University of Cambridge
Local Examinations Syndicate

GCE Examinations June 1994

MARKING SCHEME
for
MATHEMATICS

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1. EITHER: State or imply $2\left(-\frac{1}{2}\right)^2 + a\left(-\frac{1}{2}\right)^2 + 16\left(-\frac{1}{2}\right) + 6 = 0$  
   Obtain given answer $a = 9$  
   OR: Evaluate $2\left(-\frac{1}{2}\right)^2 + 9\left(-\frac{1}{2}\right)^2 + 16\left(-\frac{1}{2}\right) + 6$  
   Show correctly that this comes to zero  
   Carry out division by $(2x + 1)$ far enough to obtain 3-term quotient, or equivalent (e.g. factorise by inspection)  
   Obtain correct quotient $x^2 + 4x + 6$ and no remainder  
   Write as $(x + 2)^2 + k$, or differentiate, equate to zero and solve for $x$, or solve $x^2 + 4x + 6 = 0$, or sketch U-shaped quadratic graph, or consider $b^2 - 4ac$  
   Demonstrate given result correctly  
   [Not much need be said for the last A1, but something is needed; e.g. they could just state without further explanation that $(x + 2)^2 + 2$ is always positive. Algebraic (or other) details need to be correct for the mark to be given.]  
   [Candidates who appear to omit the first part and start on the second part will forfeit the first 2 marks for showing $a = 9$ unless they state explicitly that they’ve shown it by the fact that $(2x + 1)$ divided exactly; if they say this they can get B2 as a special case.]  
   [The question can be done back-to-front, with the first part appearing as a result of working to find the quadratic factor, though perhaps not many will try this. Stating $(2x + 1)(px^2 + qx + r)$ and finding numerical values for $p$ and $r$: M1; using coefficient of $x$ to find $q$: M1, $q = 4$: A1; deducing $a = 9$: A1. The last two marks for the question are then as normal.]  
   OR: Differentiate and substitute $x = 0$ at least once  
   Obtain first two terms $1 + \frac{1}{2}x$  
   Obtain $-\frac{1}{2}x^2$ correctly  
   Obtain $+\frac{1}{6}x^3$ correctly  
   [There could be numerical errors in the unsimplified coefficients with the M1 being earned; e.g. $\frac{1}{2} - 1$ mentally calculated as $-\frac{1}{2}$. The details of the $(2x)^2$ and $(2x)^3$ don’t matter for the M mark, except that it should be the correct integer powers of $x$ involved.]  
   OR: Differentiate and substitute $x = 0$ at least once  
   Obtain first two terms $1 + \frac{1}{2}x$  
   Obtain $-\frac{1}{2}x^2$ correctly  
   Obtain $+\frac{1}{6}x^3$ correctly  
   [Single uncancelled fractions OK for A1, A1, or exact decimals.]  

2. EITHER: State any series of positive integer powers of $x$ beginning $1 + \frac{1}{2}x$  
   Show correct method for either binomial coefficient $\frac{\binom{\frac{1}{2}}{1}}{2}$ and/or $\frac{\binom{\frac{1}{2}}{1}}{3!}$  
   OR: Differentiate and substitute $x = 0$ at least once  
   Obtain first two terms $1 + \frac{1}{2}x$  
   Obtain $-\frac{1}{2}x^2$ correctly  
   Obtain $+\frac{1}{6}x^3$ correctly  
   [Single uncancelled fractions OK for A1, A1, or exact decimals.]  

3. EITHER: State or imply $x \log 2 = y \log 3$ (any base, or none at this stage)  
   Obtain simplified equation in one unknown, e.g. $x \log 2 = (1 - x) \log 3$  
   Carry out correct processes to solve a linear equation  
   Obtain given answer $x = \frac{\ln 3}{\ln 6}$  
   OR: Substitute e.g. $y = 1 - x$ and use rules of indices to get $2^x = \frac{3}{3^x}$  
   Simplify to $(2 \times 3)^x = 3$  
   Obtain $x \ln 6 = \ln 3$ or $x = \log_6 3$  
   Obtain given answer $x = \frac{\ln 3}{\ln 6}$  
   [No credit for numerical verification of given answer by calculator. For exact verification, they might find $y$ from $x + y = 1$, getting M1 A1 for $y = \frac{\ln 2}{\ln 6}$, and then M1 A1 for checking this (via $x \ln 2 = y \ln 3$ presumably) in the other equation. Other possibilities seem to involve an M1 A1 for log/index rules as in one of the regular methods, then M1 A1 for checking the given $x$ rather than finding it.]  
   [Writing $2^x - 3^x = 0$ followed immediately by $x \ln 2 - y \ln 3 = 0$ without any explanation is M0 (and therefore A0) for use of log rules; however they can then recover and get the next M1 A1 if the solution is completed.]  

4. State or imply arc length $= 2\pi r$  
   Calculate sector area via value of $\theta$ found from $s = r\theta$, or via $\pi r^2 \times \frac{2\pi}{2\pi r}$  
   Obtain $\pi r^2$  
   [Single uncancelled fractions OK for A1, A1, or exact decimals.]
5 Use \( T^2 = a^2 + b^2 - 4ab \cos \theta \) and exact \( \cos \theta \) to show given result correctly

Make recognisable attempt at quadratic formula or completing the square, or carry out any complete 'otherwise' method for \( b \)

Show exact working at least as far as \( \left( 2, \sqrt{3} \pm \sqrt{192} \right) \) (or equivalent result given by other methods) and state the single correct answer for \( b \) (though maybe not in exact form)

[It's no good picking out the correct one in the next part; the uniqueness and the exactness (not necessarily fully simplified) must both be demonstrated (but not justified) in this part to get the A1]

use sine rule with \( B, 30^\circ, 7 \) and previous answer for \( b \), or any other complete method

Obtain given result

[Allow the final A1 if the given answer is reached, even if some decimal working is involved in this part.
Special case: \( b \) not evaluated earlier, but \( \frac{b}{\sin B} = \frac{7}{\sin 30^\circ}\) stated: allow the M1.]

6 [No penalty for minor rounding errors; e.g. answers for angles (rounding to) within \( \pm 0.1^\circ \) of correct values are OK.]

State or imply \( R = \sqrt{34} \) (which is 5.83...)

State or imply \( \alpha = \arctan \frac{5}{3} \), or equivalent, (which is 59.0...°)

**EITHER:** Evaluate \( \arccos \left( \frac{2}{R} \right) \)

Obtain value 10-9 and/or \( -129-0 \) or any other single correct value

Use correct general formula \( 360^\circ \pm \) something before subtracting \( \alpha \)

Obtain answer 360° + 10-9 and 360° - 129-0 or equivalent

**OR:** Use correct tan \( \frac{1}{2} \theta \) substitutions and obtain quadratic (it's \( 5t^2 + 10t - 1 = 0 \))

Find any one correct value for \( \theta \), e.g. 10-9

Use correct general process, i.e. \( 2(\arctan(t) + 180^\circ) \)

Obtain answer 360° + 10-9 and 360° - 129-0 or equivalent

**OR:** Carry out some mad squaring method and reduce to a soluble equation in one trig function

(e.g. \( 5t^2 + 20t = 5 = 0 \))

Find any one correct value for \( \theta \), e.g. 10-9

Use relevant correct process for a general form of solution

Obtain answer 360° + 10-9 and 360° - 129-0 or equivalent (and no extras)

[N.B. Answer 360° ± 69-9 - 59-0 is fully acceptable; if this is seen, ignore any subsequent wrong simplification. However, the common wrong answer 360° ± 10.9 does not of itself imply the correct version and will not normally score the last two marks.]

[No penalty for degree/radian muddles; values corresponding to 69-9 and 59-0 are 1.22 and 1.03; acceptable accuracy ±0.01.]

7 State \( x = 1 + e^t \) and/or \( y = 1 - e^{-t} \)

State \( \frac{dy}{dx} = \frac{1 - e^{-t}}{1 + e^t} \)

Equate gradient (or \( y \) to zero

Obtain \( t = 0 \), or correct explicit unsimplified expression. e.g. \( t = -\ln 1 \)

State coordinates (1, 1) fully simplified

8 Use factor formula in expression of the form \( \frac{y_2 - x_1}{x_2 - x_1} \)

Show given result correctly

[No explanation really required for the A1; they can just write down the answer following a correct factor formula statement.]

State \( (\phi - \theta) \) is small and use \( \sin x \approx x \)

Identify \( \frac{1}{2}(\phi + \theta) \) as being (approximately) \( \theta \)

[For the first B1, it's no good if they talk about zero angles, or try to say that \( \frac{0}{0} = 1 \). The second B1 is independent of first; no explanation required for it.]
9 State \( B = -1 \) and/or \( C = 1 \)
Carry out any complete method for finding \( A \)
Obtain \( A = -1 \) correctly
[Obtaining the false identity \( 1 \equiv A(x + 1) + Bx + Cx = M_0 \)]
State terms \(- \ln x + \ln(x - 1)\)
State term: \(+x^{-1}\)
[Follow through on non-zero values of \( A, B, C \) only. Special case: if no values for \( A, B, C \) were found, the last two B marks can be earned if \( A \ln x + C \ln(x - 1) \) and \(-Bx^{-1}\) are stated.]

10 (i) State answer 15 504
(ii) EITHER: State expression involving \( (\frac{1}{3}) \times (\frac{1}{3}) \)
Obtain answer 10 800 correctly
OR: State expression involving both \( (\frac{1}{3}) \times (\frac{1}{3}) \) and \( (\frac{1}{3}) \), and subtract from the previous answer
Obtain answer 10 800 correctly

11 (i) Expand LHS completely, or divide RHS by \( (k + 1)^2 \) (in 1 or 2 steps)
Demonstrate the identity correctly
[The marks for (i) may be earned in (ii), but only if exactly equivalent work is fully carried out in the course of doing the induction.]
(ii) Check \( 1^3 = 1^2(2 \times 1^2 - 1) \)
Consider \( k^3(2k^2 - 1) + (2k + 1)^2 \) or equivalent
Obtain \( 2k^2 + 8k^3 + 11k^2 + 6k + 1 \) from the above expression, and complete
[\( M_1 \) requires \( S_n + T_{k+1} \) attempt; not allowed to count as valid attempts at \( T_{k+1} \), are e.g. \( (k + 1), (k + 1)^3, (2k - 1)^3 \).]

12 (a) State equation \( \frac{a(1 - r^n)}{1 - r} = 1 \times \frac{a}{1 - r} \) or \( \frac{a(1 - r^n)}{1 - r} = ar^n \)
Eliminate \( a \) to obtain equation in \( r \) only
Obtain answer (rounding to) 0.917 correctly
Use \( ar^n = 10 \) with numerical \( r \) to find \( a \)
Obtain answer \( a = 40 \) (no penalty for using decimal working)
[N.B. Using \( S_n \) instead of \( T_{17} \) is not to be counted as MR.]
(b) Equate (reasonable attempt at) sum to \( n \) terms of an AP with \( d = 10 \) to 10 000
Obtain \( a = \frac{10000}{n} - 5(n - 1) \) or equivalent
Deduce given result \( \frac{10000}{n} + 5(n - 1) \) for \( n \)th term
State \( \frac{10000}{n} + 5(n - 1) < 500 \) and multiply through by \( n \)
Simplify correctly to given form
Use (recognisable attempt at) quadratic formula, or e.g. trial and error
State answer 73
[If no working shown for value of \( n \): correct answer 73 stated gets \( M_1 \) \( A_1 \); integer answer 74 or decimal answer 73.96 stated gets \( M_1 A_0 \); any other answer gets \( M_0 \).]
GCE ADVANCED LEVEL EXAMINATIONS
REVISED MARKING SCHEME JUNE 1994

13 (a) (i) State \( f(x) = \ln(1 + x) \) or imply this e.g. by 1st quadrant graph starting at \( O \), concave down
Sketch correct graph, i.e. quads 1 and 3, through \( O \), (implied) vertical asymptote on left
B1
B1 2

The second B1 implies the first B1 in this case.]

(ii) Identify \( g^{-1}(x) = e^x \)
Identify \( h^{-1}(x) = x - 1 \)
B1
B1 2

(iii) Identify \( g^{-1}h^{-1}(x) = e^{x-1} \)
B1
B1 1

(iv) Sketch an appropriate exponential shape for the graph
Show curve correctly located, e.g. through (1, 1) or (0, \( e^{-1} \))
B1
B1 2

The only expressions we follow through on are ones they could 'reasonably' have found for (iii);
i.e. \( e^{x-1} \) or \( \pm 1 \leq e^x \)]

(b) Sketch the (relevant part of the) parabola
Use correct graph to explain the one-one property
Use quadratic formula to solve \( x^2 - 4x - y = 0 \) for \( x \), or equivalent
Use known point, e.g. \( x = 0, y = 0 \), to select correct sign
Obtain answer \( q^{-1}(x) = 2 - \sqrt{4 + x} \) or equivalent
M1
A1
M1
M1
A1 5

First M1 to be given generously, but the next A1 will be hard to earn. They must say (somehow) that each \( y \)-value corresponds to just one \( x \)-value and their sketch must support this assertion.]

14 [No penalties for misuse of vector notation, so long as the meaning is clear.]

(i) Carry out all calculations with coordinates needed for \( \overline{PM} \)
Obtain the given equation correctly
M1
A1 2

[Don't worry if the details they show are a bit sketchy, so long as it's clear they know that it's \( \overline{PM} \) that's the direction vector.]

(ii) Carry out all calculations with coordinates needed for \( \overline{QN} \)
State answer \( r = \begin{pmatrix} 2 \\ -3 \\ -1 \\ h \end{pmatrix} \)
A1 2

[If no working is shown, the M1 would be implied by an answer with e.g. one sign error.]

(iii) Equate at least two components from the equations found
Show that \( t = s = \frac{1}{3} \) works for all three components
Derive given position for \( X \), i.e. \( \left( \frac{1}{3}, -\frac{1}{3}, \frac{1}{3} \right) \) from \( t \) or \( s \)
M1
A1
A1 3

[If they use the same letter for the parameter in both lines, they'll get the 'right' answer; however, max M1 out of 3 in this case. Note that this part can be done otherwise, e.g. by verification or by noting that \( X \) is the mid-point of each of PM, QN. However, any method must deal with all three components if full marks are to be earned.]

(iv) Show correct processes for the calculation of any scalar product
Equate the scalar product of the two relevant vectors to zero
Obtain \( h = \sqrt{2} \) correctly
Carry out the correct processes to evaluate \( \cos^{-1} \left( \frac{a \cdot b}{||a|| ||b||} \right) \) for the two relevant vectors
M1
M1
A1
M1

Obtain answer 70.5° or 109.5°

[It appears to me not obvious, at the start of (iv), that \( VB \) is inclined at 45°; they need to say e.g. that \( OX \) produced goes to the mid-point of \( VB \) or that \( OX \) is parallel to \( O'V' \) to justify this, I think. Hence using this 'fact' to find \( h = \sqrt{2} \) gets M1 M0 M0 A0; however, the last 2 marks remain available. Thinking that \( OX \) is the x-axis leads to such obvious impossibilitys so quickly that I think we can (and should) do nothing about it.]}
15 Attempt product rule to differentiate $\cos(x + \alpha) \cos^2 x$ and equate to zero

Obtain $-\sin(x + \alpha) \cos^2 x - 2 \cos(x + \alpha) \cos x \sin x = 0$ M1

Identify the solution $\cos x = 0$ A1

Divide through simplified equation by $\cos(x + \alpha) \cos x$ A1

Obtain $\tan(x + \alpha) + 2 \tan x = 0$ correctly (ignore any other ‘possibilities’ that they turn up) A1

[If they expand $\cos(x + \alpha)$ first, they might still get the first 3 marks, but the last 2 will almost certainly be out of reach. The equation for the first A1 is $-3 \cos^2 x \sin x \cos \alpha - \cos^2 x \sin \alpha + 2 \cos x \sin^2 x \sin \alpha = 0$.]

(i) Use $\tan(A + B)$ formula to express equation in terms of $\tan x$ only M1

Obtain $(2\sqrt{2})^2 - 3 \tau - \sqrt{2} = 0$ or equally simplified exact equivalent A1

Use quadratic formula or equivalent A1

Obtain exact $\sqrt{2}$ and $-\frac{1}{2} \sqrt{2}$ or exact equivalents A1

[Ignore subsequent working once the exact values are seen.]

(ii) Use Pythagoras or equivalent to calculate $\sin x$ and/or $\cos x$ M1

Use $\cos(A + B)$ or $\cos 2A$ formula with relevant numerical values to calculate $y$ M1

Show given answer $\frac{1}{2}$ correctly A1

[If they use approximate methods (i.e. calculator evaluation of $\alpha$, etc.) please give max M1 out of 3 for the complete calculation of $y$.]

16 Obtain $\frac{1}{2}x^2 + 4x^3$ for the integral B1

Use limits 1 and 4 to find the area under the curve M1

Subtract from the area of the rectangle (or subtract the other way round?) M1

Obtain answer $\frac{1}{2}$ or equivalent A1

State equation of the form $y = \sqrt{x} + \frac{2}{\sqrt{x}} + k$ B1

State correct equation (i.e. $k = -3$) B1

State or imply required volume is $\pi \int_1^4 y^2 \, dx$ using their transformed $y$ B1

Show correct processes for squaring a trinomial $y$ B1

Obtain integrand correctly M1

Integrate at least 3 of the given terms correctly M1

Obtain $\frac{1}{2}x^2 - 4x^3 + 13x - 24x^2 + 4 \ln x$ or equivalent M1

Obtain $(4 \ln 4 - \frac{1}{2})$ or exact equivalent A1

[Some working is necessary to demonstrate the trinomial has been properly squared out; if none is shown they’ll lose M1 A1. Note that we allow the omission of $\pi$ from the final answer; for the numerical bit, powers must be evaluated and terms collected up.]

17 (i) Use double-angle formulae relating $\cos 2x$ and $\sin^2 x$

Obtain or verify given answer correctly M1

A1

(ii) Use parts, going the correct way (condone sign errors at this stage, and allow anything for the integral of $\sin^2 x$, except $\sin^2 x$ itself!) M1

Obtain $x \left(\frac{1}{2}x - \frac{1}{2} \sin 2x\right) - \left(\frac{1}{2}x^2 + \frac{1}{2} \cos 2x\right)$, or equivalent A1

Show given answer correctly A1

(iii) Differentiate, and substitute throughout for $u$ and $du$ M1

Obtain $\left(-\frac{1}{2} \sin^2 x \, dx\right.$ (any limits or none at this stage) A1

Obtain answer $\frac{1}{2} \pi$ correctly A1

(iv) Carry out all the correct substitution steps again M1

Use double-angle formula and obtain given result (condone lack of explanation over limits) A1

State or imply integral of the form $ax + b \sin 4x$, or make further substitution $\theta = 2x$ M1

Obtain answer $\frac{1}{2} \pi$, with no errors seen anywhere A1
18 (i) EITHER: Use chain rule to differentiate $\sqrt{1 + y^2}$ (at least 2 factors required)
   Obtain $\frac{1}{2}(1 + y^2)^{-\frac{1}{2}} \times 3y^2 \times \frac{dy}{dx}$
   Show given result correctly
   OR: Attempt differentiation of both sides of $(y')^2 = 1 + y^2$ (2 factors on at least one side required)
   Obtain $2y'y'' = 3y^2y'$
   Show given result correctly
   (ii) State $y''' = 3yy'$
   Use product rule to differentiate RHS of this w.r.t $x$
   Obtain answer $y^{(6)} = 3(y')^2 + \frac{3}{2}y'$, or any equivalent in terms of $y$ only or $y$ and $y'$
   [Anyone differentiating $y'''$ in the form $3y\sqrt{1 + y^2}$, for instance, might leave $y^{(6)}$ as
   $3y\sqrt{1 + y^2} + \frac{9y'y'}{2\sqrt{1 + y^2}}$.]
   (iii) State $f(0) = 0$ and evaluate $y'$, $y''$, $y'''$, $y^{(6)}$ at $x = 0$
   Use Maclaurin's series
   Show the given answer correctly (allow 'flukes' where errors in their derivatives give the correct
   Maclaurin coefficients)
   Show or imply use of at least two expressions for values of the integrand
   Obtain answer (rounding to 0.397)
1α  
Figure with T, 1.5, 0.5g, seen or implied  
Two correct eqns eg T\cos\theta=5, T\sin\theta=1.5, \tan\theta=0.3  
Two attempted eqns, 3 forces, correct mech principles  
\[ T=5.22N, \quad \theta=16.7^\circ \]  
M1  
B1B1  
6

β  
Two correct eqns eg 2=5\sin\phi, T'=5\cos\phi  
\[ T'=5\cos\phi \quad 2\sin\phi=5, \quad T'=5\sin\phi=2\cos\phi' \]  
Two attempted eqns, 3 forces, correct mech principles  
\[ T'=4.58N, \quad \phi=23.6^\circ \]  
M1  
B1B1  
5

γ  
Attempt to use change in mgh  
0.5g(0.3)(\cos\theta-\cos\phi) or (0.5g)(0.0360)(\cos 69.9^\circ); 0.062J  
M1  
A1A1  
3

2α  
\[ x=20t; \quad y=30t-qt^2/2 \quad or \quad 30t-5t^2 \]  
\[ \tan \phi=x/y; \quad =4/(6-t) \]  
AG  
M1A1  
4

β  
Set \tan\phi=4/3 and solve for t; \quad t=3;  
Sub for t in x,y and use DA=x^2+y^2 or x/sin\phi or y/cos\phi'  
\[ (x,y)=(60,45) \]  
\[ DA=75m \quad AG \]  
\[ v=20 \text{ m/s}^2 \text{ horizontally or equiv} \]  
M1A1  
A1  
B1  
5

γ₁  
\[ t=4.5; \quad \dot{y}=-15 \quad \text{or} \quad \dot{y}=225 \quad \text{or} \quad v=62.5 \]  
\[ KE=0.2(x^2+y^2)/2; \quad = 62.5J \]  
B1  
M1A1  
3

γ₂  
\[ y=135/4 \text{ or } 33.75 \text{ OR } D \text{ (depth below top) } = 45/4 \text{ or } 11.25 \]  
\[ KE= 0.1(20^2+30^2)-0.2gy \text{ or } 0.1v^2+0.2gD; \quad 62.5J \]  
B1  
M1A1  
(3)

δ  
\[ \tan \gamma=x/y \text{ or } \dot{y}/\dot{x} \text{ or equiv used } \quad (=(-3/4); \quad (-36.9^\circ) \]  
M1A1  
2
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3\alpha
0.2v \frac{dv}{dx} = -\frac{1}{5x^2} \quad \text{or} \quad v \frac{dv}{dx} = -\frac{2}{x^2} \quad \text{(""": M1) \quad M1A1}

Attempt to separate and integrate both sides \quad M1
\frac{1}{2v^2} = \frac{2}{x} \quad \text{or equiv or } (-2/x \text{ from } +2/x^2) \quad A1

"+C" and validly show C=0 \quad v = \frac{2}{x} \quad AG \quad A1 \quad 5

\beta
\frac{1}{2}(0.1)2^2 - \frac{1}{2}(0.1)\left(\frac{2}{3}\right)^2 = \frac{32}{180} - \frac{8}{45} \quad 0.178 \quad \text{aef} \quad M1A1 \quad 2

\gamma_1
\frac{dx}{dt} = \frac{2}{\sqrt{t}} \quad B1

Attempt to separate and integrate both sides \quad M1
\frac{2^{x^{3/2}}}{3} = 2t \quad (+C) \quad \text{or equiv} \quad A1

\begin{bmatrix}
1
\end{bmatrix}_1 = \begin{bmatrix}
1
\end{bmatrix}_0 \quad \text{or } +C" \quad \text{and sub } \langle x=1, \ t=0 \rangle, \ \langle x=9 \ t=t \rangle \quad M1

\text{time} = 26/3 = 8.67 \quad \text{aef} \quad A1 \quad 5

\gamma_2
\text{Mark } dv/dt = -v^4/8, \ v^2/3 = t/8 \quad (+C') \quad \text{as } \gamma_1

\delta "\text{NO" with valid reason eg } v \text{ never zero or never neg, or } P \text{ goes to infinity, or unique } t \text{ for } x=1 \quad B2 \quad 2
4 α l = (±) (0.2) (3+9), I = (±) (0.3) (2+) (or =0) Any two BIB1
I = (±) 2.4 Ns; v = 6 ms⁻¹ BIB1 4

[If 0/4 allow M1 for attempt at two of the above – ignore sign errors]

β KE lost = \( \frac{1}{2} \) (0.2) 9² + \( \frac{1}{2} \) (0.3) v² - \( \frac{1}{2} \) (0.2) 3² - \( \frac{1}{2} \) (0.3) 2²

M1

(= \( \frac{27}{2} \) - \( \frac{9}{2} \) or \( \frac{36}{5} \) + \( \frac{24}{5} \)) = 12 J A1 2

γ

Time for P to reach wall = 6/3 or 2 B1
Distance of Q from wall (ie P) = 6+2×2; = 10 m AG MIA1 3

δ₁

Time t to collision given by 3t = 10+2t; t = 10 MIA1
Distance (3×10) = 30 m B1 3

δ₂

(10+5)/3 = x/2 or x = (s-10)/2; s = 30m MIA1 MIA1 (3)

ε

Final answer based on (0.2) 3 + (0.3) 2 = 1.2 Ns MIA1 2

5α

T \cos α = mg (T = 25/4) M1A1
T \sin α = mr \omega² or mv²/r M1
r = 1+5 \sin α or 4; T \sin α = m(1+\sin α) \omega²

(\omega² = 15/8) ω = 1.37 rad s⁻¹ A1 6

β

\( \chi = \lambda (10-6)/6 \) or 2λ/3 or equiv

MIA1

1+5 \sin β = 5 or \sin β = 4/5 or \beta = 53.1

B1

P \cos \beta = 0.5g

B1

P \sin \beta + x = 0.5 (accn); (0.5) 5×2²

MIA1

Four equations for P, \( \chi \), \( \beta \), λ and find λ;

λ = 5N MIA1 8
6α 4/24 or 20/24 x some attempt at another prob;  
Validly obtaining 10/69 AG (decimals not used)  
M1 \ A1 \ 2

(i) "23×22" seen or implied in denominator  
correct unsimplified answer eg \[ \frac{3}{29} \frac{2}{29} \frac{6}{506} \] or \[ \frac{3}{253} \] or 0.0119  
A1A1 \ 3

(ii) Attempt at summing probability of correct combined events  
Correct unsimplified expression seen or implied  
\[ P(A)+P(A'C) = \frac{4}{24} + \frac{20}{24} = P(C)+P(C'A) \]  
or \[ P(A)+P(C)-P(AC) = \frac{4}{24} + \frac{4}{24} - \frac{4}{24} = \frac{3}{29} \]  
or \[ P(AC')+P(A'C)+P(AC) = \frac{4}{24} + \frac{20}{24} + \frac{4}{24} = \frac{3}{29} \]  
or \[ 1-P(A'C') = 1 - \frac{20}{24} = \frac{19}{24} = \frac{172}{952} \] or \[ \frac{86}{276} \] or \[ \frac{43}{138} \] or 0.312  
A1 \ 3

(iii) Correct method for final answer  
Correct unsimplified expression eg \[ \frac{4}{24} \frac{11}{29} \]  
\[ \frac{44}{552} \] or \[ \frac{22}{276} \] or \[ \frac{11}{138} \] or 0.0797  
A1 \ 3

(iv) Attempt \[ P(iii)/P(either) = P(iii) / \frac{12}{24} \] or \[ \frac{22}{138} \] or \[ \frac{11}{69} \] or 0.159  
M1A1A1 \ 3
7a Attempt \( F(3) = 0 \) or \( F(4) = 1 \) or \( F(4) - F(3) = 1 \)
Two correct equations \( 9a - 24a + b = 0 \) & \( 16a - 32a + b = 1 \); \( b = 15a = 1 + 16a \)
Validly showing \( a = -1 \) AG; \( b = -15 \); Validly showing \( F(3.5) = \frac{9}{4} \) AG
B1A1B1 5

(i) Attempt differentiate \( F(x) \) to obtain \( f(x) \); \( f(x) = 8 - 2x \)
(Do not insist on "3 \( \leq x \leq 4 \)" or on "\( f(x) = \) 0 otherwise")
M1A1 2

(ii) Attempt integrate \( xf(x) \) with limits 3, 4;
integral = \( 4x^2 - \frac{2}{3}x^3 \);
\( \left( 64 - \frac{128}{3} - 36 + 18 \right) \) Validly obtaining \( \frac{10}{3} \) AG
A1 3

(iii) \( \frac{3}{2} \left( \frac{1}{4} \right) \); \( \frac{27}{84} \) or 0.422
B1B1 2

(iv) \( N \left( \frac{10}{3}, \frac{1}{1800} \right) \) or in words (only two correct B1)
B2 2
8(i). Poisson mean 5 seen or implied
Correct Poisson prob (any m) used for some r ≥ 2
Final answer based on \( \sum_{k=3}^{5} p_{k} \) for k=3 or 4 or 5
Correct expression seen or implied;
\( p_{0}=0.0067, p_{1}=0.0337, p_{2}=0.0842, p_{3}=0.1404, p_{4}=0.1755 \)

8(ii). as above with "Poisson mean 5" replaced by "binomial, n=1000, p=0.005"
\( p_{0}=0.0067, p_{1}=0.0334, p_{2}=0.0839, p_{3}=0.1403, p_{4}=0.1757 \)

(iii) N(5 or 995, 4.98) seen or implied
\( \phi \left( \frac{5-4.98}{4.98} \right) \) or equiv CC used; \( \Phi(0.224) = 0.411 \)

(ii) Use of Bin(6, 0.75) seen or implied
Final answer based on correct sum of correct bin probs
Correct expression seen or implied;
\( p_{0}=0.0002, p_{1}=0.0044, p_{2}=0.0330, p_{3}=0.1318, p_{4}=0.2966, p_{5}=0.3560, p_{6}=0.1780 \)

(iii) (a) \( 0.97p(iii)= \)
\( 0.933 \)

(b) \( 0.03(p_{4}+p_{5}+p_{6}) \); \( 0.03(0.8306)=0.025 \)

(c) \( p(a)+p(b) \);
\( 0.958 \)
\[ S \sim N(500, 10^2) \quad L \sim N(1000, 15^2) \]

(i) Use of \[ Q \left( \frac{W - 500}{10} \right) \]
seen or implied \[ M1 \]
Final answer based on \[ 1 - Q \left( \frac{1}{2} \right) \] or equiv; \[ = 0.533 \] \[ A1A1 \] 3

(ii) Use of \[ Q \left( \frac{W - 1000}{15} \right) \]
seen or implied \[ M1 \]
Final answer based on \[ 1 - Q(0.707) - Q(1.414) \] or equiv; \[ = 0.681 \] \[ A1A1 \] 3

(iii) Variance 425; \[ z = \frac{25}{\sqrt{425}} \] or 1.213,
Final answer based on \( Q(\ ) \); \[ 0.113 \] \[ B1B1 \] \[ M1A1 \] 4

(iv) Variance 156.25; \[ z = 1, \]
Final answer based on \( Q(\ ) \); \[ 0.159 \] \[ B1B1 \] \[ M1A1 \] 4

NOTE A: Although \( z \) given to 3 dp above, only require 3 sf for marks
\[ 10 \alpha s^2 = \frac{1}{99} (0.5377 - \frac{1}{100} (1.21)^2) = \frac{1}{99} (0.5231) = 0.00528 = (0.0727)^2 \quad \text{MIA1} \quad 2 \]

\[ \beta_1 = \left( \frac{z}{\sqrt{\frac{1}{s^2}} \sqrt{\frac{1}{100}}} \right) \quad \text{with } s^2=0.00528; \quad z = 0.977 \quad \text{MIA2A1} \]

Compare \( z \) with 1.65 or \( \Phi(z) = 0.164 \) with 0.05

Conclusion stated, based on correct \( z \), "Accept mean = 1.005"

or explicit correct NH

\( \text{A1} \quad 5 \)

\[ \beta_2 \]

Consider \( K \sqrt{s^2/100} \), \( (K= \text{any standard normal cv}) \)

\( (c=) 1.65 \sqrt{0.00528/100} = 0.0120 \)

Compare 1.005 + 0.0120 with \( \overline{x} \) or equiv comparison

Conclusion as above

\( \text{MIA1} \quad \text{A1} \quad (5) \)

NOTE A: Allow 100 instead of 99:

\[ s^2=0.00523 = (0.0723)^2, \quad z=0.982, \quad \Phi(z)=0.163, \quad c=0.0119 \]

(i) \( p(0.65)(0.35)\times(100 \text{ or } 1/100) \) seen

\( 0.65 \times K \sqrt{(0.65)(0.35)/100} \); \( \text{with } K=1.65 \)

\( 0.65 \pm 0.078 \) or \( (0.572, 0.728) \)

\( \text{B1} \quad \text{MIA1} \quad \text{A1} \quad 4 \)

(ii) \( \% (0.65)(0.35)\times(100 \text{ or } 1/100) \) seen

\( 65 \times K \sqrt{100(0.65)(0.35)} \); \( \text{with } K=1.65 \)

\( 65 \pm 7.8 \% \) or \( (57.2 \%, 72.8 \%) \)

\( \text{B1} \quad \text{MIA1} \quad \text{A1} \quad (4) \)

(ii) \( z = \pm 2.52 \) or 2.41 (with CC);

Compare \( z \) with 1.65 or \( \Phi(z) = 0.00587 \) or \( 0.00798 \) with 0.05

Conclusion stated, based on correct \( z \), "Accept prop < 65%"

or explicit correct AH

\( \text{A1} \quad 3 \)

(ii) \( C \quad \text{Consider } (c=) K(0.65)(0.35)\times(100 \text{ or } 1/100) \)

\( (K=1.65, c=0.078) \)

Compare 0.65-0.53 with \( c \) (correct method) or equiv comparison

Conclusion as above

\( \text{M1} \quad \text{A1} \quad (3) \)

(ii) \( p \quad \text{Compare 0.53 with lower end of CI from (i)} \)

Conclusion as above

\( \text{M1} \quad \text{A2} \quad (3) \)
GCE ADVANCED LEVEL EXAMINATIONS
MARKING SCHEME JUNE

11 (a) Reasonable attempt at differentiation of product
\((4+s^2)\) \((-s) + (2sc)(c)\) aef
Validly obtaining \(-(2+3s^2)s\) AB
r decreases because \(dr/d\theta < 0\) or equiv

M1
A1
M1
A1
B1 4

(b) Sketch showing single arc in the first quadrant
with r decreasing to 0 as \(\theta\) increases from 0
One only smooth loop in rh half-plane, roughly symmetric in \(\theta=0\)
Shape correct with vertical tangent at 0 clear

M1
A1 3

\(\gamma \quad A = \frac{1}{2} \int_{-\pi/2}^{\pi/2} a^2 (4+s^2) \cos \theta \, d\theta \quad \text{inc limits}\)
\[-\pi/2 \]
Validly showing \(A = \frac{1}{2} a^2 \int_{-1}^{1} (4z^2) \, dz\) AB

\(\frac{1}{2} a^2 \left[ 4z^2 \right]_0^1 = \frac{13a^2}{3} \text{ or } 4.33a^2\)
B1B1 4

(6) Substitute \(\cos \theta = x/r\) and \(\sin \theta = y/r\), and eliminate \(\theta\)
Correct equation in terms of \(x, y\) only or \(x^2, y^2\) eg
\(r = \frac{a^2 (4+y^2/2)}{x/r} \cdot \frac{x^2, y^2}{2} = \frac{a^2}{x^2} (4x^2 + 5y^2)\)

M1
A1 3

12 (a) \(\alpha\)
When \(x = 0, y = -3b\) or \((0, -3b)\) stated
When \(y = 0, x = -3a\) or \((-3a, 0)\) stated
Asymptotes \(x = -a; y = -b\) stated

B1
B1
B1B1 4

(\(\beta\)) Hyperbola with horizontal and vertical asymptps shown, not Ox or Oy
Each correct branch (don't insist on labels on axes)

M1
B1B1 3

(b) (i) Sketch with correct gaps, roughy symm in x-axis
Correct loop in \(x < 0\),
Correct branch in \(x > 0\),

B1
B1
B1 3

(ii) Sketch correct for \(x < 0\); ... for \(x > 0\)

B1B1 2

(iii) Parts \(y > 0\) unchanged & parts \(y < 0\) reflected in x-axis
Sketch correct with cusps

M1
A1 2
13(a) Multiply out and equate real and imag parts
Both a-b=3 and -1-ab=-4 or equiv
Eliminate a or b and obtain a quadratic in b or a
\[a^2-3a-3=0\] or \[b^2+3b-3=0\] or equiv
Solve for a or b using formula
\[a=(3\pm\sqrt{21})/2\] OR \[b=\left(\pm\sqrt{21}-3\right)/2\]
\[a=(3+\sqrt{21})/2; b=\left(\sqrt{21}-3\right)/2\]
M1 A1 7

(b)(i) \[x\] or \[\text{Re}(z)=-1; y\] or \[\text{Im}(z)=-\sqrt{2}\] (surd required)
Correct method for \(x\) & \(y\) (or \(x+iy\)), both negative
B1 B1 M1 3

(i) \[x=1\] & \[y^2=3\]; Pick neg roots for \(x\) & \(y\); -1 & -\[\sqrt{3}\]
B1 M1 A1 A1 13

(ii) \[\left|\frac{w}{z^2}\right| = 5/4\] or \[1.25;\]
\[\text{arg}(w/z^2) = \text{arg}(w) - 2\text{arg}(z)\] or equiv correct method
\[= (3n/4) + 2(2n/3) = 25\pi/12; \text{arg}=\pi/12\]
B1 M1 A1 A1 4

(ii) \[w/z^2=5(3+1+1(\sqrt{3}-1))/(8\sqrt{2})\]
Exact methods for \(w/z^2\) and mod;
\[5/4; \pi/12\]
B1 M1 B1 B1 4

14(a) CF: \[\cos 3t + \sin 3t\] or equiv
PI: Put \(x=at\) (or poly in \(t\)) and equate all necessary coeffs
\[t/3\]
M1 A1
GS: own CF (2 arb cons) + own PI
M1 A1
Substitute initial values and solve for \(A, B\)
\[(A=0, B=1/3)\]
M1 A1
\[(x=) \frac{1}{3}t + \frac{2}{3}\sin 3t\]
A1 7

(b) \[\alpha\]
\[\frac{dz}{dx} = 2 + \frac{dy}{dx}\] or equiv
B1
Eliminate \(y\) and \(dy/dx\) \[\left[\begin{array}{c}
\frac{dz}{dx} = 2 + \frac{z+2}{z-1}
\end{array}\right]\]
M1
Validly obtaining \[\frac{dz}{dx} = \frac{3z}{z-1}\] AG
A1 3

\(\beta\)
Separate and attempt integration;
\[3x = z - \ln(z) (+C)\] aef
M1 A1
Substitute for \(z\);
\[x = y - \ln(2x+y) + C\] aef
M1 A1 4
15(a) \( \alpha \) (dy/dx = 5x^4 + 50) Validly obtaining least value of dy/dx = 50
Valid reason eg slope always positive, no turning points etc

Using appropriate f (eg f(x) = x^5 + 50x - 10^5) in NR formula

\[ x' = x - \frac{x^5 + 50x - 10^5}{5x^4 + 50} \text{ or equiv} \]

Final answer 9.9900 (at least 4 dp given)

Iteration continued until stable to 24 dp or root bracketed in x \( \pm 0.00005 \)
Validly showing 9.9900 to 4dp

(f(9.9999) = -1.99, f(9.99005) = 2.99)

(y(9.9999) = 99998, y(9.99005) = 100003)

(b) (i) Angle between normal vectors considered
Correct use and evaluation of scalar product and mods to get \( \cos \theta \)

\[
\left( \frac{1-3+6}{\sqrt{(1+2+2^2)} \sqrt{(1+2^2+3^2)}} \right) = \frac{4}{\sqrt{6}} = 0.375 \]

68.00°

(ii) \( \text{Projn onto } n = (1,4,0) \times \frac{\mathbf{b}}{||\mathbf{b}||} = 5/\sqrt{6} \text{ or } 2.04 \text{ aef} \)

\[
\mathbf{p} = (\text{Projn onto plane})^2 = 1^2 + 4^2 - (\text{Projn onto } n)^2
\]

\[
(\mathbf{p}) = \sqrt{77/6} \text{ or } 3.58 \text{ aef}
\]

(ii) \( \text{Correct use and evaln of scalar product and mods to get } \cos \phi \)

\[
\left( \frac{1+4}{\sqrt{(1+2^2+2^2)} \sqrt{(1+4^2)}} \right)
\]

\[
\cos \phi = 5/\sqrt{102} \text{ or } 0.495 \text{ aef} \text{ or } \phi = 60.3°
\]

\[
p = \sqrt{(1+4^2)} \sin \phi; \quad \sqrt{(77/6)} \text{ or } 3.58 \text{ aef}
\]