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# MATHEMATICS

## Report on the June 1994 Examination

## MATHEMATICS (Syllabus C)

ADVANCED LEVEL

**Subject 9205**

*Paper 9205/1*

*General Comments*

There was a substantial amount of good work from candidates on this year's paper, and overall levels of achievement seemed very comparable with those seen in most previous years. Many candidates found the majority of the short questions in *Section I* to be quite straightforward, but there were also quite widespread difficulties over certain points, notably in *Q.4*, *Q.8* and *Q.10*. The longer questions in *Section II* were not equally popular, with *Q.14* and *Q.15* generally proving to be less often chosen than the others. All the *Section II* questions proved to have some part or parts that were quite accessible to all but the very weakest candidates, and most proved to have some part or parts which allowed the strongest or most careful to demonstrate their superiority.

Quite a number of the questions this year had one or more answers given within the question. This, on the whole, seemed to prove helpful to candidates (as was the intention, of course), but Examiners continue to find instances where candidates do not give sufficient working in their solutions to show without doubt that a given result has been properly demonstrated. This point has been mentioned in previous Reports, but it is evidently one that needs to be continually stressed; some examples of the level of detail that Examiners ideally hope to see are given at relevant points in the comments on particular questions that follow. Another feature shared by a number of the questions in this paper was the requirement to give an answer to an exact form, this being signalled either by the use of the word 'exact' or 'exactly' in the question itself (e.g. *Q.5*, *Q.15*, *Q.16*), or by the form in which a result given in the question itself was expressed (e.g. *Q.3*, *Q.5*). Many candidates, even some whose work was otherwise very good indeed, either overlooked or did not understand the implications of this. The implications are twofold: firstly, that an 'exact' answer should be given in terms of fractions, surds, logarithms, $\pi$ (or whatever is relevant in the particular case) and should not involve decimal approximations, or if it is given in the question itself should be reached in precisely the printed form; secondly, that an exact form of answer cannot properly be obtained if there are any stages in the working leading to it that involve approximations. It is the second point that most often trips candidates up, as they do not seem to realise that *any* use of a calculator may very possibly have introduced an approximation into their work, thereby technically rendering it invalid.

Mention was made in last year's Report of some candidates' lack of care over matters of algebraic notation. To some extent criticisms over such matters as omission of brackets could be repeated this year, although there seemed to be fewer instances in this paper where candidates were led into actual errors by the use of less-than-perfect notation. However, there certainly were cases this year where many candidates seemed unable to *recognise* the simple algebraic structure of the result to which their working had led them, and Examiners found this to be a disappointing aspect of a lot of the work that was presented. Instances of this occurred in *Q.3*, where a simple linear equation in $x$ was often not dealt with correctly, and in *Q.4*, where a simple relation involving two algebraic quantities was often abandoned without any attempt to simplify it, or to express one quantity in terms of the other. Further details appear in the specific comments below.

Most candidates appear to have had sufficient time to attempt the required number of questions, though Examiners noticed a few scripts where it seemed clear that the candidate had run out of time while still working productively on a problem. It was sometimes clear that candidates had spent an excessive amount of time on one of the longer questions when something had failed to work out properly, or when an unfortunate choice of method had prolonged the question somewhat. This seemed to happen most often this year in *Q.15* or *Q.17*, with the latter in particular proving rather more difficult for candidates to give succinct solutions to than Examiners had anticipated.

*Comments on Individual Questions*

*Section I*

*Q.1* Most of the work on the first two parts of this question was very good. The factor theorem was normally used to answer the first part, and failures to demonstrate the given result $a = 9$ correctly were rare. A small number of candidates did not seem to understand how to find the quadratic factor of the expression, or perhaps did not know what this request meant, but the great majority were able to carry out the required algebraic division correctly, or were able to write down the factorisation without needing to show any working. A minority used 'synthetic division' or the method of 'detached coefficients' for carrying out this step, generally successfully, though for some reason that is not altogether clear, quite a number using this method obtained the answer $2x^2 + 8x + 12$ for the quotient instead of $x^2 + 4x + 6$. Some candidates ran the first two parts of the question together, and started their solution by dividing $2x + 1$ into $2x^3 + 9x^2 + 16x + 6$. There was never any problem in assigning marks for obtaining the quadratic factor when a candidate started off in this way, but it was not always clear to Examiners whether the candidate had simply omitted the first part of the question and just assumed the given answer to carry on with the later parts, or whether the candidate had answered the first part by implication in showing that the division worked out with no remainder. In this particular instance the mark scheme did not allow the candidates the benefit of any doubt, and the marks for showing $a = 9$ were only given to these candidates if they included an explicit statement that the division working out exactly demonstrated the result; otherwise they were assumed to have omitted the first part of the problem. The last part of the question proved to be the part with which candidates had most difficulty, although a good number were able to do it without any problem. Most candidates wrote something, generally following the hint given and attempting to complete the square. The standard of accuracy in doing this was generally good, but many candidates failed to draw any conclusion from their result, or else wrote down something which was plainly unsatisfactory (e.g. 'which is positive if $x > -2$'). A few candidates may genuinely have thought that, having correctly completed the square, the expression they had obtained made the result so obvious that there was literally nothing else to say. However, Examiners did not take this view, and expected candidates to write *something* down, even if only something very brief, to indicate that they had understood what they had shown. For example, an observation that the final expression had a least value 2, or that it was the sum of two positive terms, was the sort of conclusion expected (the second of these examples is not strictly accurate, but Examiners would certainly not fuss over the distinction between 'positive' and 'non-negative' in this sort of situation!). Alternatives to completing the square were seen quite often, but, as with the standard method, the argument was frequently left incomplete or was wrong. Sketches of $y = x^2 + 4x + 6$ were sometimes attempted, and these gained full credit provided that the minimum point appeared to be correctly located; not infrequently, however, the minimum was shown at (0, 6). Also seen quite often were attempts at solving the equation $x^2 + 4x + 6 = 0$. Candidates who took this approach almost always showed successfully that the equation had complex roots, but usually claimed that this of itself demonstrated the required result, so failing to gain full marks.

$x^2 + 4x + 6.$

*Q.2* This binomial expansion question was generally well done, and it seemed to Examiners that there were fewer very weak attempts than has sometimes been the case with similar questions in previous years. There were, as usual, a very small number of attempts at the expansion by the direct use of McLaurin's method, but the great majority used the standard series. The most common errors were using $x$ in place of $2x$ throughout, failing to square and cube the factor 2 in $2x$, and failure to evaluate a binomial coefficent accurately (including in particular the subtraction error $\frac{1}{4} - 1 = -\frac{1}{4}$). A few weak candidates were not satisfied to leave the correct answer with fractional coefficients, but insisted on 'multiplying through', by 16 in this case, to produce a series of terms with integer coefficients.

$$1 + \tfrac{1}{2}x - \tfrac{3}{8}x^2 + \tfrac{7}{16}x^3.$$

*Q.3* Most candidates showed quite a good understanding of logarithms and indices, and solutions often proceeded correctly as far as $x \ln 2 = (1 - x) \ln 3$, or some equivalent equation in one unknown. However, Examiners were very surprised to note the number who got this far, and then clearly did not realise that what they had was a straightforward linear equation in $x$ to solve. A good number of attempts simply stopped at this point, or after one more (not very useful) step, with $\frac{x}{1-x} = \frac{\ln 3}{\ln 2}\cdot$ Others tried to solve the linear equation in unnecessarily difficult ways, for example dividing through by $x$ and finding $\frac{1}{x}$ first. Such unconventional methods sometimes worked out all right in the end, though equally there was sometimes a good deal of doubt about whether the answer had been properly shown; for example when a candidate went straight from the statement $x = \dfrac{1}{1 + \dfrac{\ln 2}{\ln 3}}$ to the given answer $x = \dfrac{\ln 3}{\ln 6}$ with no intervening simplifying step. Most candidates, no doubt prompted by the form of the given answer, took logarithms at an early stage of their solution; relatively few used rules of indices first, to obtain $2^x = \frac{3}{3^x}$ and hence $6^x = 3$, and, of those who tried this approach, quite a number were unable to carry it through correctly, owing to errors in expressing $3^{1-x}$ as a quotient or the inability to simplify $2^x\,3^x$.

*Q.4* The work on this question was rather disappointing. Many of the candidates who could do it found it very short and easy indeed, but many others seem to have been completely baffled and were unable to make any sort of start. One of the difficulties, presumably, was that candidates had to develop some sort of strategy for themselves if they were to solve the problem, since the question itself gave no clue as to how to start. However, this was not the only difficulty, because Examiners noted that there were many who *did* make a correct start but who were nevertheless unable to complete the solution. Typically, such candidates would introduce $\theta$ to denote the angle at $O$, and would then translate the information about the length of the arc into an equation involving $r$ and $\theta$, usually quite correctly. The resulting equation was often written in the form $r\theta = \frac{1}{2}(r + r + r\theta)$, or in some slightly simplified equivalent form. But very often this was where the solution stopped; many candidates could not see that this equation can be used to find $\theta$, after which the solution can be quickly completed. Those who pressed on often made errors in simplifying this equation, and in particular Examiners were very disappointed to see the number of candidates who obtained $r\theta = 2r$ correctly and were unable to deduce $\theta = 2$; for some inexplicable reason $\theta = r$ was a regrettably common next step. Some candidates provided a very neat solution to the question without using the angle at $O$ at all. Their first step was to observe that the arc length must be $2r$ (since the other half of the perimeter is $2r$), and that the area of the sector is thus a fraction $\frac{2r}{2\pi r}$ of the area of the complete circle. In general, knowledge of the formulae $r\,\theta$ and $\frac{1}{2}r^2\theta$ was good and there were only very occasional mistakes; there was occasional confusion between degrees and radians, and the meaning of the word 'perimeter' was evidently not known by an occasional candidate.

$r^2$.

*Q.5* Work on the trigonometrical aspects of this question was generally quite good, but many candidates were much less convincing on the 'exact' aspects of what was required. The perhaps rather unfamiliar use of the cosine formula at the start of the question was handled competently by most candidates, though some were put off by the fact that the unknown side $b$ seemed to be opposite the 'wrong' angle of the triangle and so were not able to get started. There were very few mistakes in the formula itself, and the only other point worth mentioning about this first part of the question is that quite a number of candidates did not indicate clearly enough exactly where the $\sqrt{3}$ came from; i.e. they never stated clearly that $\cos 30 = \frac{1}{2}\sqrt{3}$. (In fact, a few had clearly had to work back from the given answer in order to see where the surd went in their original cosine formula!) Most were able to solve the quadratic equation for $b$ correctly as far as $b = \dfrac{2\sqrt{3} \pm \sqrt{192}}{2}$, or thereabouts, and most understood that only the positive solution was required, but rather few could produce a good solution in which the simplified exact value $5\sqrt{3}$ was convincingly derived; many resorted to calculators and obtained $b = 8{\cdot}66$. In fact, the decimal value 8·66 was interchanged, without any explanation, with $5\sqrt{3}$ or $\sqrt{75}$ by many candidates at various stages of the question, and this was certainly a disappointing feature of many solutions – perhaps even most solutions. On the other hand, a small number rather neatly factorised the quadratic, getting $(b - 5\sqrt{3})(b + 3\sqrt{3}) = 0$ (apparently done mentally, but probably in fact with a bit of intelligent working backwards from the last answer in the question). Most candidates had no trouble realising that the sine rule was appropriate for the last part of the problem, and the work here was usually carried out correctly, apart from the use of a non-exact value for $b$. Only occasionally did a candidate find it necessary to try an 'otherwise' method here, though a few who had not been able to make any progress with the earlier parts of the question tried to start afresh on the last part. Very often the work in other methods was correct in the sense that the value of $\sin B$ was being validly found, but only very occasionally was there any *exact* work in evidence.

$5\sqrt{3}$.

*Q.6* There was much competent work at the start of this question, but disappointingly few candidates could deal correctly with the general solution of the trigonometrical equation at the end. The first part was usually well done, and the correct values of $R$ and $\alpha$ were very often found. A few were unable to start at all, but most knew the proper method, and arithmetical slips were fairly rare. The only error that was at all common involved a confusion of signs, leading to $\tan \alpha = -\frac{5}{3}$; those who made this error had to indulge in some judicious 'fudging' to get their value of $\alpha$ to comply with the statement $0 < \alpha < 90°$ printed in the question. The relevance of the '$R\cos(\theta + \alpha)$' method for solving the given equation was generally well understood, and most were able to proceed correctly as far as finding one particular value for $\theta$, which was usually 10·9°, or (less commonly) two particular values, such as 10·9° and –129·0°. However, very many candidates unfortunately did not know how to proceed to the general solution. Some just ignored this request altogether, and stopped when they had found one or two specific angles. Of those who tried, but failed, to give the proper general solution, the most common wrong answer was $360n \pm 10{\cdot}9$; this was given even by some who had correctly written $\theta + 59{\cdot}0 = 360n \pm 69{\cdot}9$ previously. The idea of a general solution, and the correct sequence of operations, is clearly something that candidates find very difficult, and indeed there were comments about very similar errors in a very similar question in last year's Report. One other comment from last year that regrettably needs to be repeated concerns the confusion between degrees and radians. This particular question was a 'degrees' question, but many evidently learn their formulae for general solutions in radians, and it seemed to many Examiners that they saw $2n\pi \pm 10{\cdot}9$ just as often as $360n \pm 10{\cdot}9$.

$$R = \sqrt{34},\ \alpha = \tan^{-1}\left(\tfrac{5}{3}\right);\ \theta = 360n + 10{\cdot}9,\ 360n - 129{\cdot}0.$$

*Q.7* A few candidates did not know about parametric differentiation, and consequently made fruitless attempts to eliminate $t$ between the pair of equations, but the majority knew what had to be done and were generally able to carry out at least some of the steps quite well. The standard of differentiation was actually a little disappointing, though of course many of the stronger candidates found this to be a very easy question, and were able to score full marks with only a few lines of working. However, many did not differentiate both $x$ and $y$ correctly. The main errors were loss of a minus sign (i.e. $\dot{y} = 1 - e^{t}$ or $\dot{y} = 1 + e^{-t}$), loss of one complete term in each derivative (i.e. $\dot{x} = e^{t}$ and $\dot{y} = -e^{-t}$), and complete nonsense (i.e. $\dot{x} = 1 + te^{t-1}$). The overall method, of equating $\dfrac{\dot{y}}{\dot{x}}$ to zero, was generally well understood, though candidates who had made differentiation errors usually found themselves with an equation for $t$ that they could not solve. Algebraic errors sometimes arose in this question also, for example the deduction $\dot{y} = \dot{x}$ instead of $\dot{y} = 0$ when cross-multiplying. A few candidates showed their shaky grasp of basic algebra at an earlier stage, by transforming $\dot{x} = 1 + e^{t}$ into $\dfrac{1}{\dot{x}} = 1 + e^{-t}$. However, a fair number managed to find $t = 0$ at the stationary point, and most of these managed to go on to find the values of $x$ and $y$ correctly.

$$\frac{dy}{dx} = \frac{1 - e^{-t}}{1 + e^{t}};\ (1, 1).$$

*Q.8* Few were able to score full marks on this question, and indeed it was often omitted entirely by the weaker candidates (and by some of the not-so-weak ones). However, those who knew the definition of gradient, and who had the confidence to get started and get something written down, generally managed, with the help of the List of Formulae, to demonstrate the displayed result. The deduction for the case $\phi \approx \theta$ proved very difficult, however. A good number of candidates were able to explain correctly that the $\cos\theta$ arose from the factor $\cos\frac{1}{2}(\phi + \theta)$, but only the very best were able to explain convincingly what happened to the other factors. Most, perhaps not surprisingly, could only say (in effect) that they gave $\dfrac{0}{0}$, and consequently 'cancelled out' or 'could be ignored'. Some were obviously not very happy with this, and tried to write a couple of sentences about how the zeros were only nearly zero and so cancelling them out was all right really. But in order to get full credit, candidates had to refer explicitly to properties of small angles in radians before they began to talk of cancelling things out.

*Q.9* The method for finding the partial fractions was generally well known, and there was much good work on this first part of the question. Slips in the working were relatively rare, but a noticeable number of candidates made a basic error at the outset, and did not multiply through correctly by the proper LCM, obtaining the impossible identity $1 \equiv Ax^2(x-1) + Bx(x-1) + Cx^3$. The use of the partial fractions for finding the integral in the second part of the question was, on the whole, rather less well done. All but a very few realised that the integration should be tackled by making use of the partial fractions, and the first and third of the fractions were usually integrated correctly. Integration of the second term was surprisingly often wrong, however. Many of the wrong versions involved logarithms, so perhaps candidates were unconsciously prompted by the other logarithm terms into thinking that all the integrations would involve logs. Perhaps – but whatever the context, an answer such as $\dfrac{1}{2x}\ln x^2$ for the integral of $\dfrac{1}{x^2}$ is extremely bad. Of those who recognised that they had to integrate a simple power of $x$, by no means all could get the sign right. It was probably fortunate for most candidates that the values of $A$, $B$ and $C$

were all integers, and that they were mostly correctly found. Those few who obtained fractional values for one or more of these constants often made further errors when it came to the integration, such as $\int \frac{1}{2x}\,dx = \ln(2x)$.

$$-\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x-1}\,;\; -\ln x + \frac{1}{x} + \ln(x-1) + c.$$

*Q.10* A fair proportion of candidates were able to answer the first part of this question on selections correctly, though a good number of others were in error by a factor of 5!, having calculated the number of permutations instead of the number of combinations. The second part of the question proved to be too difficult, and very few were able to do any relevant correct calculations on this part at all. There proved to be two main hurdles standing in the way of successful solutions. The first was the idea that two separate cases needed to be considered, i.e. 3 + 2 and 2 + 3, and unfortunately many candidates did not realise that this was the way to go about the problem. A common approach was to argue that there needed to be 2 students from the first year-group, 2 from the second, and then 1 other student who could be any of those not already chosen. This led to the calculation $\binom{10}{2}\times\binom{10}{2}\times\binom{16}{1}$; it is temptingly plausible, but incorrect on account of multiple counting of the same selection. A variation on this, also quite plausible but also incorrect, that was quite often seen, led to the calculation $\frac{10\times 9\times 10\times 9\times 16}{5!}$. The second hurdle lay in knowing when to multiply and when to add, and a regrettably large number of candidates seemed to be totally confused about this aspect of the calculation.

(*i*) 15 504; (*ii*) 10 800.

*Q.11* Many were able to show the required result in part (*i*) of this question, but a minority did not see that it was just a matter of simple algebraic expansion. Perhaps the inclusion of the phrase 'for all values of $k$' at the end was unhelpful, since it certainly seems to have turned the thoughts of some candidates towards induction. For those who carried out the correct method, the standard of algebraic accuracy was usually good, and few failed to demonstrate the result because of a slip in the working. A few candidates showed dangerously little working, thereby leaving Examiners in some doubt as to whether they really had shown what was required. For the most common method, in which the left-hand side was expanded, at least 3 lines of working were expected. For one method these could be: first, the removal of the outside brackets, giving $2(k+1)^4 - (k+1)^2$; second, the complete binomial expansions, with all 8 terms visible separately; and third, the required answer. For another method they could be: first, the expansion to $(k^2 + 2k + 1)(2k^2 + 4k + 1)$; second, the complete expansion of this pair of trinomials, with all 9 terms visible separately; and third, the required answer. (Incidentally, it was noticeable that a few candidates using the second of these methods chose to multiply the first trinomial by the unsimplified version $2k^2 + 4k + 2 - 1$ of the second; some candidates are quite extraordinarily prone not to notice obvious simplifications that could be made, and often make questions much harder than they need be for this reason.) There was a fair amount of success with the induction problem in part (*ii*), though the algebra was perhaps a little trickier than in some of the examples that have been set in the past. Some did not notice that the first part of the question was relevant here, though this did not necessarily prevent them from completing the induction part correctly. As has been remarked in previous Reports, candidates are expected to demonstrate that the induction starts properly at (in this case) $n = 1$; merely stating, as a few still do, 'True for $n = 1$' with no supporting evidence, is not sufficient.

*Section II*

*Q.12* This was a popular question, and it proved to be one where, on the whole, good and average candidates could score at least some of the marks quite easily. However, it was also noticeable that some of the weaker candidates who tackled the question scored very poorly, sometimes because they simply did not know the required AP and GP formulas, and sometimes because of extremely weak algebraic skills. In part (*a*), the great majority were able to set up the correct initial equation, usually in the form $\frac{a(1-r^8)}{1-r} = \frac{a}{2(1-r)}$. Most were able to cancel out the factor *a* without too much trouble, but quite a number were not able to deal with the factor $1-r$ in the denominators nearly so easily; this was even more liable to cause a problem where candidates used the alternative form $\frac{a(r^8-1)}{r-1}$ for the left-hand side of the equation. It was quite surprising to see how often candidates cross-multiplied and obtained a 9th degree equation for *r*; and unfortunately some were unable to proceed any further. However, most who attempted the question eventually arrived at the correct simplified equation $r^8 = \frac{1}{2}$, and this was almost always solved accurately. There was a certain amount of carelessness over the second request in part (*a*), with a number of candidates using the sum of the first 17 terms instead of the 17th term, but once again very many found no difficulties, and obtained the correct result. It was pleasing to see that quite a few of the better candidates realised, and showed, that the value of *a* was an exact integer (though there was no penalty if this point was not appreciated). Work on part (*b*) was generally quite good, and once again many of the better candidates found it easy to complete all the parts quite quickly. However, others tended to make rather heavy weather of the algebra, and also sometimes did not read carefully enough what the question actually asked. The great majority set up the initial equation correctly, and most made quite a fair attempt to rearrange their equation so as to express *a* in terms of *n*. However, a small, though quite noticeable, minority appeared to think that the question asked them to show that *a* was equal to the displayed result; this led them to make algebraic errors in trying to get this to work out, and also to omit the part which asked about the *n*th term. Most of those candidates who did what the question actually asked made quite good progress, though inevitably the algebra proved too much for some, and there were a fair number of sign and other errors in the working. A few did not understand how to get started on the last part of the question, and omitted it, but most set up the correct inequality, using the previous given answer, and were then generally able to simplify it correctly into the required form. A few gave an unsatisfactory argument at this stage, and started by *equating* the *n*th term to 500 and deducing $n^2 - 101n + 2000 = 0$. They then almost always changed the equals sign to the inequality shown in the answer without any proper explanation or justification. Solutions of the quadratic inequality were generally accurate, and it was pleasing to see that most candidates who had got this far with the question were able to choose the correct root and were able to give the appropriate integer answer for *n*, quite often with a good supporting argument. However, answers left in non-integer form, or values that had been rounded up instead of down, were not uncommon.

(*a*) 0·917, 40; (*b*) $\frac{10\,000}{n} - 5(n-1)$, 73.

*Q.13* This was another popular question. It turned out to be relatively easy to score most or all of the marks for part (*a*), but to be very difficult, even for strong candidates, to make much satisfactory progress with part (*b*). In some ways this question may therefore have proved a poor choice for some good candidates, who may very well have dropped three or four marks without realising it, when they might have done better with another of the *Section II* problems. Weaker candidates, on the other hand, seem to have benefited from the relatively straightforward part (*a*), and often seem to have done just as well, or even better, on this question as on their other *Section II* choices. Most of the parts of (*a*) were well done, and only occasionally did candidates show confusion over the order in which functions should be composed, or mistakenly take the inverse function notation to denote a reciprocal. In (*i*), the correct function was usually identified, and sketches of the graph were almost always of the correct general shape, and correctly shown to pass through the origin. However, the asymptotic behaviour at $x = -1$ was quite often not indicated at all clearly, and this was a definite shortcoming of many solutions. A less common error, but one which arose often enough to be noticeable, was shown by candidates who drew the graph of f in the first quadrant only, so that their curve just stopped at the origin. The inverses in part (*ii*) and the composition in part (*iii*) were generally accurately found, although those few who did not understand the notation naturally tended to make similar errors at various different points in the question. Perhaps the most common slip, for those candidates who basically knew what was required, was when the answer for $h^{-1}(x)$ appeared as $1 - x$ instead of $x - 1$. In part (*iv*) most attempts at the sketch graph were sound as regards the overall shape, but not all were able to locate the curve correctly. Examiners expected candidates to show in their sketch both the general overall shape of the curve, and also to give some indication of location or scale, and the better candidates did this, either by noting some convenient point on the curve, usually $(0, e^{-1})$ or $(1, 1)$, or by referring to the amount by which the standard curve $y = e^x$ had to be translated (or scaled) to obtain the required graph. Nearly all the work on part (*b*) was most disappointing, and unfortunately this part of the question seems to have been just too hard. Examiners had expected that many candidates would be able to score well on the part regarding the one-one nature of the function q, and that most would make some progress in finding the inverse; it had been expected that getting the details of the inverse function correct would provide a couple of discriminating marks that only the best candidates would earn. However, in the event, the great majority found it difficult to get anything right at all. Most attempted to give an argument based on a graph (as the question suggested), but almost always the graph consisted just of the parabola $y = x^2 - 4x$ with no attention being paid to the restricted domain $|x| < 1$. The concept of a one-one function seemed to be very unfamiliar to all but a very small number of candidates; or if it was familiar to them, most proved to be quite incapable of stating their reasoning accurately. The concept must surely have been unfamiliar to those who answered along the lines 'the function is one-one because the graph is a parabola', or 'the function is one-one because the graph has an axis of symmetry'. The great majority of those who seemed to have *some* idea of the topic said, often in these words, that 'each value of $x$ maps to one value of $y$'. Just occasionally, Examiners wondered, perhaps because of the exact phraseology used, whether the candidate was actually trying to say that each value of $y$ arose from precisely one value of $x$; but whatever their intentions may have been, hardly any candidates wrote down a form of words that actually said the correct thing. Those few who really knew about one-one properties of course found nothing very difficult in this example; they sketched the appropriate part of the curve and explained clearly how the critical point was demonstrated, namely that each $y$-value corresponded to one and only one $x$-value. Most candidates had shown in part (*a*)(*ii*) that they could carry out the process for finding an inverse function in simple cases, but very few indeed could carry out the same process for the more complicated case in part (*b*). Candidates mainly use one of two methods for finding inverses; either they let $y$ equal the original formula, and turn the equation round to express $x$ in terms of $y$, finally interchanging the letters, or else they construct the sequence of operations by which $x$ is transformed to the original function value and then form the reverse sequence. Very few could make the first method work when applied to the function q. A good many were able to write down $y = x^2 - 4x$, but almost all attempts at algebraic rearrangements made no progress at all. The problem seemed to be that hardly any candidates recognised that they had a quadratic equation from which $x$ could be expressed in terms of $y$, and recognising this is the essential preliminary step in this method. The 'reverse operations' method might perhaps be thought harder to apply in this case, since q itself has to be thought of as $q : x \mapsto (x-2)^2 - 4$ before any progress can really be made. However, a few did see this, and identified the operations for q as 'subtract 2', 'square', 'subtract 4'. Their reverse sequence was virtually always 'add 4', 'square root', 'add 2', with unfortunately never a thought as to the sign of the square root, so although these candidates made some worthwhile progress, their final answer was almost never correct.

(*a*)(*ii*) $e^x$, $x - 1$; (*iii*) $e^{x-1}$; (*b*) $2 - \sqrt{(4 + x)}$.

*Q.14* This vector question was one of the less popular choices, but many who did select it made quite good progress. There were also a few very weak attempts; perhaps there is some tendency for questions on vectors to be chosen only by those candidates who feel quite confident about the topic, or else by those who are desperately casting round for a last question to try! In part (*i*), candidates often found the given equation rather hard to explain coherently, but their work nearly always showed that they understood the key features of the equation of the line, namely that $\begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix}$ is the position vector of $P$ and that $\begin{pmatrix} 1 \\ 3 \\ h \end{pmatrix}$ is $\overrightarrow{PM}$. Answers to part (*ii*) were generally accurate, with just occasional slips of sign, for example. The work on part (*iii*), concerning the intersection of the lines $PM$ and $QN$ was often not wholly satisfactory, particularly from the majority of candidates using the standard method based on algebraic working with the equations of the two lines. Candidates generally knew that they should make simultaneous equations by equating components, and then solve their equations. Some made a very basic error at the outset, by using the same parameter $t$ in both lines. Because of the symmetry inherent in this particular question, this happens to 'work' in the sense of leading to the correct answer for the point of intersection, but such a very serious error of method was penalised in the marking. Those working with two parameters almost always solved a pair of equations, and thus obtained the given point of intersection all right, but disappointingly few showed that their values for the parameters really did satisfy the third equation; this was a very definite weakness in many answers to this part. A few candidates noticed, and exploited, the geometrical symmetry underlying the algebra, and were able to dispose of this part of the problem very neatly by noting that the given point is the mid-point of each of $PM$ and $QN$. The necessary scalar product methods for part (*iv*) were generally well known, but for some reason candidates found it very difficult to carry out the work accurately. There was one quite extraordinary common error in the evaluation of $\begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2}h \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ -2h \end{pmatrix}$, which was very frequently written as $1 + 1 - h$ instead of $1 + 1 - h^2$.

(*ii*) $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} -3 \\ -1 \\ h \end{pmatrix}$; (*iv*) $\sqrt{2}$, 70·5°.

*Q.15* This was not one of the more popular questions in *Section II*. Many of those candidates who attempted it were able to make quite good progress with one or more of the parts, but not very many were able to cope completely successfully with the various demands made. The method needed for the first part, involving the differentiation of a product, equating the result to zero and then doing some algebra, was understood by virtually all those who tackled the question. However, a fair number made things much more difficult for themselves than was necessary by expanding the factor $\cos(x + \alpha)$ before starting on the differentiation. The form of the given answer, which involves $\tan(x + \alpha)$, really ought to have indicated that this expansion was probably not a good idea; in fact, once the expansion is done, it really becomes very difficult indeed to get the algebra back into the particular form required in the answer. The accuracy with which the differentiation was carried out (whether in an expanded version, or as intended) was fair; there were a good number of sign errors, and also quite a few instances of candidates treating $\alpha$ as a variable rather than a constant, and this again hopelessly complicated matters. A reasonable number managed to produce the first part of the given result ($\cos x = 0$) from their working, since this tended still to appear even if there were slips in the differentiation, but only those who were able to employ the proper method accurately were able to get anywhere near the other part of the solution. As is often the case in questions which the less good candidates find very complicated, the best were able to make rapid progress, and could often obtain the required results in just a few lines. All were able to make a fresh start at part (*i*), and use of the $\tan(A+B)$ formula was generally good. Many candidates were able to obtain the correct quadratic equation for $\tan x$ and solve it, though sometimes the work was not carried through in an exact form right to the end, as was required. There was some evidence of carelessness in this part of the problem, with a number using $\alpha = \sqrt{2}$ instead of $\tan \alpha = \sqrt{2}$, and with some writing $x$ in their equation where they meant $\alpha$ and *vice-versa*. With some simplification errors as well, not all candidates by any means actually finished up with a quadratic equation at all for $\tan x$. A good number unfortunately lost marks in part (*ii*) by not understanding that the requirement to show the result exactly precluded them from finding the angle $\alpha$ by calculator, and then simply evaluating $\cos 2\alpha \cos^2 \alpha$ by the same means. The great majority who appreciated what was required found little difficulty in deducing the exact value of $\cos \alpha$ and in using an appropriate double-angle formula to demonstrate the exact value of $y$.

(*i*) $\sqrt{2}$, $-\frac{1}{4}\sqrt{2}$.

*Q.16* This question, on mainly quite elementary topics, was popular, and many candidates scored well. However, certain aspects were rather disappointingly handled by a good number, and many lost marks through seemingly rather careless mistakes. The first part of the question is a case in point. All candidates were able to make a reasonable attempt at integrating $y$ and using the limits 1 and 4. A good proportion did this accurately, though there were also rather many slips, particularly with the integral of $2x^{-\frac{1}{2}}$ (e.g. $x^{\frac{1}{2}}$ or $2x^{\frac{1}{2}}$ instead of $4x^{\frac{1}{2}}$), or real blunders with the same integral (e.g. $\frac{2}{\frac{2}{3}x^{\frac{3}{2}}}$). However, many candidates failed to subtract their value from the area of the square, and so never made any attempt to find the area of the region actually asked for, and this does seem careless. The majority were able to write down the correct equation for the translated curve, though both $y = \sqrt{x} + \frac{2}{\sqrt{x}} + 3$ and $y = \sqrt{(x+3)} + \frac{2}{\sqrt{(x+3)}}$ appeared now and again. A good number understood the connection between the equation of the translated curve and the volume of rotation of $R$ about the line $y = 3$, and were able to demonstrate the given result convincingly. As in *Q.11*, Examiners expected to see some details of the algebraic working if full credit was to be given. The work on the evaluation of the volume at the end of the question was really only fair. There were a number of careless errors in integrating one or more of the five terms (by no means always repeats of slips that may have been made in the corresponding integrations in the first part of the question!) and, in particular, the integral of $\frac{4}{x}$ was wrong disgracefully often. In addition, though this is perhaps a minor point by comparison, very many candidates neglected to give their final answer in the exact form required, having introduced decimals for their logarithm term at an early stage. The overwhelming majority attempting this question did what was required, in that they used algebraic integration to evaluate the area and the volume. Examiners noticed the occasional candidate who produced values for the integrals that occurred in the problem without showing any explicit indefinite integration step. Such candidates were either quite recklessly omitting essential working from their solutions, or more likely were using numerical integration facilities on their calculators. Candidates should be made aware that the questions set out to test their knowledge of particular syllabus items, and that questions may be worded so that credit can only be gained if knowledge of a particular topic is demonstrated. In the present case, the requirement for 'exact' answers was there precisely to ensure that they were tested on their ability to integrate powers of $x$, which is in the syllabus, and not on their ability to use approximate numerical integration via a built-in calculator routine, which isn't.

$$\tfrac{1}{3};\ y = \sqrt{x} + \frac{2}{\sqrt{x}} - 3;\ \pi(4 \ln 4 - \tfrac{11}{2}).$$

*Q.17* Many candidates attempted this question, including unfortunately a few who had no understanding of any of the integration methods required. However, most were able to make some progress, even if full marks were quite rare. On the whole, candidates found the question rather long, and often did not seem to appreciate that earlier results could be used at later stages to shorten the working somewhat. For example, having found or verified the integral of $\sin^2 x$ in the first part of the question, it is available for quoting in the second part; or having worked through the details of using the substitution $2u = \cos x$ in part (*iii*), it is really not necessary to write out the same working all over again in part (*iv*) where the same substitution is repeated. Many candidates, using basically sound methods, made errors of sign in one or more parts, and perhaps rather surprisingly, the presence of given answers in both parts (*ii*) and (*iv*) does not seem to have been particularly helpful in locating and correcting these. Most were able to do part (*i*) all right, and the given answer here no doubt did prove useful in preventing sign and coefficient errors appearing at an early stage. The great majority attempting the question recognised that part (*ii*) required integration by parts, and most were able to make use of the previous answer; some, however, preferred to start again with $\sin^2 x$ in terms of $\cos 2x$, and use integration by parts on $x \cos 2x$. The main errors involved signs, either in applying the parts formula and integrating $-\frac{1}{4} \sin 2x$ or else when substituting the limits, and not evaluating everything at $x = 0$ correctly, and as a result of one or both of these types of error many failed to reach the given answer. Both parts (*iii*) and (*iv*) involved integration by substitution, and although (as always seems to be the case) some candidates clearly had no idea of the required methods, Examiners felt that there seemed to be a larger proportion of attempts this year where at least some correct work was done. There was a good deal of trouble over signs again, and in the correct placement of the new limits (when there *were* any new limits, that is). Also, quite a few candidates failed to recognise that $\sqrt{(1 - 4u^2)}$ simplified to $\sin x$ when the substitution $2u = \cos x$ was made, and were thus unable to proceed. The work of a fair number was accurate enough to allow them to arrive at an integral involving $\sin^2 x$, though relatively few had all the details right, thus obtaining the correct answer to this part. Similar difficulties arose in the course of part (*iv*), where candidates first had to show the equivalence of the two integrals by means of the same substitution again. As was mentioned above, the given answer in this case did not seem to help candidates to find sign errors, and, although it may have been of some use in helping with the simplification aspect, it remains true to say that really not many could transform one integral to the other in the few lines of working that are necessary, given that most of the steps had already been carried out earlier in the question. The final evaluation was often quite well done, but was again too often marred by slips of detail in dealing with the change from $\sin^2 x$ to $\sin^2 2x$.

(*iii*) $\frac{1}{8}\pi$; (*iv*) $\frac{1}{32}\pi$.

*Q.18* Marks on this quite popular question showed some tendency to go towards the extremes. Those candidates who had a sound understanding of implicit differentiation were often able to score pretty well, while others tended not to be able to get much right at all, apart perhaps from the last part of the question about the trapezium rule. The somewhat unusual start to the question, with $\frac{dy}{dx}$ instead of $y$ being the given quantity, caused a few candidates to think that the first thing they needed to do was to integrate in order to find $y$, and generally those who were under this misapprehension failed to make any real progress. However, most realised that part (*i*) just required them to differentiate with respect to $x$, and the necessity of using the chain rule was very widely recognised. The given answer was undoubtedly very helpful to many candidates here, but quite a few others still failed to get this first part to work out. The usual cause of failure was the omission of the factor $\frac{dy}{dx}$ from the differentiation; those who were able to string together $\frac{1}{2}(1+y^3)^{-\frac{1}{2}}$ and $3y^2$ and $\frac{dy}{dx}$ were generally able to carry out the necessary simplification. In part (*ii*) a good number were able to write down the correct expression for the third derivative, but mistakes in the fourth were more common. Candidates usually recognised that the differentiation of a product was now involved, but a factor of $\frac{dy}{dx}$ was again frequently omitted from one of the terms. Another quite common error, this time rather more careless, was a failure to express the final answer in terms of $y$ and $\frac{dy}{dx}$. Almost all of those who realised that part (*iii*) involves the McLaurin series did well, though the fact that the answer was given sometimes meant that Examiners were rather doubtful about how much candidates had really understood. Some gave very sketchy explanations indeed; for example it was often very unclear whether a candidate had really appreciated the significance of the statement in the question that the graph of $y = f(x)$ passes through the origin, or whether the leading term f(0) had been quietly omitted just because it wasn't there in the given answer. A lot of work on the trapezium rule in part (*iv*) was good, with many candidates obtaining the correct answer. The use of $y$ in the integral in place of the more conventional $x$ appeared to cause no trouble, but there was a certain amount of carelessness in evaluating the function values (using a different function, such as $\sqrt{(1+y^3)}$ or $\frac{1}{\sqrt{(1+y)}}$, for example), or in using four ordinates instead of five (omitting the value y = 0 usually). As always, a few very weak candidates tried to apply the rule using 'values' 0, 0·1, 0·2 etc., instead of values of the integrand.

(*ii*) $3y\frac{dy}{dx}$, $3\left(\frac{dy}{dx}\right)^2 + \frac{9}{2}y^3$; 0·397.

*Paper 9205/2*

*General Comments*

The paper provided a good test, with opportunities for the weaker candidates to show some achievement, but also with some parts that were sufficiently testing to make full marks of 98 quite rare. Most candidates attempted seven questions at least. Much very good work was seen, indicating that many Centres are preparing their candidates well for this examination. The presentation of work from such Centres was generally good also. However some Centres do not seem to insist on clear and neat presentation and this often tends to go with poor mathematical work. The general standard of work was quite good, but mistakes in elementary algebra seem to be getting more common, and attention is drawn to some examples of this in the detailed comments. Failure to introduce brackets has been mentioned before in these reports and again occurred quite frequently e.g. $1 - p_0 + p_1 + p_2$ for $1 - (p_0 + p_1 + p_2)$ and $(4 + \sin^2\theta) - \sin\theta$ for $(4 + \sin^2\theta)(-\sin\theta)$. Despite the bad notation candidates often seemed to know what they meant and, in fact, usually went on correctly. In mechanics, the formula $mv^2/2$ was often seen for kinetic energy but, on substitution, the value of $v$ was often not squared. Another area of weakness was found where a simple explanation or reason was asked for. This was rarely answered convincingly. All too often the answer made no sense at all and, even when it did, it often bore no logical relation to the question asked. This does not promise well for forthcoming developments at A-level where understanding and ability to communicate this in words will have more emphasis. A related problem is the common failure to explain the method being used. Equations appear and the Examiner is often left to infer what is being attempted. This may be easy if the equations are correct but when they are incorrect method marks can be lost if the method is not clear. Confusion between decimal places and significant figures is another recurring theme. The significance of the word 'exact' was quite often missed. This is a signal that decimal working will not earn marks. Use of decimals is not appropriate when an exact answer is asked for, even if the answer is given to all the figures held in the calculator! 'Exact' answers should be left in a form which may involve fractions, surds, $\pi$, logarithms, exponentials etc., as appropriate, and they should of course be simplified as far as possible.

The most popular questions in the Mechanics Option were *Qs 1, 2* and *4*, and in the Statistics Option, *Qs 6, 8* and *9*. In the Pure Mathematics Option all the questions attracted roughly equal attention.

*Comments on Individual Questions*

*Option (a): Particle Mechanics*

*Q.1* The first part was generally well answered, usually with a clear diagram showing the forces acting. In the second part there were usually two correct resolutions, but many of those who resolved horizontally and vertically could not solve the resulting equations to find either $\phi$ or the tension in *OP*. The main problem appeared to be that elimination of the tension led to a trigonometrical equation which candidates had difficulty in simplifying to $5 \sin \phi = 2$. Resolution along *PQ* would of course have led directly to this equation. Resolution along *OP* would then have easily given the tension. Candidates often seem to think that the only permissible directions for resolution are horizontal and vertical. A small number had difficulty finding the angle between *PQ* and the horizontal. In the last part most candidates did not realise that all that is needed is the change in gravitational potential energy of *P*. Attempts to answer the question by multiplying the force by the distance made little progress, and failed to take into account the fact that the force is varying in magnitude and direction and the displacement is not in a straight line. Curiously the length 0·3 m given in the question was quite frequently misread as 3 m. Despite the clear request for 2 significant figures, many gave their answer as 0·06 J.

$\theta = 16{\cdot}7°$, Tension = 5·22 N; $\phi = 23{\cdot}6°$, Tension = 4·58 N; 0·062 J.

*Q.2* This question was answered well by a good proportion of candidates. In the first part some candidates wasted time by trying to find the speed of projection and the angle of projection, and this was often taken to be $90° - \phi$, leading to great confusion. Those who realised that the given initial velocity components actually make things easier had little difficulty with the question. In the second part a surprising number described a horizontal velocity as 'due east'. In the third part most candidates found $\dot{y}$ at *B*, and found the kinetic energy correctly, but there were some who merely considered the kinetic energy of the vertical motion. Most candidates answered the last part correctly, but there were some who thought that the angle required had something to do with the value of $\phi$ when $t = 4{\cdot}5$.

$x = 20t$, $y = 30t - 5t^2$; velocity at *A* is 20 ms$^{-1}$ horizontally; 62·5 J, 36·9° below the horizontal.

*Q.3* This was not a popular question but the request for a differential equation seemed to mean that a smaller number of candidates than usual thought that the constant acceleration formulae were appropriate. Many good answers were seen, but the most common error was omission of the minus sign needed to take account of the direction of the force. These candidates sometimes attempted to compensate by omission of another minus sign on integration, thereby obtaining the given answer. Of course, to obtain full credit in this part of the question, where the answer is given, it is necessary either to introduce a constant of integration and show that it is zero or to use appropriate limits of integration. Using the given expression for $v$, the loss in kinetic energy was nearly always correct, even when candidates had not been able to establish the result in the first part. Presumably because a differential equation was not asked for, some candidates attempted the third part using the constant acceleration formulae, even though they had used a differential equation in the first part. The third part can, of course, be answered by using the given answer to the first part to obtain a differential equation relating either $x$ and $t$ or $v$ and $t$. Those who obtained such a differential equation usually answered this part correctly. The last part was not well answered. Most candidates thought that the question could be answered by reference to the direction of the force, and deduced that $P$ would return to $x = 1$. In fact the answer to the first part shows that $v$ is never zero, so the direction of motion of $P$ is never reversed.

$v\,dv/dx = -2/x^2$; 8/45 J; 26/3 s.

*Q.4* The fact that the particles are moving in opposite directions before the collision, and also afterwards, gave problems to many candidates, particularly those who did not make up their minds clearly as to what direction they were going to take as positive for velocity and momentum. With the given initial velocities the momentum before the collision is $1{\cdot}8-0{\cdot}3v$, measured in the direction of the initial motion of $P$. Quite a large number of candidates left their answer for the magnitude of the impulse as a negative number. Most candidates answer the second part correctly, but there was a small number of very confused attempts. Some found only the loss of kinetic energy for one particle. Others took the initial kinetic energy to be 8·1–5·4 and the final kinetic energy to be 0·9–0·6, presumably thinking that kinetic energy changes sign with the direction of motion. A few took the loss in kinetic energy of $P$ to be $0{\cdot}1(9-3)^2$. The construction of the question meant that mistakes in the first two parts did not lead to difficulties in the third and subsequent parts. The third and fourth parts were answered correctly by most candidates. In the last part there were signs that some candidates did not feel they could answer the question without knowing more about the second collision. Of course the momentum after the second collision is equal to the momentum beforehand, which is easily found, and many candidates seemed to appreciate this.

2·4 Ns, 6 $\text{ms}^{-1}$; 12 J; 30 m; 1·2 Ns (or $\text{kgms}^{-1}$).

*Q.5* This was the least popular question in the Mechanics Option and it was not generally answered well. In the first part the vertical resolution was usually correct. The most common error was to use an incorrect value for the radius of the motion in the horizontal equation of motion. Many took this radius to be 1 m. A few felt that it was necessary to consider both $P$ and $Q$. In the second part, Hooke's Law was usually correctly used to find the tension in $PQ$. There were several common sources of error in this part. Firstly the inclination of $AP$ to the vertical was taken to be the same as in the first part, whereas it needs to be found from $1 + 5 \sin \theta = 5$. Secondly the two tensions were often taken to be the same. Thirdly the horizontal component of the tension in $AP$ was often omitted. Lastly there were some attempts to treat $P$ and $Q$ together.

$\omega = 1{\cdot}37$ rad $\text{s}^{-1}$; $\lambda = 5$.

*Option (b): Probability and Statistics*

*Q.6* Most candidates answered the first part correctly, obtaining the given answer by an acceptable method. Confusion about conditional probability often led to errors in parts (*i*) and (*iv*). In (*i*) many correct solutions were seen, but the most common error was based on (3/23) (2/22)/(4/24). The required probability can, of course, be written down directly as (3/23) (2/22). In (*ii*), many correct answers were seen, but the error (commented on in previous reports) of taking the required probability to be $P(A) + P(C) + P(A \,\&\, C)$ was again common. A rarer error was '$P(A \,\&\, C) = 2(4/24)(3/23)$'.

Correct answers were nearly always based on either $P(A) + P(C) - P(A \& C)$ or $P(A \& C') + P(A' \& C) + P(A \& C)$. The quickest method using $1 - (20/24)(19/23)$ was very rare. Part (*iii*) was often answered correctly – it does not of course involve conditional probability. In part (*iv*) many candidates did not realise that all that is required is the answer to (*iii*) divided by 1/2. Some had the basic idea but divided by 11/23.

(*i*) 3/253; (*ii*) 43/138; (*iii*) 11/138; (*iv*) 11/69.

*Q.7* Although there was a good number of correct answers to this question (apart from parts (*iii*) and (*iv*)), quite a number of candidates did not recognise that the given function is in fact the cumulative distribution function of $X$. The most disastrous error was to take the given function to be a density function. Many candidates showed $a = -1$, using $P(X \leqslant 4) - P(X \leqslant 3) = 1$, but often they could not find $b$, except by using the given value for $P(X \leqslant 3{\cdot}5)$. Two simple equations for $a$ and $b$ can of course be obtained by using $P(X \leqslant 3) = 0$ and $P(X \leqslant 4) = 1$. Parts (*i*) and (*ii*) were usually well answered. In (*iii*), failure to recognise a binomial situation was very common, leading to answers based on $(3/4)^2(1/4)$, with the factor '3' omitted. In part (*iv*) some candidates did not realise that the question was a request for the probability distribution of $M$. Others did not appreciate that $M$ is a random variable and gave answers in the form $\bar{X} = N(M, 1/1800)$.

$b = -15$; (*i*) $f(x) = 8 - 2x$ $(3 < x < 4)$, $f(x) = 0$ otherwise; (*iii*) 27/64;
(*iv*) N(10/3, 1/1800).

*Q.8* Correct solutions to part (*i*) based on either the Poisson distribution with mean 5 or the binomial distribution with $n = 1000$, $p = 0{\cdot}005$ were common. The normal approximation is not very accurate, particularly if no continuity correction is used. Part (*ii*) was usually answered correctly, using a binomial distribution with $n = 6$ and $p = 0{\cdot}75$, although there were some attempts to use the Poisson or normal distributions. A fairly common error was to take the required probability to be $1 - (p_0 + p_1 + p_2 + p_3)$. In part (*iii*)(*a*) the answer was usually correctly based on 0·97 times the answer to (*ii*), and in (*b*) the method was often correct. In (*c*) many did not realise that all that is needed is the sum of the answers to (*a*) and (*b*).

(*i*) 0·440; (*ii*) 0·962; (*iii*)(*a*) 0·934, (*b*) 0·025, (*c*) 0·958.

*Q.9* As always, the main difficulty that candidates have with this type of question is in finding the correct variances for the various combinations of random variables. Many answered two or three parts correctly, but correct answers to all four parts were rare. Generally speaking, the normal tables were used correctly. In (*ii*) the variance of the total weight is $(100 + 100)$ and not $2^2(100)$, since the question involves the sum of two independent observations. In (*iii*) the variance is $225 + 200$ and in (*iv*) $225/2^2 + 100$ (if the variable considered is (large)/2 – (small)). In (*iv*) a common incorrect variance was 212·5 $(= 100 + 225/2)$. Many other values for the variances were seen, including some that involved subtraction. A small proportion of candidates took the given standard deviations to be variances.

(*i*) 0·533; (*ii*) 0·681; (*iii*) 0·113; (*iv*) 0·159.

*Q.10* The first part was found to be difficult, with many candidates unable to deal with the fact that the data was given in terms of $(x - 1)$. Plainly many did not realise that $\text{mean}(x - 1) = \text{mean}(x) - 1$, and $\text{variance}(x - 1) = \text{variance}(x)$. Perhaps as a result of this, many candidates omitted the factor $1/n$ in passing from the variance of $x$ to the variance of the mean, while others tried to test 0·021 against 1·005. Part (*i*) was usually well done. In (*ii*) candidates often used the sample proportion 0·53 instead of the null hypothesis proportion in calculating the variance to use in the test. In fact (*ii*) can be answered directly once (*i*) has been answered.

$z = 0{\cdot}977$: accept mean quantity = 1·005 litres; (*i*) 0·572, 0·728; (*ii*) $z = 2{\cdot}52$ (or $-2{\cdot}41$, with continuity correction), accept proportion is less than 65%.

*Option (c): Pure Mathematics*

*Q.11* The differentiation was quite well done, but candidates often thought that the correct technique was to use the cos $2\theta$ formula. While this could be followed through correctly, it often led to errors. Very few were able to give a coherent reason why $r$ decreases. Consideration of the sign of the derivative given in the question was all that was required. The sketch was usually symmetric in the initial line, and often showed a loop closing at the pole. Some candidates did not realise that there is a vertical tangent at the pole, but showed a cusp of some kind. Some sketched a two-looped curve, symmetrical in $\theta = \pm \pi/2$, and a few gave a sketch with more than one loop in the given range of values for $\theta$. The justification of the given result for $A$ was often careless, with no attention paid to the limits of integration. The value of $A$ was usually correct, although some felt obliged to work in terms of the original variable $\theta$, and a few of these thought that what was required was $\int(4 + \sin^2 \theta)\,d\theta$. Mistakes were quite common in the algebra in the last part, the most common source of error being the difficulty of dealing with powers of $(x^2 + y^2)^{\frac{1}{2}}$. It was not uncommon to see $\cos \theta = y/r$ and $\sin \theta = x/r$.

$A = 13a^2/3$; $m = 5/2$, $b = 4$, $c = 5$.

*Q.12* This popular question was often answered well, but there was some weak algebra in (*a*) and some confusion, presumably due to misremembered procedures for obtaining the graphs, in (*b*). In (*a*) a surprising number left $y = -3ab/a$ without simplification. Answers should be simplified as much as possible. The asymptote $x = -a$ was usually correct, but $y = -1$ often appeared also. Candidates plainly do not recognise a graph of this kind as a hyperbola, and some seemed quite content to find only one asymptote. A few thought that there were three asymptotes. Despite the explicit request for the asymptotes to be shown, they were quite often omitted from the sketch. In (*b*), many sketched (*i*) correctly, but sketches which were essentially $y^2 = |f(x)|$ were quite often seen. The correct sketch for (*ii*) was often shown as the answer to (*i*). Others seemed to think that the graph of (*ii*) is the same as the graph of (*i*). In (*i*) there are two values of $y$ corresponding to each value of $x$ for which $f(x)$ is positive. In (*ii*) there is only one value of $y$ for each such value of $x$, and this value of $y$ is positive. Many candidates seem to think that two numbers are denoted by $\sqrt{a}$, where $a > 0$. The 'Mathematical Notation' section of the syllabus clearly defines $\sqrt{a}$ 'as the positive square root of the real number $a$', i.e. the positive number $x$ such that $x^2 = a$, so e.g. $\sqrt{4} = +2$ only, and $\sqrt{3} = +1{\cdot}732....$ $\sqrt{4}$ does not mean $\pm 2$. It would be better if the symbol '$\sqrt{\ }$' was read as 'positive square root', rather than 'square root'. In (*i*) and (*ii*), many sketches showed curves with cusps on the $x$-axis, instead of a vertical tangent at each such point. Some candidates seemed to think that because, in passing to $y^2$, values of $y < 1$ are decreased and values of $y > 1$ are increased, the resulting loop, for $x < 0$ in (*i*) for example, was 'wavy'. The sketch of (*iii*) was usually correct except that it was often shown as having U-shaped minima touching the $x$-axis, instead of cusps. The 'tails' of the sketches were often shown as straight, without any justification.

(*a*) $(0, -3b)$, $(-3a, 0)$; $x = -a$, $y = -b$.

*Q.13* (*a*) was generally answered well, with real and imaginary parts being equated and either $a$ or $b$ eliminated. The significance of the word 'exact' was lost on many candidates, who seemed to think that a decimal answer was sufficient. Many offered two values for $a$ and two for $b$ without realising that in each case one of the values was negative. In (*b*) (*i*) many candidates failed to use the quick method: $\text{Re}(z) = |z|\cos\{\arg(z)\}$ and $\text{Im}(z) = |z|\sin\{\arg(z)\}$. Instead the given information was often translated into $x^2 = 1$, $y^2 = 3$, and thence into $x = 1$, $y = \sqrt{3}$. Many seemed unaware that with $\arg(z)$ as given, in the third quadrant, both the real and imaginary parts of $z$ must be negative. A simple diagram should have shown this. Some gave '$z = 1 - i\sqrt{3}$', which is correct, but does not fully answer the question. Some gave $\text{Im}(z) = -i\sqrt{3}$, which is not correct since $\text{Im}(z)$ is a real number and not a ('pure imaginary') complex number. In (ii) many candidates felt that the only way to tackle it was to find $w$ and $z^2$ in cartesian form, and hence $w/z^2$, and finally its modulus and argument. The exact complex arithmetic gets pretty complicated, and many went to decimal approximations, which have no hope of providing exact values as requested in the question. Those who realised that the modulus and argument approach is best for multiplication and division generally obtained the modulus correctly, but the argument was often left as $25\pi/12$. Some incorrect variants of the argument formula were seen: e.g. $\arg(w)/\arg(z^2)$, $\arg(w) - \{\arg(z)\}^2$ and even $\arg(w) - \tan^{-1}(2y/x)$, where $z = x + iy$.

(*a*) $a = (\sqrt{21} + 3)/2$, $b = (\sqrt{21} - 3)/2$; (*b*) (*i*) $\text{Re}(z) = -1$, $\text{Im}(z) = -\sqrt{3}$;
(*ii*) 5/4, $\pi/12$.

*Q.14* In (*a*) the complementary function was rarely correct. Candidates often obtained the incorrect auxiliary equation $m^2 + 9m = 0$, presumably due to the missing '$dx/dt$ term'. Others correctly obtained $m^2 + 9 = 0$ and proceeded to solve this by factorisation to give $m = \pm 3$. Alternatively the formula was sometimes used, often with equally bizarre results. Candidates were plainly unaware that the equation $d^2x/dt^2 + k^2x = 0$ has complementary function $A \cos kt + B \sin kt$ (or some equivalent expression). Plainly some had learned the complementary function when the auxiliary equation has complex non-real roots $p \pm iq$, but found the roots $\pm i3$ hard to deal with. A factor $e^{0t}$ often appeared for example, while others produced some very garbled expressions e.g. $\cos(3Ax) + \sin(3Bx)$. Many had failed to notice that the independent variable is $t$ and not $x$. In finding the particular integral, many knew they had to try $x = at + b$ or even $x = 3at$, but not everyone knew this had to be substituted in the equation. Some merely equated the trial solution to the right hand side. A common error was correctly to try $x = at + b$, correctly obtaining $a = 1/3$ and $b = 0$ (though this was often ignored), but the particular integral was then taken to be 1/3, rather than $t/3$. The general solution was usually obtained by adding a particular integral to the complementary function, but was rarely correct and often with mixed independent variables $x$ and $t$. Most tried to find the values of the unknown constants by a valid method, but because the general solution was so rarely accurate, correct solutions were very rare. A small number tried to find the arbitrary constants using the complementary function only. In (*b*) most candidates found $dz/dx = 2 + dy/dx$, and went on to establish the required differential equation, although the algebra required to pass from $2 + (z + 2)/(z - 1)$ to the given expression $3z/(z - 1)$ proved too difficult for some. Many candidates went on to solve the equation correctly, but some had difficulty integrating $(z - 1)/(3z)$. Not all candidates remembered to substitute for $z$ in terms of $x$ and $y$, to obtain a solution relating $x$ and $y$.

(*a*) $x = (3t + 2 \sin 3t)/9$; (*b*) $y - x = \ln|2x + y| + C$.

*Q.15* (*a*) Few candidates answered the first part correctly, and many gave the least value as 0. Others said there was no least value as the equation $5x^4 + 50 = 0$ has no real root. There were plainly muddles between $dy/dx$ having a least value and $y$ having a stationary value. Few candidates gave a convincing reason for the equation having exactly one real root. Reasons were often unrelated to the positivity of the gradient, and often concluded that 'it cuts the $x$-axis exactly once', without stating what 'it' is. In the second part of (*a*), although many acceptable answers were seen, many candidates failed to state clearly what they were taking $f(x)$ to be. Often they were tacitly solving $x^5 + 50x = 0$, and the method took quite a long time to converge to $x = 0$! In this sort of question it is essential to state explicitly what $f(x)$ is being used. Examiners cannot deduce this from a string of decimal numbers. Very few candidates spotted that $x = 10$ is a good first approximation (ignoring '$50x$' in the equation), and most used complicated methods to try to find a first approximation, which often turned out to be a long way from 10. There was some confusion as to what 'correct to 4 decimal places' meant. Some gave the answer as 9·990, which is correct to 4 significant figures, others gave it as 9·9899, which is truncated to 4 decimal places and not correct to 4 decimal places. In (*b*) most candidates answered part (*i*) correctly. In (*ii*) however, most candidates had no idea how to tackle the problem and often found other lengths without realising it. For example some found the perpendicular distance from $\mathbf{i}+4\mathbf{j}$ to the plane $\Pi_1$, while others found the distance from O to the point of intersection of the line $r=t(\mathbf{i}+4\mathbf{j})$ with $\Pi_1$. Neither of these are much help in answering the question. These misguided attempts might have been avoided if the candidate had drawn a simple figure. There are two closely related and straightforward methods of tackling this question. One is to find the length ($5/\surd 6$) of the projection of $\mathbf{i}+4\mathbf{j}$ onto the normal to $\Pi_1$, and then use Pythagoras' Theorem to find the required length ($\surd\{17-25/6\}$). The other is to find $\cos\alpha$, where $\alpha$ is the angle between the vector $\mathbf{i}+4\mathbf{j}$ and the normal to $\Pi_1$, and then find $\surd 17\sin\alpha$.

(*a*) 50, 9·9900; (*b*) 68·0 °, $\surd\{77/6\}$.

*Paper 9205/0*

*General Comments*

Much ingenuity was displayed by this year's candidates, and not only by the top performers. Build-up methods were applied in curve sketching. A variety of identities was brought into play to prove a trigonometrical result. Scalar products were used to show that one plane bisects the angle between two others. Different geometrical properties were used to show that a certain quadrilateral was a square. All candidates tried six questions, and some more, from which the best six marks were taken for their total.

Although some candidates appeared not to have been adequately prepared for a paper at this level, it is not only the best who appear to have gained some advantage from sitting it. From the time and space devoted to them, some questions seem to have spurred some candidates into much creative thought. This reaction reflects very favourably on their teachers, as well as on themselves.

In terms of marks, this has not been a particularly strong year, but neither have there been many disastrously low scores. The setting out of work has been careful, except that several new answers have not been started on a fresh page, as the rubric asks. Diagrams have been helpfully drawn and annotated. It is appreciated that sheets have not been fastened together too tightly. One possible improvement would be that, when a second attempt is made at a question, a note to this effect is added to the end of the first attempt.

*Comments on Individual Questions*

*Q.1* *(i)* This was only moderately popular, and no doubt some candidates were wary of limits. Many knew that the limit of $\dfrac{\sin\theta}{\theta}$ as $\theta\to 0$ is 1, but did not think of writing the given expression as $n\left(\dfrac{\sin 2n\theta}{2n\theta}\right)\left(\dfrac{\theta}{\sin\theta}\right)$, from which its limit is clearly $n$. Using the approximation $\sin\theta\approx\theta$ is not quite enough. In the second part, some found the correct limit without observing that the $n$ must be integral, but they received full credit nevertheless.

*(ii)* This popular part-question attracted many attempts, but some work was unnecessarily heavy; one use of the product/sum formula for sines does all that is required. The structure of proof by induction was well understood.

*(iii)* Some brave attempts here, and some successes. The differentiation was well managed, but the tidying-up was found to be taxing.

*(i)* $-n$.

*Q.2* This question was not only the most popular, but also the best answered, over half the candidates scoring 12 marks or more out of 16. A common fault was omitting a reason why $x = 4$ gives a maximum value of $A$, rather than a minimum. A reference to the graph would be sufficient, or a physical reason could be given.

*(ii)* $0 \leqslant x \leqslant 12(\sqrt{2} - 1)$.

*Q.3* Equally popular was this integration question. In *(i)*, many found the best substitution $x = \sin u$, some integrating by parts first. A great variety of correct answers was given, and the standard of integration was high, only spoilt by weak algebra. In *(ii)* the substitution $u = 1 + x^{\frac{1}{2}}$ worked well, and the simpler $u = x^{\frac{1}{2}}$ sufficed for those who did not fall into the trap of assuming that $\dfrac{2u}{(1+u)(1-u^2)}$ could be expressed as $\dfrac{A}{1+u} + \dfrac{Bu + C}{(1-u^2)}$. Part *(iii)* required a simple rearrangement.

*(i)* $(2x^2 - 1)\sin^{-1} x + x\sqrt{(1 - x^2)} + c$, *(ii)* $\frac{1}{2}\ln\left|\dfrac{1+\sqrt{x}}{1-\sqrt{x}}\right| + \dfrac{1}{1+\sqrt{x}} + c$,

*(iii)* $\ln|x + e^x| - x + c$.

*Q.4* Coordinate geometry does not seem to be as widely studied as other parts of the syllabus, but over a third of those who tried this question scored well. In *(iv)*, candidates revealed a variety of ways of showing that $RTSU$ is a square. The shortest was showing that the diagonals (already perpendicular) bisect each other and that $RT$ is perpendicular to $RU$.

*Q.5* Over 80% of the field went for this question, attracted by its familiar opening. Too little attention was paid to finding the stationary points in *(iii)*, and the inflexions at $x = \pi$ and $-\pi$ were not always clear. The functional implications of parts *(iv)* to *(vi)* were not often perceived. Even though $a$ was given to be positive, an answer such as $-1 \leqslant a \leqslant 1$ to *(iv)* received full credit.

*(i)* $(\frac{1}{2}\pi, a)$, $(-\frac{1}{2}\pi, -a)$, *(ii)* $(\pi/4, \sqrt{2}a)$, $(-3\pi/4, -\sqrt{2}a)$, *(iii)* $(-\pi, 0)$, $(-\pi/3, -3\sqrt{3}a/2)$, $(\pi/3, 3\sqrt{3}a/2)$, $(\pi, 0)$, *(iv)* $0 \leqslant a \leqslant 1$, *(v)* $0 \leqslant a \leqslant \frac{1}{2}\sqrt{2}$, *(vi)* $0 \leqslant a \leqslant 2\sqrt{3}/9$.

*Q.6* The easy start to this question was enticing, and those who eliminated $\alpha$ between their expressions for $H$ and $R$ found little to delay them. The time-wasters were those who embarked on the first paragraph without a clear idea of what they were being asked to do, and who brought in unnecessary extra variables.

Many went on to the second paragraph without noticing that $x$ was now being used differently from the text-book way. Full credit was allowed to those who said that the path is parabolic, and so is of the form $y = p + qx^2$, passing through $(0, H)$ and $(\pm R, 0)$. The last part proved troublesome for those who went for $dV/dH$ instead of $dV^2/dH$, and for those who thought that $H = a - \frac{1}{2}b$ must give a minimum $V$ because $H = a + \frac{1}{2}b$ is larger.

$\sqrt{(2g(a + b))}$.

*Q.7* Although there were several reasonable starts and finishes, part *(ii)* was often wrong because, as the question stressed, the plate's displacement is measured by $x + y$, and its acceleration by $\ddot{x} + \ddot{y}$. The reasoning and explaining in *(iv)* and *(v)* were both good, though in *(iv)* it was too rarely said that, while both objects are moving, $\ddot{x} < 0$ and $\ddot{y} > 0$ with $y_0 = 0$, so $\dot{x} = 0$ first.

*Q.8* The number of statistics questions answered in this paper is usually low, and this one received scant attention. The trickiest part was finding Var($S$), but an alternative approach was offered by arguing that, as $X_1, X_2, \ldots, X_k$ are mutually independent, $\mathrm{Var}(S) = \sum_{r=1}^{k} \mathrm{Var}(X_r)$, which leads to the given expression.

A continuity correction is needed in the last part.

0·0781.

*Q.9* Only a few answered this completely, the main difficulty being to provide a watertight argument that $p_n$ is the greatest probability. Many devices were used here and many were valid, but it is not enough to say that, since the expected number of faults is $n$, $p_r$ is greatest when $r = n$.

*Q.10* It was gratifying to see so many attempts at this question on complex numbers, often deemed unattractive. Parts *(i)* and *(ii)* were very well answered, but trouble began in *(iii)* with the sketching of the loci of $U$, $V$ and $W$. $U$ could be seen to lie on the circle $u - \frac{1}{2} = \frac{1}{2}e^{i\theta}$, but any similar curve was accepted. Of course $V$ lies on the straight line $x = 1$, a tangent to the circle, as most candidates correctly stated. $W$ lies on the parabola whose equation was found in *(ii)*, but not many got so far.

There was confusion in *(iv)* where it was not noticed that

$$\frac{v - w}{v} = 1 - v = \mathrm{i}\tan\left(\tfrac{1}{2}\theta\right),$$

although some eventually reached this conclusion after many lines of working. The last part of all follows from the fact that $\arg(z_1/z_2)$ = the angle between the vectors that represent $z_1$ and $z_2$ in the Argand diagram.

*(i)* $|v| = \sec(\tfrac{1}{2}\theta)$, $\arg(v) = -\tfrac{1}{2}\theta$, $|w| = \sec^2(\tfrac{1}{2}\theta)$, $\arg(w) = -\theta$.

## MATHEMATICS 'C' 9205

*Component Threshold Marks*

| Component | Maximum Mark | A(1,2) | B(3) | C(4) | D(5) | E(6) | N(7) | U(8) |
|---|---|---|---|---|---|---|---|---|
| 1 | 98 | 78 | 68 | 59 | 50 | 42 | 34 | 0 |
| 2 | 98 | 78 | 68 | 58 | 49 | 40 | 31 | 0 |

*Special Paper*

| | |
|---|---|
| 1 | 59 |
| 2 | 40 |

*Overall Threshold Marks*

| Combination | Maximum Mark | A | B | C | D | E | N | U |
|---|---|---|---|---|---|---|---|---|
| All | 196 | 154 | 135 | 117 | 99 | 82 | 65 | 0 |

The percentage of candidates awarded each grade was as follows:

| GRADE | A | B | C | D | E | N | U |
|---|---|---|---|---|---|---|---|
| Cumulative % | 24·9 | 40·0 | 53·7 | 67·6 | 79·1 | 87·8 | 100 |

The total candidature was 6341

These statistics are correct at the time of publication.