x^{\frac{1}{2}} + 4\ln x$ or equivalent	A1	
	Obtain $(4\ln 4 - \frac{11}{2})$ or exact equivalent	A1	6
	[Some working is necessary to demonstrate the trinomial has been properly squared out; if none is shown they'll lose M1 A1. Note that we allow the omission of π from the final answer; for the numerical bit, powers must be evaluated and terms collected up.]		
17	**(i)** Use double-angle formula relating $\cos 2x$ and $\sin^2 x$	M1	
	Obtain or verify given answer correctly	A1	2
	(ii) Use parts, going the correct way (condone sign errors at this stage, and allow anything for the integral of $\sin^2 x$, except $\sin^2 x$ itself!)	M1	
	Obtain $x(\frac{1}{2}x - \frac{1}{4}\sin 2x) - (\frac{1}{4}x^2 + \frac{1}{8}\cos 2x)$, or equivalent	A1	
	Show given answer correctly	A1	3
	(iii) Differentiate, and substitute throughout for u and du	M1	
	Obtain $-\int \frac{1}{2}\sin^2 x\,dx$ (any limits or none at this stage)	A1	
	Obtain answer $\frac{1}{8}\pi$ correctly	A1	3
	(iv) Carry out all the correct substitution steps again	M1	
	Use double-angle formula and obtain given result (condone lack of explanation over limits)	A1	
	State or imply integral of the form $ax + b\sin 4x$, or make further substitution $\theta = 2x$	M1	
	Obtain answer $\frac{1}{32}\pi$, with no errors seen anywhere	A1	4

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18	(i) *EITHER*: Use chain rule to differentiate $\sqrt{(1+y^3)}$ (at least 2 factors required)	M1	
	Obtain $\frac{1}{2}(1+y^3)^{-\frac{1}{2}} \times 3y^2 \times \frac{dy}{dx}$	A1	
	Show given result correctly	A1	
	OR: Attempt differentiation of both sides of $(y')^2 = 1 + y^3$ (2 factors on at least one side required)	M1	
	Obtain $2y'y'' = 3y^2y'$	A1	
	Show given result correctly	A1	3
	(ii) State $y''' = 3yy'$	B1	
	Use product rule to differentiate RHS of this w.r.t x	M1	
	Obtain answer $y^{(4)} = 3(y')^2 + \frac{9}{2}y^3$, or any equivalent in terms of y only or y and y'	A1	3
	[Anyone differentiating y''' in the form $3y\sqrt{(1+y^3)}$, for instance, might leave $y^{(4)}$ as $3y'\sqrt{(1+y^3)} + \frac{9y^3y'}{2\sqrt{(1+y^3)}}$.]		
	(iii) State f(0) = 0 and evaluate y', y'', y''', $y^{(4)}$ at $x = 0$	M1	
	Use Maclaurin's series	M1	
	Show the given answer correctly (allow 'flukes' where errors in their derivatives give the correct Maclaurin coefficients)	A1	3
	Show or imply use of at least two expressions for values of the integrand	M1	
	Use correct trapezium rule formula with $h = 0{\cdot}1$ and 5 function values	M1 (dep)	
	Obtain answer (rounding to) 0·397	A1	3

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1α	Figure with T, 1.5, 0.5g, seen or implied	M1	
	Two correct eqns eg $T\cos\theta=5$, $T\sin\theta=1.5$, $\tan\theta=0.3$	B1B1	
	Two attempted eqns, 3 forces, correct mech principles	M1	
	$T=5.22N$, $\theta=16.7°$	B1B1	6
β	Two correct eqns eg $2=5\sin\phi$, $T'=5\cos\phi$, $T'\cos\phi+2\sin\phi'=5$, $T'\sin\phi=2\cos\phi'$	B1B1	
	Two attempted eqns, 3 forces, correct mech principles	M1	
	$T'=4.58N$, $\phi=23.6°$	B1B1	5
γ	Attempt to use change in mgh	M1	
	$0.5g(0.3)(\cos\theta-\cos\phi)$ or $(0.5g)(0.0360)(\cos 69.9)$; 0.062J	A1A1	3
2α	$x=20t$; $y=30t-gt^2/2$ or $30t-5t^2$	B1B1	
	$\tan\phi=x/y$; $=4/(6-t)$ AG	M1A1	4
β	Set $\tan\phi=4/3$ and solve for t; $t=3$;	M1A1	
	Sub for t in x,y and use $OA^2=x^2+y^2$ or $x/\sin\phi$ or $y/\cos\phi$	M1	
	$((x,y)=60,45))$ $OA=75m$ AG	A1	
	$v_A=20\ ms^{-1}$ horizontally or equiv	B1	5
γ_1	$t=4.5$: $\dot{y}=(-)15$ or $\dot{y}^2=225$ or $v^2=62.5$	B1	
	$KE=0.2(\dot{x}^2+\dot{y}^2)/2$; $=62.5J$	M1A1	3
γ_2	$y=135/4$ or 33.75 OR D (depth below top) $=45/4$ or 11.25	B1	
	$KE=0.1(20^2+30^2)-0.2gy$ or $0.1v_A^2+0.2gD$; 62.5J	M1A1	(3)
δ	$\tan\psi=\dot{x}/\dot{y}$ or $\dot{y}/\dot{x}$ or equiv used $(=(-)3/4)$; $(-)36.9°$	M1A1	2

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3α	$0.1v\frac{dv}{dx} = -\frac{1}{5x^2}$ or $v\frac{dv}{dx} = -\frac{2}{x^2}$ ("+": M1)	M1A1	
	Attempt to separate and integrate both sides	M1	
	$\frac{1}{2}v^2 = \frac{2}{x}$ or equiv or ($-2/x$ from $+2/x^2$)	A1	
	"+C" and validly show C=0 $v = \frac{2}{\sqrt{x}}$ AG	A1	5
β	$\frac{1}{2}(0.1)2^2 - \frac{1}{2}(0.1)\left(\frac{2}{3}\right)^2$; $\frac{32}{180}$, $\frac{8}{45}$, 0.178 aef	M1A1	2
γ_1	$\frac{dx}{dt} = \frac{2}{\sqrt{x}}$	B1	
	Attempt to separate and integrate both sides	M1	
	$\frac{2}{3}x^{3/2} = 2t$ (+C) or equiv	A1	
	$\left[\ \right]_1^9 = \left[\ \right]_0^T$ or "+C" and sub (x=1, t=0), (x=9 t=t)	M1	
	time = 26/3 = 8.67 aef	A1	5
γ_2	Mark $dv/dt=-v^4/8$, $v^{-3}/3=t/8$ (+C′) as γ_1		
δ	"NO" with valid reason eg v never zero or never neg, or P goes to infinity, or unique t for x=1	B2	2

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4α I=(±)(0.2)(3+9), I=(±)(0.3)(2+v), (signs must be consistent)
(0.2)9 −(0.3)v = −(0.2)3 + (0.3)2 (or =0) Any two — B1B1
I=(±)2.4 Ns; v=6 ms^{-1} — B1B1 — 4
[If 0/4 allow M1 for attempt at two of the above - ignore sign errors]

β KE lost = $\frac{1}{2}(0.2)9^2 + \frac{1}{2}(0.3)v^2 - \frac{1}{2}(0.2)3^2 - \frac{1}{2}(0.3)2^2$ — M1
$(= \frac{27}{2} - \frac{3}{2}$ or $\frac{36}{5} + \frac{24}{5})$ = 12 J — A1 — 2

γ Time for P to reach wall = 6/3 or 2 — B1
Distance of Q from wall (ie P) = 6+2x2; = 10m AG — M1A1 — 3

δ_1 Time t to collision given by 3t=10+2t; t=10 — M1A1
Distance (=3x10) =30m — B1 — 3

δ_2 (10+x)/3=x/2 or s/3=(s−10)/2; s=30m — M1A1A1(3)

ε Final answer based on (0.2)3 + (0.3)2; = 1.2 Ns — M1A1 — 2

5α Tcosα = mg (T=25/4) — M1A1
Tsinα = mrω² or mv²/r — M1
r = 1+5sinα or 4; Tsinα=m(1+sinα)ω² — B1A1
(ω²=15/8) ω=1.37rad s^{-1} — A1 — 6

β X = λ(10−6)/6 or 2λ/3 or equiv — M1A1
1+5sinβ = 5 or sinβ = 4/5 or β=53.1 — B1
Pcosβ = 0.5g — B1
Psinβ+X = 0.5(accn); = (0.5)5x2² — M1A1
Four equations for P,X,β,λ and find λ; λ=5N — M1A1 — 8

GCE ADVANCED LEVEL EXAMINATIONS
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6a	4/24 or 20/24 x some attempt at another prob;	M1	
	Validly obtaining 10/69 AG (decimals not used)	A1	2
(i)	"23x22" seen or implied in denominator	M1	
	correct unsimplified answer eg $\frac{3}{23}\frac{2}{22}$; $\frac{6}{506}$ or $\frac{3}{253}$ or 0.0119	A1A1	3
(ii)	Attempt at summing probability of correct combined events	M1	
	Correct unsimplified expression seen or implied	A1	
	eg $P(A)+P(A'C) = \frac{4}{24} + \frac{20}{24}\frac{4}{23} = P(C)+P(C'A)$		
	or $P(A)+P(C)-P(AC) = \frac{4}{24} + \frac{4}{24} - \frac{4}{24}\frac{3}{23}$		
	or $P(AC')+P(A'C)+P(AC) = \frac{4}{24}\frac{20}{23} + \frac{20}{24}\frac{4}{23} + \frac{4}{24}\frac{3}{23}$		
	or $1-P(A'C') = 1 - \frac{20}{24}\frac{19}{23}$		
	$\frac{172}{552}$ or $\frac{86}{276}$ or $\frac{43}{138}$ or 0.312	A1	3
(iii)	Correct method for final answer	M1	
	Correct unsimplified expression eg $\frac{4}{24}\frac{11}{23}$	A1	
	$\frac{44}{552}$ or $\frac{22}{276}$ or $\frac{11}{138}$ or 0.0797	A1	3
(iv)	Attempt P(iii)/P(either); = P(iii) / $\frac{12}{24}$; $\frac{22}{138}$ or $\frac{11}{69}$ or 0.159	M1A1A1	3

7α	Attempt F(3)=0 or F(4)=1 or F(4)−F(3)=1 Two correct equations 9a−24a+b=0 & 16a−32a+b=1 ; (b=15a=1+16a) Validly showing a=−1 AG; b=−15 ; Validly showing $F(3.5)=\frac{3}{4}$ AG	M1 A1 B1A1B1	5
(i)	Attempt differentiate F(x) to obtain f(x); f(x)=8−2x (Do not insist on "$3\le x\le 4$" or on "f(x)= 0 otherwise")	M1A1	2
(ii)	Attempt integrate xf(x) with limits 3,4; integral $= 4x^2-\frac{2}{3}x^3$; $(64-\frac{128}{3}-36+18)$ Validly obtaining $\frac{10}{3}$ AG	M1 B1 A1	3
(iii)	$3\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)$; $\frac{27}{64}$ or 0.422	B1B1	2
(iv)	$N\left(\frac{10}{3}, \frac{1}{1800}\right)$ or in words (only two correct B1)	B2	2

GCE ADVANCED LEVEL EXAMINATIONS
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8(i)$_P$	Poisson mean 5 seen or implied		B1	
	Correct Poisson prob (any m) used for some $r \geq 2$		M1	
	Final answer based on $\sum_0^k P_r$, for k=3 or 4 or 5		M1	
	Correct expression seen or implied;	0.440	A1A1	5
	(p_0=0.0067, p_1=0.0337, p_2=0.0842, p_3=0.1404, p_4=0.1755)			
(i)$_B$	as above with "Poisson mean 5" replaced by "binomial, n=1000, p=0.005"			
	(p_0=0.0067, p_1=0.0334, p_2=0.0839, p_3=0.1403, p_4=0.1757)			
(i)$_N$	N(5 or 995 , 4.98) seen or implied		B1	
	$\Phi\left(\frac{5-4.5}{\sqrt{4.98}}\right)$ or equiv CC used; $\Phi(0.224)$ =)	0.411	M1A1	(3)
(ii)	Use of Bin(6, 0.75) seen or implied		B1	
	Final answer based on correct sum of correct bin probs		M1	
	Correct expression seen or implied;	0.962	A1A1	4
	(p_0=0.0002, p_1=0.0044, p_2=0.0330, p_3=0.1318, p_4=0.2966, p_5=0.3560, p_6=0.1780)			
(iii) (a)	(0.97p(iii)=)	0.933	B1	1
(b)	$0.03(p_4+p_5+p_6)$; (0.03(0.8306)=)	0.025	M1A1	2
(c)	p(a)+p(b);	0.958	M1A1	2

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9	$S \sim N(500, 10^2)$ $L \sim N(1000, 15^2)$			
(i)	Use of $Q\left(\pm\frac{w-500}{10}\right)$ seen or implied		M1	
	Final answer based on $1-Q(\frac{1}{2})-Q(1)$ or equiv;	= 0.533	A1A1	3
(ii)	Use of $Q\left(\pm\frac{w-1000}{\sqrt{200}}\right)$ seen or implied		M1	
	Final answer based on $1-Q(0.707)-Q(1.414)$ or equiv;	= 0.681	A1A1	3
(iii)	Variance 425; $z = 25/\sqrt{425}$ or 1.213,		B1B1	
	Final answer based on Q( );	0.113	M1A1	4
(iv)	Variance 156.25; z= 1,		B1B1	
	Final answer based on Q( );	0.159	M1A1	4

NOTE A: Although z given to 3 dp above, only require 3 sf for marks

GCE ADVANCED LEVEL EXAMINATIONS
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10 α $s^2 = \frac{1}{99}\{0.5377 - \frac{1}{100}(1.21)^2\} = \frac{1}{99}\{0.5231\} = 0.00528 = (0.0727)^2$ M1A1 2

β_1 $z = (\pm)\dfrac{\bar{x} - 1.005}{\sqrt{(s^2/100)}}$; with s^2=0.00528; = 0.977 M1A1A1

Compare z with 1.65 or Q(z) (=0.164) with 0.05 M1

Conclusion stated, based on correct z, "Accept mean= 1.005" or explicit correct NH A1 5

β_2 Consider $K\sqrt{(s^2/100)}$, (K= any standard normal cv) M1

(c=) $1.65\sqrt{(0.00528/100)}$ (= 0.0120) A1

Compare 1.005 + 0.0120 with $\bar{x}$ or equiv comparison M1A1

Conclusion as above A1 (5)

NOTE A: Allow 100 instead of 99:
s^2=0.00523 =$(0.0723)^2$, z=0.982, Q(z)=0.163, c=0.0119

$(i)_p$ (0.65)(0.35)x(100 or 1/100) seen B1

$0.65 \pm K\sqrt{((0.65)(0.35)/100)}$; with K=1.65 M1A1

0.65 ± 0.078 or (0.572, 0.728) A1 4

$(i)_{\%}$ (0.65)(0.35)x(100 or 1/100) seen B1

$65 \pm K\sqrt{(100(0.65)(0.35))}$; with K=1.65 M1A1

$65 \pm 7.8\%$ or (57.2%, 72.8%) A1 (4)

$(ii)_N$ z=($\pm$)2.52 or 2.41 (with CC); B1

Compare z with 1.65 or Q(z) (=0.00587 or 0.00798) with 0.05 M1

Conclusion stated, based on correct z, "Accept propn <65%" or explicit correct AH A1 3

$(ii)_C$ Consider (c=) K(0.65)(0.35)x(100 or 1/100) B1

(K=1.65, c=0.078)

Compare 0.65-0.53 with c (correct method) or equiv comparison M1

Conclusion as above A1 (3)

$(ii)_D$ Compare 0.53 with lower end of CI from (i) M1

Conclusion as above A2 (3)

GCE ADVANCED LEVEL EXAMINATIONS
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11α	Reasonable attempt at differentiation of product	M1	
	$(4+s^2)(-s)+(2sc)(c)$ aef	A1	
	Validly obtaining $-(2+3s^2)s$ AG	A1	
	r decreases because $dr/d\theta<0$ or equiv	B1	4
β	Sketch showing single arc in first quadrant with r decreasing to 0 as θ increases from 0	M1	
	One only smooth loop in rh half-plane, roughly symmetric in θ=0	M1	
	Shape correct with vertical tangent at 0 clear	A1	3
γ	$A=\frac{1}{2}\int_{-\pi/2}^{\pi/2} a^2(4+s^2)c\,d\theta$ inc limits	B1	
	Validly showing $A=\frac{1}{2}a^2\int_{-1}^{1}(4+z^2)dz$ AG	B1	
	$\frac{1}{2}a^2\left[4z+\frac{1}{3}z^3\right]_{-1}^{1}$; $=13a^2/3$ or $4.33a^2$	B1B1	4
δ	Substitute $\cos\theta=x/r$ and $\sin\theta=y/r$, and eliminate θ	M1	
	Correct equation in terms of x,y only or x,y,r eg $r^2=a^2(4+y^2/r^2)(x/r)$	A1	
	$(x^2+y^2)^{5/2}=a^2x(4x^2+5y^2)$	A1	3
12(a)α	When x=0, y=-3b or (0,-3b) stated	B1	
	When y=0, x=-3a or (-3a,0) stated	B1	
	Asymptotes x=-a; y=-b stated	B1B1	4
β	Hyperbola with horizontal and vertical asymps shown, not Ox or Oy	M1	
	Each correct branch (don't insist on labels on axes)	B1B1	3
(b) (i)	Sketch with correct gaps, roughly symm in x-axis	B1	
	Correct loop in x<0,	B1	
	Correct branch in x>0,	B1	3
(ii)	Sketch correct for x<0; ... for x>0	B1B1	2
(iii)	Parts y>0 unchanged & parts y<0 reflected in x-axis	M1	
	Sketch correct with cusps	A1	2

GCE ADVANCED LEVEL EXAMINATIONS
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13(a)	Multiply out and equate real and imag parts	M1	
	Both a−b=3 and −1−ab=−4 or equiv	A1	
	Eliminate a or b and obtain a quadratic in b or a	M1	
	$a^2-3a-3=0$ or $b^2+3b-3=0$ or equiv	A1	
	Solve for a or b using formula	M1	
	$a=(3\pm\sqrt{21})/2$ OR $b=(\pm\sqrt{21}-3)/2$	A1	
	$a=(3+\sqrt{21})/2$; $b=(\sqrt{21}-3)/2$	A1	7
(b)(i)$_1$	x or Re(z)= −1; y or Im(z)= $-\sqrt{3}$ (surd required)	B1B1	
	Correct method for x & y (or x+iy), both negative	M1	3
(i)$_2$	$x^2=1$ & $y^2=3$; Pick neg roots for x & y; -1 & $-\sqrt{3}$	B1M1A1(3)	
(ii)$_1$	$\lvert w/z^2\rvert = 5/4$ or 1.25;	B1	
	$\arg(w/z^2) = \arg(w) - 2\arg(z)$ or equiv correct method	M1	
	$(= (3\pi/4) + 2(2\pi/3)) = 25\pi/12$; $\arg=\pi/12$	A1A1	4
(ii)$_2$	$w/z^2=5\{(\sqrt{3}+1)+i(\sqrt{3}-1)\}/(8\sqrt{2})$	B1	
	Exact methods for w/z^2 and mod; 5/4; $\pi/12$	M1B1B1(4)	
14(a)	CF: Acos3t+Bsin3t or equiv	B1	
	PI: Put x=at (or poly in t) and equate all necessary coeffs	M1	
	t/3	A1	
	GS: own CF (2 arb consts) + own PI	M1	
	Substitute initial values and solve for A,B	M1	
	(A=0, 3B+1/3=1) A=0 & B=2/9	A1	
	$(x=)\ \frac{1}{3}t + \frac{2}{9}\sin 3t$	A1	7
(b) α	$\frac{dz}{dx} = 2 + \frac{dy}{dx}$ or equiv	B1	
	Eliminate y and dy/dx; $\left[\frac{dz}{dx} = 2 + \frac{z+2}{z-1}\right]$	M1	
	Validly obtaining $\frac{dz}{dx} = \frac{3z}{z-1}$ AG	A1	3
β	Separate and attempt integration; $3x = z - \ln z\ (+C)$ aef	M1A1	
	Substitute for z; $x = y - \ln(2x+y) + C$ aef	M1A1	4

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GCE ADVANCED LEVEL EXAMINATIONS
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15(a) α ($dy/dx = 5x^4+50$) Validly obtaining least value of $dy/dx = 50$ B1
Valid reason eg slope always positive, no turning points etc B1 2

β Using appropriate f (eg $f(x)=x^5+50x-10^5$) in NR formula M1

$$x' = x - \frac{x^5+50x-10^5}{5x^4+50} \text{ or equiv}$$ A1

Final answer 9.9900 (at least 4 dp given) A1

Iteration continued until stable to ≥ 4 dp
or root bracketed in $x \pm 0.00005$ M1
Validly showing 9.9900 to 4dp A1 5
($f(9.98995)=-1.99$, $f(9.99005)=2.99$)
($y(9.98995)=99998$, $y(9.99005)=100003$)

(b) (i) Angle between normal vectors considered M1
Correct use and evaluation of scalar product and mods to get $\cos\theta$ M1

$$\left[\frac{1-3+6}{\sqrt{(1^2+1^2+2^2)}\sqrt{(1^2+3^2+3^2)}} = \frac{4}{\sqrt{6}\sqrt{19}} = 0.375\right] \quad 68.0°$$ A1 3

(ii)$_1$ Projn onto $\underline{n} = (1,4,0).\underline{n}/|\underline{n}|$; $= 5/\sqrt{6}$ or 2.04 aef M1A1
$p^2 =$ (Projn onto plane)$^2 = 1^2+4^2 -$ (Projn onto $\underline{n}$)^2 M1
(p=) $\sqrt{(77/6)}$ or 3.58 aef A1 4

(ii)$_2$ Correct use and evaln of scalar product and mods to get $\cos\phi$ M1

$$\left[\frac{1+4}{\sqrt{(1^2+1^2+2^2)}\sqrt{(1^2+4^2)}}\right]$$

$\cos\phi = 5/\sqrt{102}$ or 0.495 aef or $\phi = 60.3°$ A1
$p = \sqrt{(1^2+4^2)}\sin\phi$; $\sqrt{(77/6)}$ or 3.58 aef M1A1 (4)