CAMBRIDGE
UNIVERSITY PRESS & ASSESSMENT

A Level

Mathematics

Session: 2000 June
Type: Question paper
Code: 9200

OCR
RECOGNISING ACHIEVEMENT

General Certificate of Education Advanced Level
former Cambridge linear syllabus

MATHEMATICS 9200/1

PAPER 1

Thursday **8 JUNE 2000** Afternoon 3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 7 printed pages and 1 blank page.

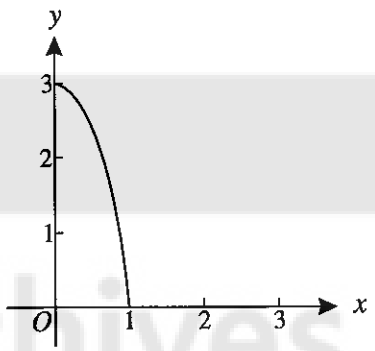
[Turn over

1 The circle C has equation $x^2 + (y - 1)^2 = 25$. Write down the radius of C and the coordinates of the centre of C. [2]

2 The graph of y against x is a straight line which has gradient 3 and passes through the point $(0, -1)$. Express y in terms of x. [1]

Given also that $y = \frac{t}{s^3}$ and $x = \frac{1}{s}$, express t in terms of s. [2]

3



The diagram shows the graph of $y = \mathrm{f}(x)$ for $0 \leqslant x \leqslant 1$. Sketch, on separate diagrams, the graphs of

(i) $y = \frac{1}{2}\mathrm{f}(x)$ for $0 \leqslant x \leqslant 1$, [1]

(ii) $y = -\mathrm{f}(x)$ for $0 \leqslant x \leqslant 1$, [1]

(iii) $y = \mathrm{f}^{-1}(x)$ for $0 \leqslant x \leqslant 3$. [1]

4 A sequence with nth term u_n is defined by $u_1 = 3$ and

$$u_{n+1} = u_n + 4, \quad \text{for } n \geqslant 1.$$

(i) Write down the values of u_2, u_3 and u_4. [1]

(ii) By identifying the type of sequence, or otherwise, express u_n in terms of n. [2]

(iii) The sequence v_n is defined by $v_n = \frac{1}{u_n}$. State, with a reason, whether the sequence v_n is convergent. [1]

5 Solve the inequality $|y - 1| < 6$. [2]

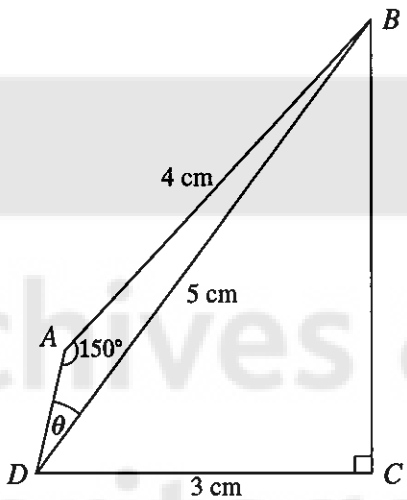
Hence solve the inequality $|2^x - 1| < 6$, expressing any numerical value in the form $\frac{\ln p}{\ln q}$, where p and q are integers. [3]

6 Show by calculation that the equation $2x - \tan x = 0$ has a root, α, between $x = 1$ and $x = 1.2$, where x is in radians. [2]

Taking $x = 1.1$ as a first approximation, use the Newton-Raphson method to obtain a second approximation to α, giving your answer correct to 2 decimal places. [3]

7 Use partial fractions to find $\int \frac{1}{x(x-2)}\,dx$. [5]

8



The diagram shows a plane quadrilateral $ABCD$, where $AB = 4$ cm, $CD = 3$ cm, angle $BAD = 150°$ and angle $BCD = 90°$. The diagonal BD has length 5 cm and angle ADB is denoted by θ.

(i) Find the exact value of $\sin\theta$. [2]

(ii) Hence find the exact value of $\cos\theta$. [2]

(iii) Write down the exact values of $\cos BDC$ and $\sin BDC$. [1]

(iv) Hence find the exact value of $\sin ADC$. [3]

9 **(a)** It is given that $4x^3 + 60x^2 + 299x + 495$ can be expressed in the form

$$A(x + B)^3 + C(x + B).$$

Find the integers A, B and C. [4]

(b) Solve the equation $9(y - 2)^3 - 4(y - 2) = 0$. [4]

[Turn over

10 Two signposts are located at the points $A(-3, 4, 0.1)$ and $B(1, 2, 0.5)$, where the origin O is at sea level and the z-axis is taken vertically upwards. Lengths are in kilometres.

(i) Find angle AOB in degrees, correct to 3 decimal places. [3]

(ii) The points A' and B' are at sea level and vertically below the points A and B respectively. Write down the coordinates of A' and B'. [1]

(iii) Show that the relative error which results when angle AOB is used as an approximation for angle $A'OB'$ is 0.66%, correct to 2 significant figures. [4]

11 **(i)** Express $7\sin\theta + 24\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $0° < \alpha < 90°$, stating the values of R and α. [2]

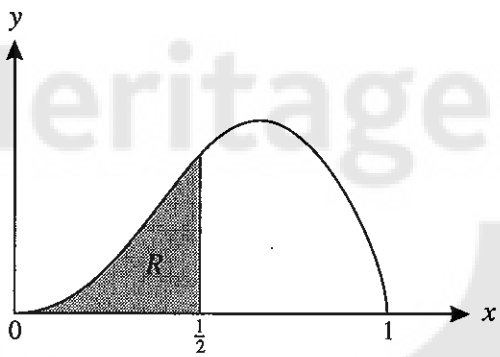
Solve the equation $7\sin\theta + 24\cos\theta = -3$, for θ in the interval $-90° < \theta < 270°$. [4]

(ii) Using your solutions from part **(i)**, or otherwise, solve the equations

(a) $7\sin(x - 10°) + 24\cos(x - 10°) = -3$, for x in the interval $0° < x < 180°$, [2]

(b) $7\cos y + 24\sin y = -3$, for y in the interval $0° < y < 180°$. [2]

12



The diagram shows the shaded region R enclosed between the curve $y = x^2\sqrt{(1 - x^2)}$, the x-axis, and the line $x = \frac{1}{2}$. The area A of this region is given by

$$A = \int_0^{\frac{1}{2}} x^2\sqrt{(1 - x^2)}\,dx.$$

By using the substitution $x = \sin\theta$, show that

$$A = \tfrac{1}{4}\int_0^{\frac{1}{6}\pi} \sin^2 2\theta\,d\theta.$$ [3]

Hence evaluate A exactly. [3]

Find the exact volume generated when the region R is rotated through four right angles about the x-axis. [4]

13 The straight line l has equation $2y - x + 7 = 0$. The straight line l' passes through the point $P(-1, 6)$ and is perpendicular to l.

(i) Find the equation of l', giving your answer in the form $ax + by + c = 0$. [3]

(ii) Find the coordinates of the point of intersection of l and l'. [2]

(iii) Show that the perpendicular distance from P to l is $4\sqrt{5}$. [2]

(iv) It is given that the points $Q(-7, -7)$ and $R(9, 1)$ lie on l. Find the exact area of the triangle PQR. [3]

14 During a chemical reaction, the mass m of one of the chemicals involved decreases at a rate which is proportional to m. Express this information as a differential equation involving m and the time t. [2]

When $t = 0$ the mass of the chemical is m_0. Show by integration that the solution of the differential equation is

$$m = m_0 e^{-kt},$$

where k is a positive constant. [4]

A sketch of the graph of $m = m_0 e^{-kt}$ is shown in the diagram. The mass M of another chemical involved in the reaction varies in such a way that $m + M = A$, where A is a constant.

(i) Sketch the graph of M against t. [2]

(ii) Describe what happens to the value of M as t becomes large. [1]

(iii) Show that $\dfrac{dM}{dt} = k(A - M)$. [2]

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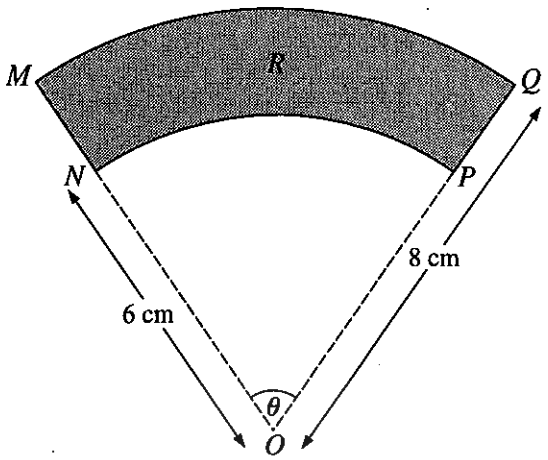

The diagram shows a sector OMQ of the circle with centre O and radius 8 cm. An arc of the circle with centre O and radius 6 cm joins the points N on OM and P on OQ. Angle NOP is θ radians, where $0 < \theta < \pi$, and the area of the shaded region R between the two arcs is $A\,\text{cm}^2$. Express A in terms of θ. [2]

It is given that θ is increasing at a rate of $0.1\,\text{rad}\,\text{s}^{-1}$.

(i) Find the rate of increase of A with respect to time. [2]

(ii) Find the rate of increase of the perimeter of the region R with respect to time. [2]

(iii) The length of the straight line NQ is L cm. Show that

$$L^2 = 100 - 96\cos\theta.$$ [2]

Hence find the rate of increase of L with respect to time when $\theta = \frac{1}{3}\pi$. [5]

16

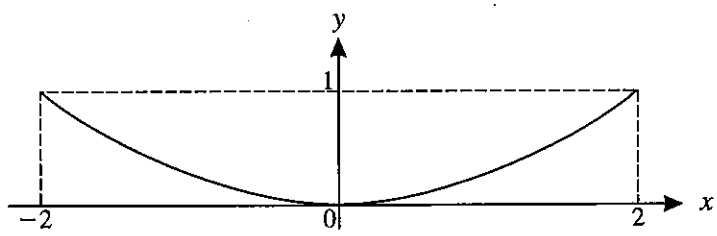

Fig. 1

A suspended chain is represented by a curve C which passes through the origin and the points $(-2, 1)$ and $(2, 1)$, as shown in Fig. 1.

(i) Various possible equations for C are to be considered.

(a) The first equation to be considered is $y = px^2 + qx^4$, where p and q are constants. Write down an equation satisfied by p and q. [1]

Hence show that, for any suitable curve of the form $y = px^2 + qx^4$, the gradient when $x = \sqrt{2}$ is $\frac{1}{2}\sqrt{2}$. [3]

(b) The second equation to be considered is $y = \dfrac{ax^2}{1 - \frac{1}{9}x^2}$, where a is a constant. Find the value of a. [1]

Find the series expansion of y up to and including the term in x^4. [2]

(c) The third equation to be considered is $y = k(e^x + e^{-x} - 2)$, where k is a constant. Find the value of k, giving your answer correct to 3 significant figures. [1]

By using the standard series, find the series expansion of y in this case, up to and including the term in x^4, expressing the coefficients in terms of k. [2]

(ii)

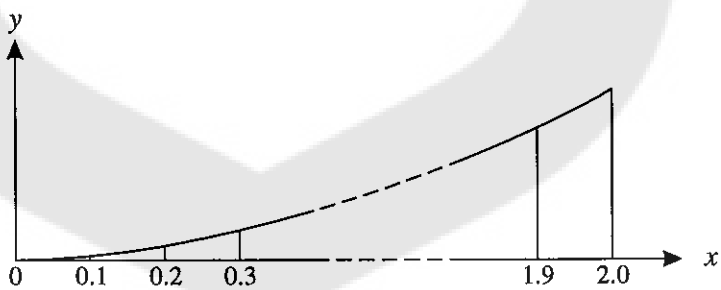

Fig. 2

Wires are suspended vertically from the chain, for $0 \leqslant x \leqslant 2$, at horizontal intervals of 0.1 (see Fig. 2). The length of the wire suspended from the point (x, y) on the curve is y, for $x = 0.1, 0.2, 0.3, \ldots, 1.9, 2.0$. The total length T of the wires is to be found using the equation $y = k(e^x + e^{-x} - 2)$ from part **(i) (c)**. Show that

$$T = k\left[\sum_{r=1}^{20} e^{0.1r} + \sum_{r=1}^{20} e^{-0.1r} - 40\right]. \quad [2]$$

By summing the progressions, calculate the value of T using the value of k found in part **(i) (c)**. [3]

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MATHEMATICS 9200/2

PAPER 2

Tuesday **13 JUNE 2000** Afternoon 3 hours

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[Turn over

Section (a): Pure Mathematics

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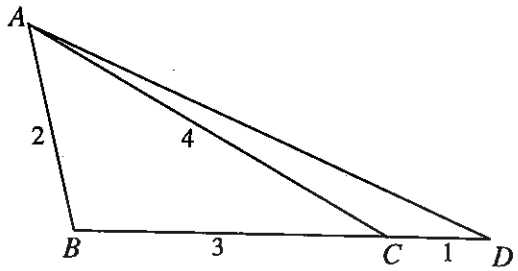
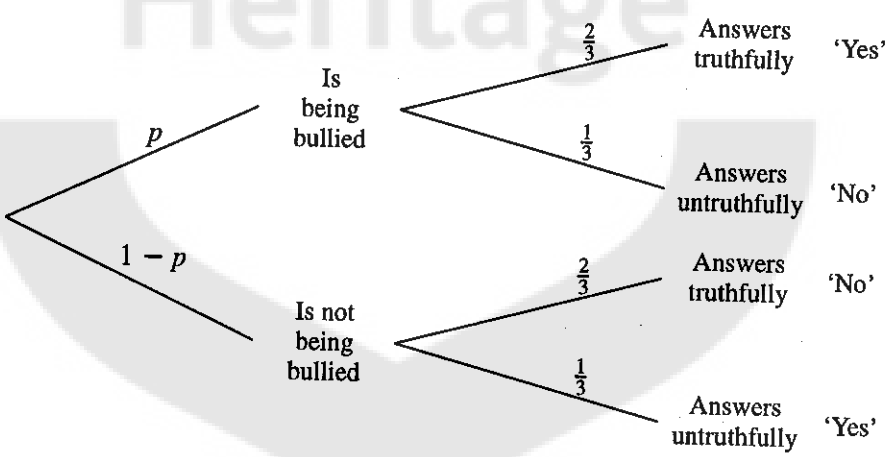
2



In the diagram, BCD is a straight line and $AB = 2\,\text{cm}$, $BC = 3\,\text{cm}$, $CA = 4\,\text{cm}$, $CD = 1\,\text{cm}$. Calculate the length of AD. [4]

3 A researcher is investigating the proportion p of children who are being bullied at school. To overcome any reluctance children might have to answering questions about being bullied, the following procedure is used. The researcher asks 'Are you being bullied at school?'. Before answering, the child being interviewed throws an unbiased die (unseen by the researcher); if the score on the die is 1, 2, 3 or 4 the child answers the question truthfully and if the score is 5 or 6 the child answers untruthfully. This procedure is illustrated in the tree diagram below.



(i) Show that the probability that a child answers 'Yes' to the researcher's question is $\frac{1}{3}(1+p)$. [3]

(ii) The researcher finds that 35% of children, on average, answer 'Yes'. Calculate the value of p. [1]

Hence find the conditional probability that a child who answers 'No' is answering truthfully. [3]

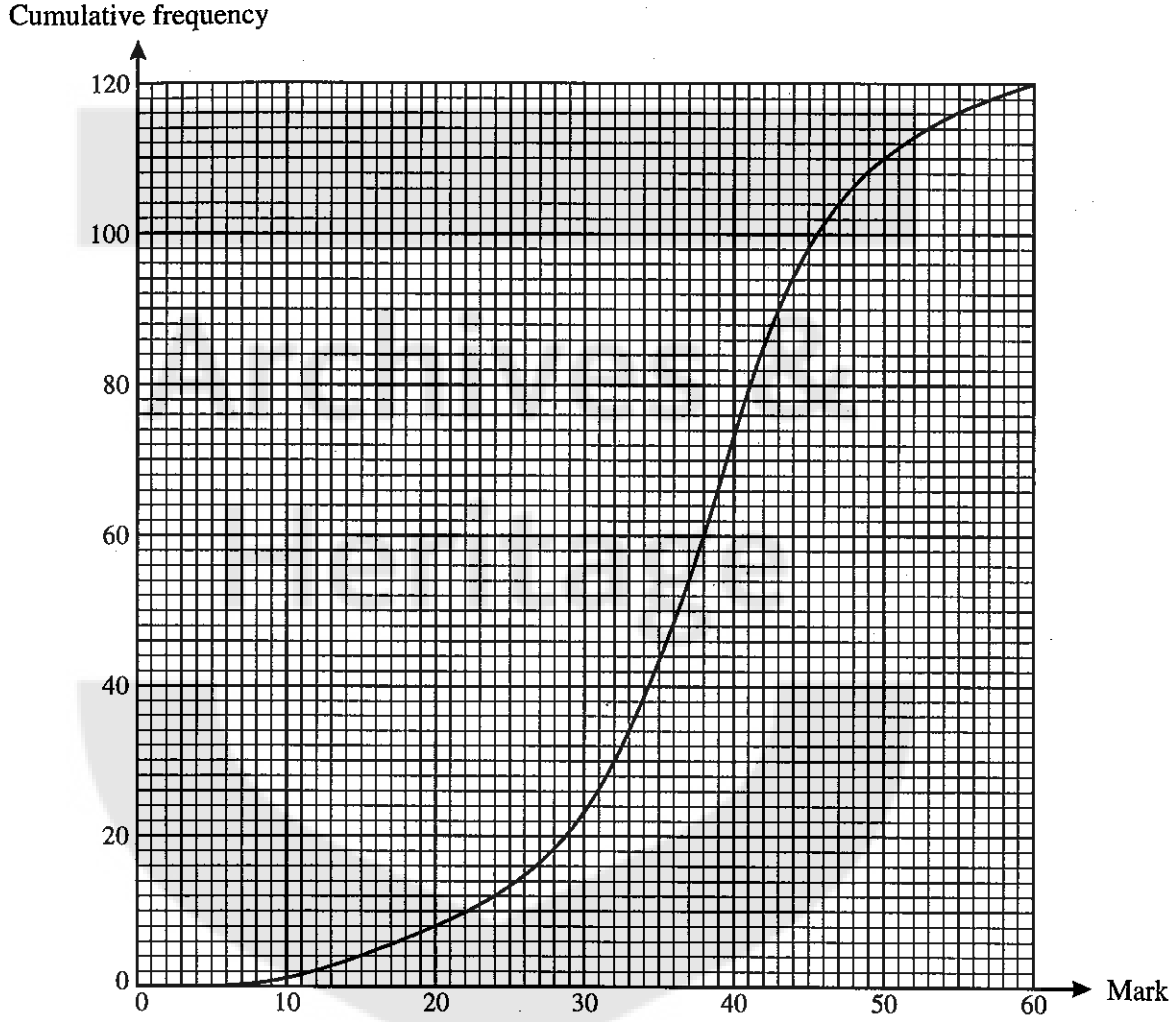
4 **(a)** For each of the following sets of data, state a type of statistical diagram that gives a suitable representation.

(i) The proportions of the electorate that supported various political parties at an election. [1]

(ii) The ages, in completed years at the times of their marriage, of women who married for the first time during 1999. [1]

(iii) The numbers of boys and the numbers of girls achieving grades A, B, C, D, E, N, U in A level Mathematics in 1999. [1]

(b)



The marks scored (out of 60) by the 120 candidates in an examination are illustrated by the cumulative frequency graph in the diagram.

(i) Estimate the median mark and the interquartile range. [4]

(ii) The best candidate scored 60 and the worst candidate scored 5. Illustrate the data by means of a box-and-whisker plot. [3]

Section (b): Mechanics

5 A ball is thrown vertically upwards, from ground level, with speed $15\,\mathrm{m\,s^{-1}}$ at time $t = 0$. Air resistance to the motion of the ball may be neglected. Find the values of t when the ball is 6 m above ground level. [4]

6 A racing car is driven on a straight horizontal test track as follows. Starting from rest, the car accelerates for 10 s up to a speed of $40\,\mathrm{m\,s^{-1}}$. The car then immediately slows down for 5 s, its speed dropping from $40\,\mathrm{m\,s^{-1}}$ to $30\,\mathrm{m\,s^{-1}}$. Finally, the car comes to rest after another 5 s, so that the whole of the motion lasts for 20 s.

(i) Assume that the acceleration or deceleration in each of the three stages of the motion is constant. Sketch a (t, v) graph to illustrate the motion, and calculate the total distance travelled by the car. [4]

(ii) In practice, the acceleration of the car during the first 10 s of its motion is unlikely to be constant. Assuming that the driving force from the car's engine decreases as the car's speed increases, sketch a possible shape for the (t, v) graph for the first 10 s of the motion. [1]

7 Two railway trucks A and B are moving in the same direction on the same straight horizontal track, and they collide. Truck A has mass 20 000 kg and truck B has mass 10 000 kg, and immediately before the collision their speeds are $1.5\,\mathrm{m\,s^{-1}}$ and $1\,\mathrm{m\,s^{-1}}$ respectively. When the trucks collide they are automatically coupled together. Find the speed of the trucks immediately after they collide. [3]

After they collide, a braking mechanism exerts a resisting force of magnitude 5000 N on the leading truck B, and the trucks slow down. Calculate

(i) the deceleration of the trucks, [2]

(ii) the magnitude of the force exerted on the leading truck B by the second truck A while the trucks are decelerating. [2]

8

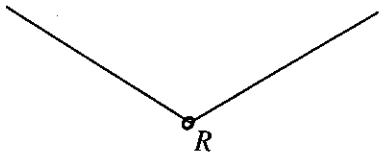


Fig. 1

Fig. 1 shows a small smooth ring R, of weight W, threaded on a light inextensible string. The ends of the string are attached to fixed points, and in the equilibrium position the tension in the string is $\frac{2}{3}W$. Calculate the angle that each part of the string makes with the vertical. [2]

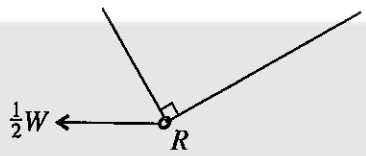


Fig. 2

A horizontal force of magnitude $\frac{1}{2}W$ is now applied to the ring. The line of action of this force is in the vertical plane containing the string. In the new equilibrium position the two parts of the string are at right angles to each other, as shown in Fig. 2. Calculate the angles that the two parts of the string make with the vertical and find the tension in the string. [7]

9 A particle is projected from a point O with speed V at an angle θ above the horizontal. Prove that the range R on the horizontal plane through O is given by

$$R = \frac{V^2 \sin 2\theta}{g}.$$ [3]

State any assumption that you have made in establishing this result. [1]

A gun fires a shell at a target which is at the same horizontal level as the gun and at a distance of D metres from the gun. The speed of projection of the shell is $U\,\mathrm{m\,s^{-1}}$. When the angle of elevation at which the shell is fired is 30° the shell falls short of the target by a horizontal distance of 100 m. When the angle of elevation is 45° the shell overshoots the target by a horizontal distance of 100 m.

(i) Show that $U = 121$, correct to 3 significant figures. [3]

(ii) Find D, and hence calculate the angle of elevation, between 30° and 45°, required for the shell to hit the target. [3]

10 Particles P and Q, of masses 0.3 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string.

(a)

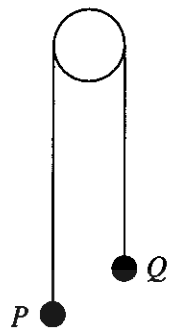


Fig. 1

The string passes over a fixed smooth peg, as shown in Fig. 1. The system is released from rest with the parts of the string between the peg and the particles taut and vertical. In the subsequent motion air resistance may be neglected. Find

(i) the tension in the string, [4]

(ii) the time taken for the particles to reach a speed of $4\,\text{m s}^{-1}$. [3]

(b)

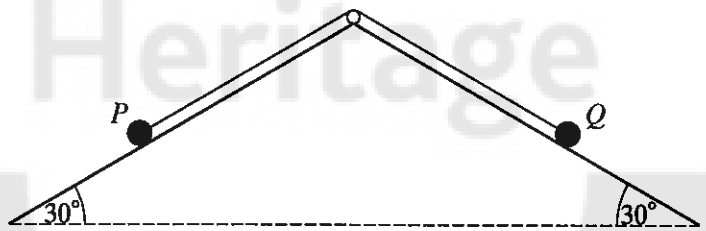


Fig. 2

The string passes over a fixed smooth peg with the particles P and Q on planes inclined at 30° to the horizontal, as shown in Fig. 2. The planes are equally rough and the coefficient of friction between each particle and the corresponding plane is μ. The system is released from rest with the parts of the string between the peg and the particles taut and parallel to lines of greatest slope. All resistances to motion, apart from friction between the particles and the planes, are negligible. Given that the particles move, show that

$$\mu < \frac{1}{5\sqrt{3}}.$$ [6]

Section (c): Statistics

11 A shopkeeper who owns a small shop wishes to find out what customers think about the goods that she sells. She has been advised that she should take a *random* sample of her customers for this purpose. State, with reasons, which of the following sampling procedures is preferable.

A: Select every 10th customer on each day in a typical week.

B: Select the first 20 customers on each day in a typical week. [2]

Give a reason why the shopkeeper should perhaps not restrict the sample to her present customers. [1]

12 The discrete random variable X takes values 0, 1, 2 only. The probability distribution of X is shown in the table, where p is a constant ($0 < p < \frac{1}{3}$).

x	0	1	2
$P(X = x)$	$1 - 3p$	$2p$	p

Given that $\text{Var}(X) = \frac{1}{2}$, find the two possible values of $E(X)$. [6]

13 In a certain country 12% of school leavers have no educational qualifications. In a random sample of 50 school leavers, X denotes the number who have no educational qualifications.

(i) Calculate $P(X \leqslant 2)$, using a binomial distribution. [3]

(ii) Calculate $P(X \geqslant 10)$, using a suitable approximation. [4]

14 The daily amount of petrol sold at a garage may be modelled by a normal distribution with mean μ litres and standard deviation σ litres. The amount sold, x litres, was recorded on each day of a random sample of 75 days, and the data is summarised (correct to 4 significant figures) by

$$\Sigma x = 6.675 \times 10^5, \quad \Sigma x^2 = 5.978 \times 10^9.$$

Calculate, correct to 3 significant figures,

(i) unbiased estimates of μ and σ^2, [3]

(ii) a symmetric 95% confidence interval for μ. [3]

Assume that the values found in part **(i)** can be taken to be the true values of μ and σ^2. Find the value of a such that at least a litres of petrol are sold on 90% of days on average. Give your answer correct to 3 significant figures. [3]

[Turn over

15 In safety tests, the distance required to bring a car to a stop was measured for various speeds of the car. The table shows corresponding values of the car's speed, v miles per hour, and the stopping distance, d metres.

v	20	30	40	50	60	70
d	13	22	37	51	72	95

[$\Sigma v = 270$, $\Sigma d = 290$, $\Sigma v^2 = 13\,900$, $\Sigma d^2 = 18\,832$, $\Sigma vd = 15\,920$.]

Calculate the product moment correlation coefficient between v and d for the data. [3]

What information does the value of this coefficient provide about a possible relationship between v and d? [1]

Calculate the equation of the regression line of d on v, giving your answer in the form $d = a + bv$, with the constants a and b correct to 3 significant figures. [2]

Plot the data, together with this regression line, on graph paper. Use scales of 2 cm for 10 miles per hour for v and 2 cm for 20 metres for d. [3]

Use the regression line to estimate the distance required to stop a car travelling at 55 miles per hour. [1]

Comment briefly on the suitability of using the regression line to estimate this distance. [1]

16 A darts player aims repeatedly for the bull's-eye. He counts it a success if his dart hits the bull's-eye. It may be assumed that, on each throw, the probability of success is $\frac{1}{4}$, independently of all other throws. Find the probability that, in a series of throws,

(i) the first success occurs on the fourth throw, [2]

(ii) the first success occurs between the fourth and eighth throws (inclusive), [2]

(iii) the second success occurs on the third throw. [3]

The player keeps a record of the number of throws, X, up to and including the first one to hit the bull's-eye. The mean of a random sample of 60 values of X is denoted by $\bar{X}$. Find $P(\bar{X} > 5)$. [5]

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MATHEMATICS 9200/3
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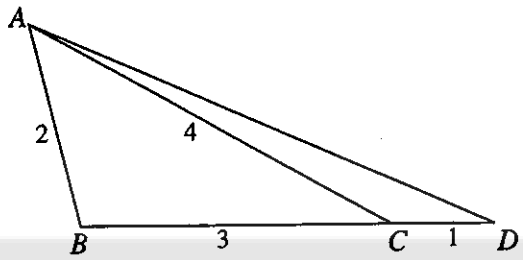
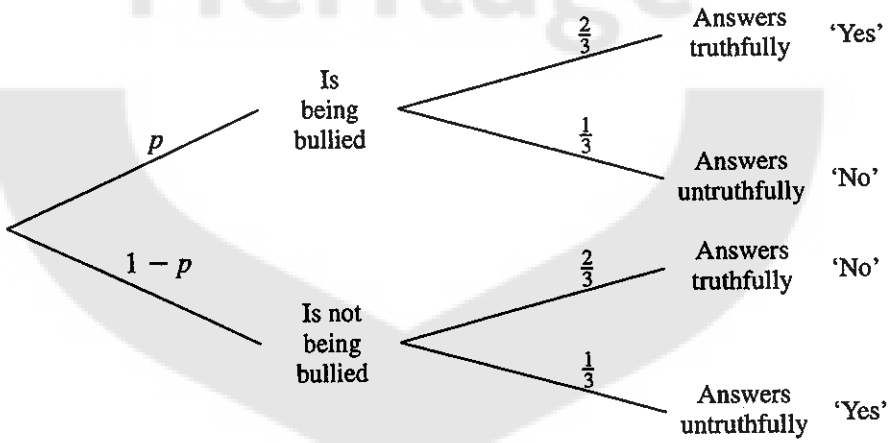


In the diagram, BCD is a straight line and $AB = 2$ cm, $BC = 3$ cm, $CA = 4$ cm, $CD = 1$ cm. Calculate the length of AD. [4]

3 A researcher is investigating the proportion p of children who are being bullied at school. To overcome any reluctance children might have to answering questions about being bullied, the following procedure is used. The researcher asks 'Are you being bullied at school?'. Before answering, the child being interviewed throws an unbiased die (unseen by the researcher); if the score on the die is 1, 2, 3 or 4 the child answers the question truthfully and if the score is 5 or 6 the child answers untruthfully. This procedure is illustrated in the tree diagram below.



(i) Show that the probability that a child answers 'Yes' to the researcher's question is $\frac{1}{3}(1+p)$. [3]

(ii) The researcher finds that 35% of children, on average, answer 'Yes'. Calculate the value of p. [1]

Hence find the conditional probability that a child who answers 'No' is answering truthfully. [3]

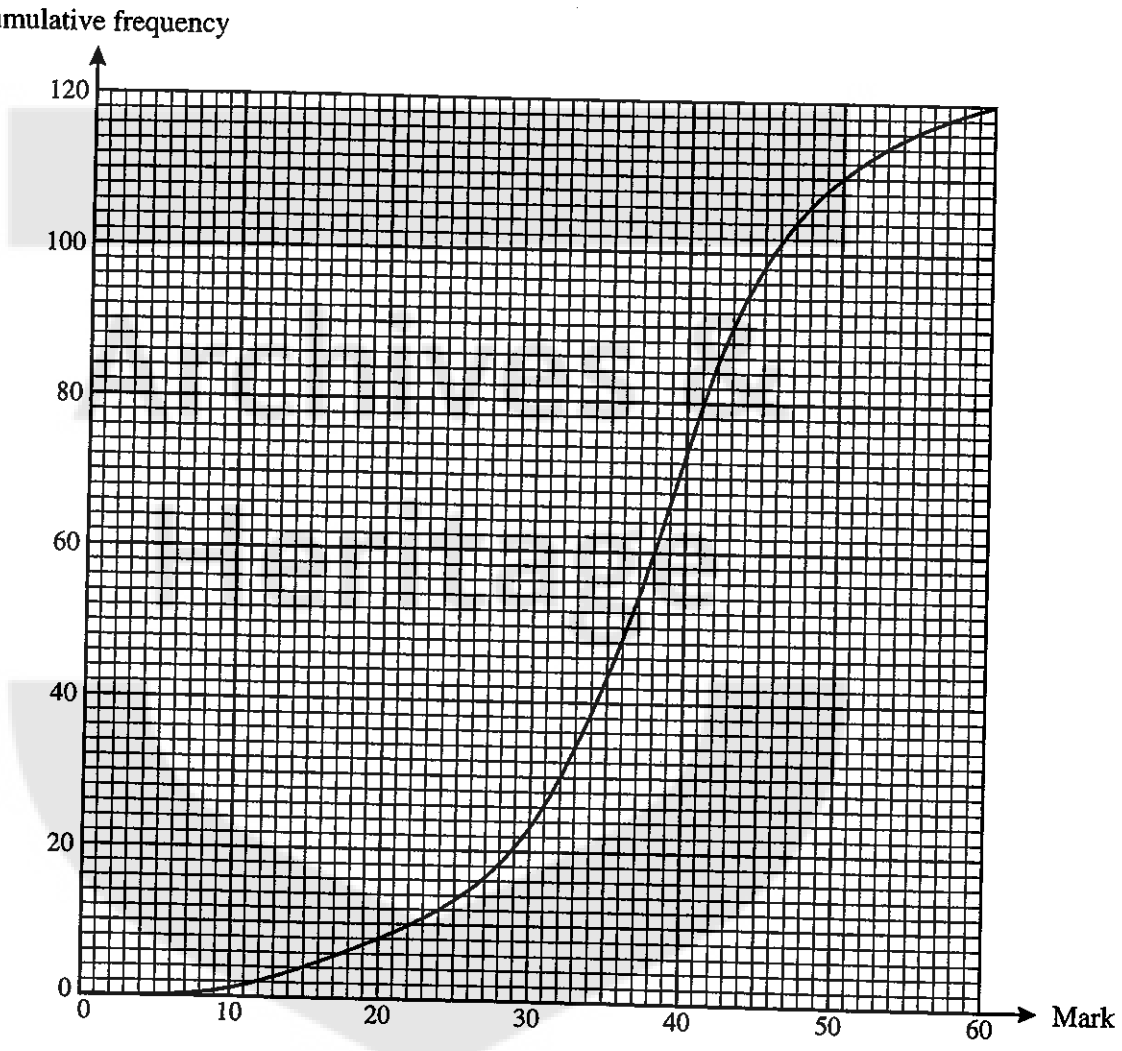
4 **(a)** For each of the following sets of data, state a type of statistical diagram that gives a suitable representation.

(i) The proportions of the electorate that supported various political parties at an election. [1]

(ii) The ages, in completed years at the times of their marriage, of women who married for the first time during 1999. [1]

(iii) The numbers of boys and the numbers of girls achieving grades A, B, C, D, E, N, U in A level Mathematics in 1999. [1]

(b)



The marks scored (out of 60) by the 120 candidates in an examination are illustrated by the cumulative frequency graph in the diagram.

(i) Estimate the median mark and the interquartile range. [4]

(ii) The best candidate scored 60 and the worst candidate scored 5. Illustrate the data by means of a box-and-whisker plot. [3]

Section (b): Mechanics

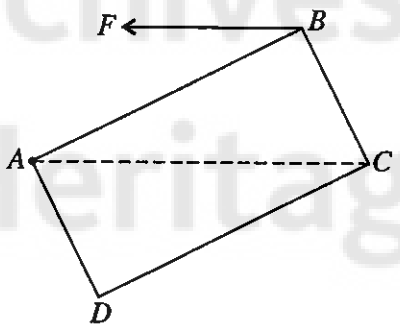
5 A ball is thrown vertically upwards, from ground level, with speed $15\,\mathrm{m\,s^{-1}}$ at time $t = 0$. Air resistance to the motion of the ball may be neglected. Find the values of t when the ball is 6 m above ground level. [4]

6 A racing car is driven on a straight horizontal test track as follows. Starting from rest, the car accelerates for 10 s up to a speed of $40\,\mathrm{m\,s^{-1}}$. The car then immediately slows down for 5 s, its speed dropping from $40\,\mathrm{m\,s^{-1}}$ to $30\,\mathrm{m\,s^{-1}}$. Finally, the car comes to rest after another 5 s, so that the whole of the motion lasts for 20 s.

(i) Assume that the acceleration or deceleration in each of the three stages of the motion is constant. Sketch a (t, v) graph to illustrate the motion, and calculate the total distance travelled by the car. [4]

(ii) In practice, the acceleration of the car during the first 10 s of its motion is unlikely to be constant. Assuming that the driving force from the car's engine decreases as the car's speed increases, sketch a possible shape for the (t, v) graph for the first 10 s of the motion. [1]

7



A uniform rectangular lamina $ABCD$ has weight W. The length of AB is 2 units and the length of BC is 1 unit. The lamina is free to rotate, in a vertical plane, about a fixed smooth horizontal axis through A. The lamina is held in equilibrium with AC horizontal by means of a horizontal force of magnitude F acting at B. The line of action of this force is in the plane of the lamina (see diagram).

(i) Find F in terms of W. [3]

(ii) Find the magnitude, in terms of W, of the force acting on the lamina at A. Find also the angle which this force makes with the line AC. [3]

8 Two railway trucks A and B are moving in the same direction on the same straight horizontal track, and they collide. Truck A has mass 20 000 kg and truck B has mass 10 000 kg, and immediately before the collision their speeds are $1.5\,\mathrm{m\,s^{-1}}$ and $1\,\mathrm{m\,s^{-1}}$ respectively. When the trucks collide they are automatically coupled together. Find the speed of the trucks immediately after they collide. [3]

After they collide, a braking mechanism exerts a resisting force of magnitude 5000 N on the leading truck B, and the trucks slow down. Calculate

(i) the deceleration of the trucks, [2]

(ii) the magnitude of the force exerted on the leading truck B by the second truck A while the trucks are decelerating. [2]

9 The unstretched length of a light elastic string is 0.6 m. One end of the string is attached to a fixed point O, and the other end is attached to a particle of mass 0.5 kg. When the particle hangs freely in equilibrium the total length of the string is 1.8 m. Calculate the modulus of elasticity of the string. [2]

The particle is released from rest at O, and in the subsequent motion air resistance may be ignored. Find the speed with which the particle passes through the equilibrium position. [5]

10

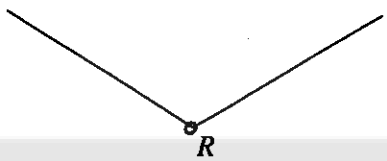


Fig. 1

Fig. 1 shows a small smooth ring R, of weight W, threaded on a light inextensible string. The ends of the string are attached to fixed points, and in the equilibrium position the tension in the string is $\frac{2}{3}W$. Calculate the angle that each part of the string makes with the vertical. [2]

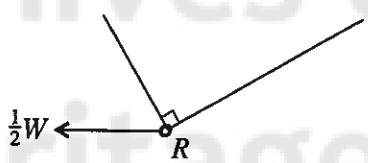


Fig. 2

A horizontal force of magnitude $\frac{1}{2}W$ is now applied to the ring. The line of action of this force is in the vertical plane containing the string. In the new equilibrium position the two parts of the string are at right angles to each other, as shown in Fig. 2. Calculate the angles that the two parts of the string make with the vertical and find the tension in the string. [7]

11 A particle is projected from a point O with speed V at an angle θ above the horizontal. Prove that the range R on the horizontal plane through O is given by

$$R = \frac{V^2 \sin 2\theta}{g}.$$ [3]

State any assumption that you have made in establishing this result. [1]

A gun fires a shell at a target which is at the same horizontal level as the gun and at a distance of D metres from the gun. The speed of projection of the shell is $U\,\mathrm{m\,s^{-1}}$. When the angle of elevation at which the shell is fired is 30° the shell falls short of the target by a horizontal distance of 100 m. When the angle of elevation is 45° the shell overshoots the target by a horizontal distance of 100 m.

(i) Show that $U = 121$, correct to 3 significant figures. [3]

(ii) Find D, and hence calculate the angle of elevation, between 30° and 45°, required for the shell to hit the target. [3]

12

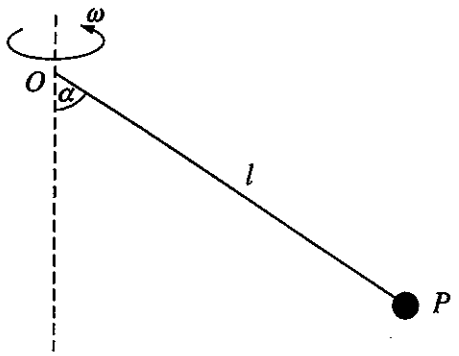


Fig. 1

A particle P of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point O. The particle moves, with constant angular speed ω, in a horizontal circle whose centre is vertically below O. The string OP makes a constant angle α with the vertical (see Fig. 1). Show that

$$\cos\alpha = \frac{g}{\omega^2 l}.$$ [4]

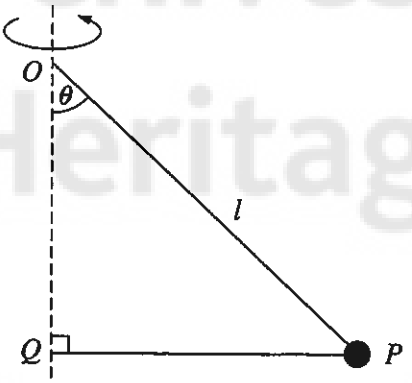


Fig. 2

The particle P is now joined to a point Q on the axis of rotation by a light elastic string of natural length $\frac{1}{2}l$ and modulus of elasticity mg. The particle moves, with constant angular speed and with the string PQ taut, in a horizontal circle with centre Q. The string OP makes a constant angle θ with the vertical (see Fig. 2).

(i) Use Hooke's law to express the tension in the string PQ in terms of m, g and θ. [2]

(ii) Given that the angular speed is $\sqrt{\left(\frac{2g}{l}\right)}$, find θ. [4]

13

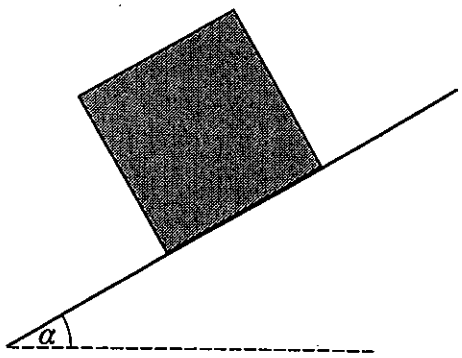


Fig. 1

A uniform cubical packing-case of weight W rests in equilibrium on a rough plane which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. The edges of the packing-case are parallel or perpendicular to a line of greatest slope of the plane (see Fig. 1). The coefficient of friction between the packing-case and the plane is μ. Show that $\mu \geqslant \frac{3}{4}$. [2]

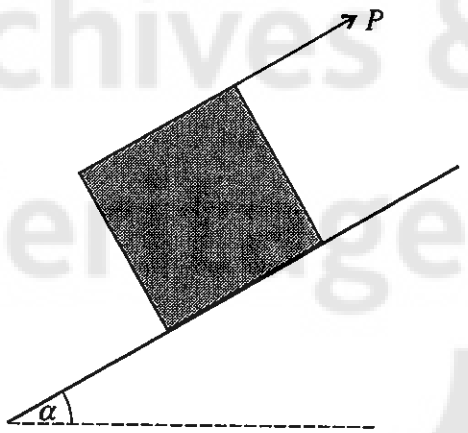


Fig. 2

A force of magnitude P, parallel to a line of greatest slope and acting upwards, is now applied at the centre of the top edge of the packing-case (see Fig. 2). Assuming that equilibrium is about to be broken by sliding, show that $P = \frac{1}{5}(3 + 4\mu)W$. [3]

Assuming instead that the packing-case is about to topple up the plane, show that $P = \frac{7}{10}W$. [3]

Deduce that if P is gradually increased from zero, then equilibrium is broken by toppling and not by sliding. [2]

14 Particles P and Q, of masses 0.3 kg and 0.2 kg respectively, are attached to the ends of a light inextensible string.

(a)

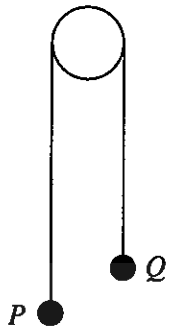


Fig. 1

The string passes over a fixed smooth peg, as shown in Fig. 1. The system is released from rest with the parts of the string between the peg and the particles taut and vertical. In the subsequent motion air resistance may be neglected. Find

(i) the tension in the string, [4]

(ii) the time taken for the particles to reach a speed of $4\,\text{m s}^{-1}$. [3]

(b)

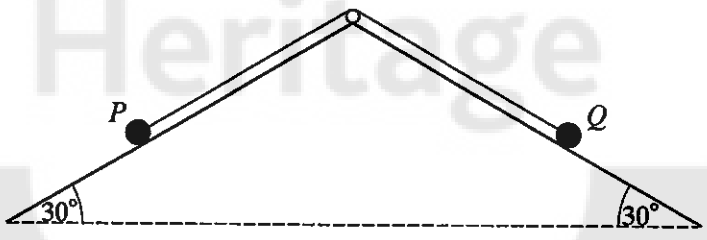


Fig. 2

The string passes over a fixed smooth peg with the particles P and Q on planes inclined at 30° to the horizontal, as shown in Fig. 2. The planes are equally rough and the coefficient of friction between each particle and the corresponding plane is μ. The system is released from rest with the parts of the string between the peg and the particles taut and parallel to lines of greatest slope. All resistances to motion, apart from friction between the particles and the planes, are negligible. Given that the particles move, show that

$$\mu < \frac{1}{5\sqrt{3}}.$$ [6]

15 [In parts **(iii)** and **(iv)** of this question the following indefinite integrals may be used without proof; a and b denote positive constants, and c is an arbitrary constant.

$$\int \frac{v}{a-bv}\,\mathrm{d}v = -\frac{v}{b} - \frac{a}{b^2}\ln|a-bv| + c,$$

$$\int \frac{v^2}{a-bv}\,\mathrm{d}v = -\frac{v^2}{2b} - \frac{av}{b^2} - \frac{a^2}{b^3}\ln|a-bv| + c.]$$

A car of mass 400 kg moves on a straight horizontal road. The engine of the car has a constant power of 10 kW, and the resistance to motion is a constant force of magnitude 200 N.

(i) Calculate the acceleration of the car at an instant when its speed is $25\,\mathrm{m\,s^{-1}}$. [3]

(ii) Estimate the time taken for the car to accelerate from $20\,\mathrm{m\,s^{-1}}$ to $30\,\mathrm{m\,s^{-1}}$ by assuming a constant acceleration, using the value found in part **(i)**. [1]

(iii) By solving an appropriate differential equation, show that the time taken for the car to accelerate from $20\,\mathrm{m\,s^{-1}}$ to $30\,\mathrm{m\,s^{-1}}$ is 20.5 s, correct to 3 significant figures. [4]

(iv) By solving an appropriate differential equation, find, correct to 3 significant figures, the distance travelled by the car as it accelerates from $20\,\mathrm{m\,s^{-1}}$ to $30\,\mathrm{m\,s^{-1}}$. [3]

(v) The distance, s metres, calculated in part **(iv)** and the time, t seconds, calculated in part **(iii)** are related by

$$50t = 500 + s.$$

Prove this result, independently of your previous calculations, by considering work and kinetic energy. [4]

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OCR
RECOGNISING ACHIEVEMENT

General Certificate of Education Advanced Level
former Cambridge linear syllabus

MATHEMATICS **9200/4**
PAPER 4

Tuesday **13 JUNE 2000** Afternoon 3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

Within each section of the paper, questions are printed in the order of their mark allocations and candidates are advised, within each section, to attempt questions sequentially.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

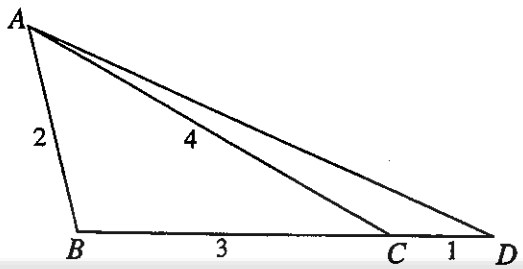
This question paper consists of 6 printed pages and 2 blank pages.

[Turn over

Section (a): Pure Mathematics

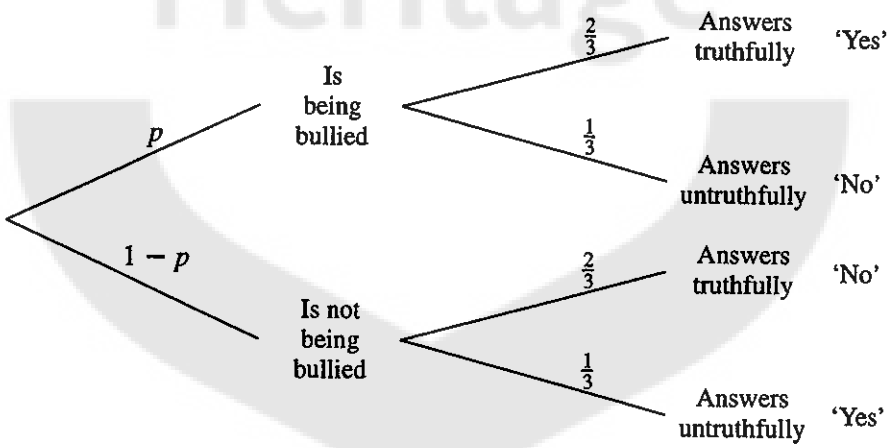
1 Differentiate $\frac{x-2}{2x+1}$ with respect to x, simplifying your answer. [3]

2



In the diagram, BCD is a straight line and $AB = 2$ cm, $BC = 3$ cm, $CA = 4$ cm, $CD = 1$ cm. Calculate the length of AD. [4]

3 A researcher is investigating the proportion p of children who are being bullied at school. To overcome any reluctance children might have to answering questions about being bullied, the following procedure is used. The researcher asks 'Are you being bullied at school?'. Before answering, the child being interviewed throws an unbiased die (unseen by the researcher); if the score on the die is 1, 2, 3 or 4 the child answers the question truthfully and if the score is 5 or 6 the child answers untruthfully. This procedure is illustrated in the tree diagram below.



(i) Show that the probability that a child answers 'Yes' to the researcher's question is $\frac{1}{3}(1+p)$. [3]

(ii) The researcher finds that 35% of children, on average, answer 'Yes'. Calculate the value of p. [1]

Hence find the conditional probability that a child who answers 'No' is answering truthfully. [3]

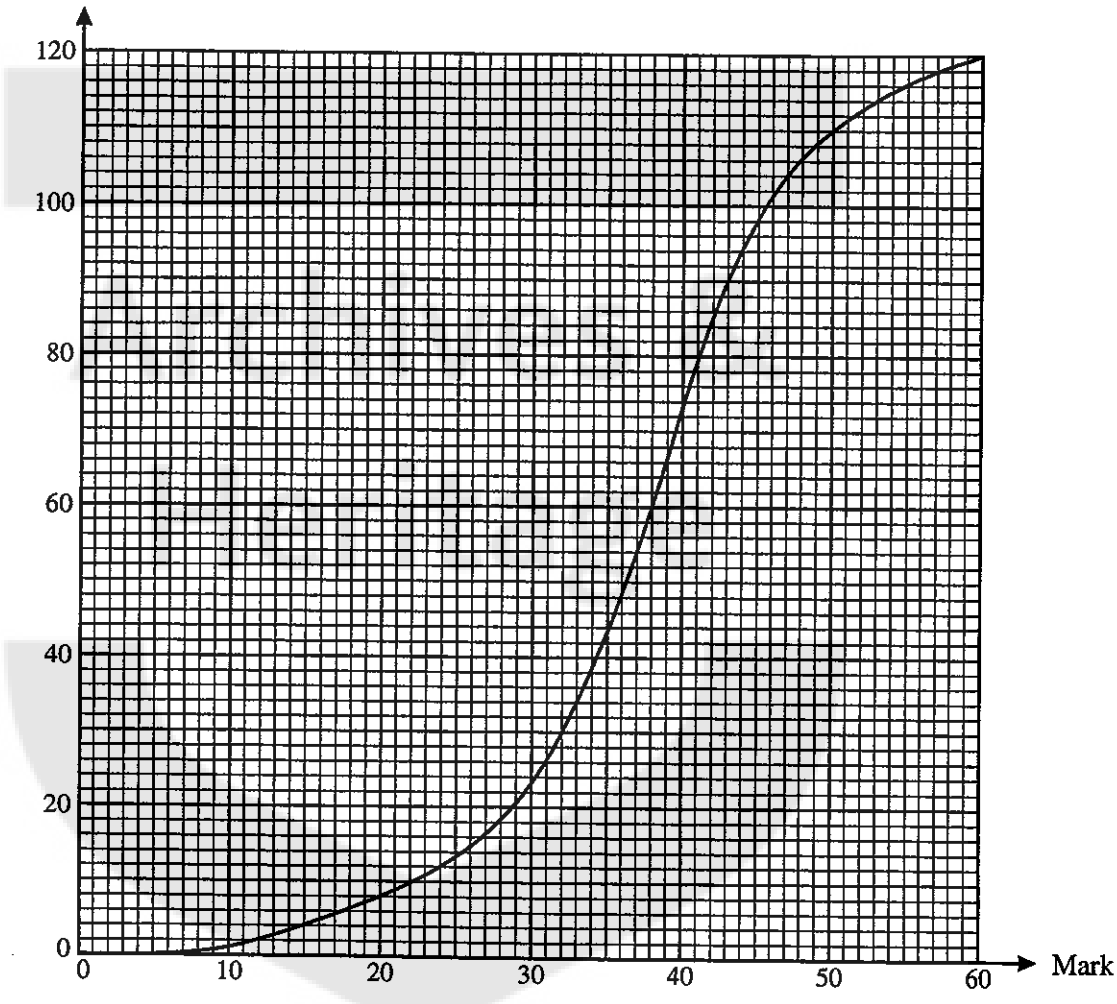
4 **(a)** For each of the following sets of data, state a type of statistical diagram that gives a suitable representation.

(i) The proportions of the electorate that supported various political parties at an election. [1]

(ii) The ages, in completed years at the times of their marriage, of women who married for the first time during 1999. [1]

(iii) The numbers of boys and the numbers of girls achieving grades A, B, C, D, E, N, U in A level Mathematics in 1999. [1]

(b)



The marks scored (out of 60) by the 120 candidates in an examination are illustrated by the cumulative frequency graph in the diagram.

(i) Estimate the median mark and the interquartile range. [4]

(ii) The best candidate scored 60 and the worst candidate scored 5. Illustrate the data by means of a box-and-whisker plot. [3]

Section (b): Statistics

5 A shopkeeper who owns a small shop wishes to find out what customers think about the goods that she sells. She has been advised that she should take a *random* sample of her customers for this purpose. State, with reasons, which of the following sampling procedures is preferable.

A: Select every 10th customer on each day in a typical week.

B: Select the first 20 customers on each day in a typical week. [2]

Give a reason why the shopkeeper should perhaps not restrict the sample to her present customers. [1]

6 The discrete random variable X takes values 0, 1, 2 only. The probability distribution of X is shown in the table, where p is a constant ($0 < p < \frac{1}{3}$).

x	0	1	2
$P(X = x)$	$1 - 3p$	$2p$	p

Given that $\text{Var}(X) = \frac{1}{2}$, find the two possible values of $E(X)$. [6]

7 In a certain country 12% of school leavers have no educational qualifications. In a random sample of 50 school leavers, X denotes the number who have no educational qualifications.

(i) Calculate $P(X \leqslant 2)$, using a binomial distribution. [3]

(ii) Calculate $P(X \geqslant 10)$, using a suitable approximation. [4]

8 A university research scientist finds that she receives e-mails from colleagues in her university at random times, at an average rate of 4 per day. Find the probability that the number of such e-mails that she receives in a randomly chosen day is

(i) exactly 3, [2]

(ii) less than 3. [2]

The scientist also receives e-mails independently from other people. These also arrive at random times, but at an average rate of 1 per day. Find the probability that the scientist receives a total of at least 4 e-mails on a randomly chosen day. [3]

9 A company specialising in emergency repairs to domestic electrical goods charges a call-out fee of £30 plus £20 per hour for each job. Thus, for example, the total charge for a job taking 1.2 hours is £54. The length of time in hours needed for a job is modelled by a normal distribution with mean 0.9 and standard deviation 0.2.

(i) Find the probability that the total charge for a randomly chosen job will be more than £40. [4]

(ii) Find the probability that the total amount charged by the company for 10 randomly chosen jobs will be more than £500. [4]

10 The daily amount of petrol sold at a garage may be modelled by a normal distribution with mean μ litres and standard deviation σ litres. The amount sold, x litres, was recorded on each day of a random sample of 75 days, and the data is summarised (correct to 4 significant figures) by

$$\Sigma x = 6.675 \times 10^5, \quad \Sigma x^2 = 5.978 \times 10^9.$$

Calculate, correct to 3 significant figures,

(i) unbiased estimates of μ and σ^2, [3]

(ii) a symmetric 95% confidence interval for μ. [3]

Assume that the values found in part **(i)** can be taken to be the true values of μ and σ^2. Find the value of a such that at least a litres of petrol are sold on 90% of days on average. Give your answer correct to 3 significant figures. [3]

11 In a random sample of 24 primary-school children in a certain region of the country, it was found that 2 suffered from asthma. The null hypothesis that the proportion of primary-school children in this region that suffer from asthma is 0.2 is to be tested against the alternative hypothesis that the proportion is less than 0.2. Use a binomial distribution to carry out the test, using a 10% significance level. [4]

In a random sample of 40 primary-school children in a different region of the country, it was found that 12 suffered from asthma. The null hypothesis that the proportion of primary-school children in this region that suffer from asthma is 0.2 is to be tested against the alternative hypothesis that the proportion is *greater* than 0.2. Carry out the test, using a 10% significance level. [5]

12 A new version of a computer operating system is said, by the manufacturers, to result in computer users being able to 'start their favourite programs up to 36% faster'. In an investigation, each of a random sample of 10 computer users determined how long it took to start their most commonly used programs before and after installing the new operating system. They then calculated an appropriate figure to represent the percentage improvement. The results, to the nearest integer, were as follows:

18, 27, 35, 20, 32, 31, 36, 25, 27, 24.

It is proposed to use these figures to carry out a t-test for the population mean, μ.

(i) State an assumption that is necessary to justify the use of a t-test. [1]

(ii) Use a t-test to show that the null hypothesis $\mu = 36$ is rejected, in favour of the alternative hypothesis $\mu < 36$, at the 0.1% significance level. [6]

Find the largest value of μ that is *not* rejected at the 5% significance level. [3]

[Turn over

13 In safety tests, the distance required to bring a car to a stop was measured for various speeds of the car. The table shows corresponding values of the car's speed, v miles per hour, and the stopping distance, d metres.

v	20	30	40	50	60	70
d	13	22	37	51	72	95

$[\Sigma v = 270,\ \Sigma d = 290,\ \Sigma v^2 = 13\,900,\ \Sigma d^2 = 18\,832,\ \Sigma vd = 15\,920.]$

Calculate the product moment correlation coefficient between v and d for the data. [3]

What information does the value of this coefficient provide about a possible relationship between v and d? [1]

Calculate the equation of the regression line of d on v, giving your answer in the form $d = a + bv$, with the constants a and b correct to 3 significant figures. [2]

Plot the data, together with this regression line, on graph paper. Use scales of 2 cm for 10 miles per hour for v and 2 cm for 20 metres for d. [3]

Use the regression line to estimate the distance required to stop a car travelling at 55 miles per hour. [1]

Comment briefly on the suitability of using the regression line to estimate this distance. [1]

14 A darts player aims repeatedly for the bull's-eye. He counts it a success if his dart hits the bull's-eye. It may be assumed that, on each throw, the probability of success is $\frac{1}{4}$, independently of all other throws. Find the probability that, in a series of throws,

(i) the first success occurs on the fourth throw, [2]

(ii) the first success occurs between the fourth and eighth throws (inclusive), [2]

(iii) the second success occurs on the third throw. [3]

The player keeps a record of the number of throws, X, up to and including the first one to hit the bull's-eye. The mean of a random sample of 60 values of X is denoted by $\overline{X}$. Find $P(\overline{X} > 5)$. [5]

15 The number of times that candidates re-sit one of the compulsory modules in a modular A level course was investigated. The table shows the data from a random sample of 400 candidates who had completed the course, classified by the number of re-sits of the module and by sex.

	Number of re-sits				
	0	1	2	3	Total
Boys	113	49	18	8	188
Girls	136	58	14	4	212
Total	249	107	32	12	400

(i) Test, at the 5% significance level, whether there is any association between the sex of a candidate and the number of re-sits. [7]

(ii) Test, at the 5% significance level, the goodness of fit of a Poisson distribution as a model for the distribution of the number of re-sits (for boys and girls together). [7]

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