CAMBRIDGE
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A Level

Mathematics

Session: 2010
Type: Specification
Code: 3890-7890; 3892-7892

Oxford Cambridge and RSA Examinations

OCR ADVANCED SUBSIDIARY GCE IN MATHEMATICS (3890)
OCR ADVANCED SUBSIDIARY GCE IN PURE MATHEMATICS (3891)
OCR ADVANCED SUBSIDIARY GCE IN FURTHER MATHEMATICS (3892)

OCR ADVANCED GCE IN MATHEMATICS (7890)
OCR ADVANCED GCE IN PURE MATHEMATICS (7891)
OCR ADVANCED GCE IN FURTHER MATHEMATICS (7892)

First AS assessment January 2005
First A2 assessment June 2005
First AS certification January 2005
First GCE certification June 2005

QAN (3890) 100/3434/1
QAN (7890) 100/3435/3
QAN (3891) 100/6021/2
QAN (3892) 100/6020/0
QAN (7891) 100/6023/6
QAN (7892) 100/6022/4

Key Features

- Simple, flexible structure.
- Clearly defined AS and A2 standards.
- Permits AS Further Mathematics to be studied in year 12.
- Firmly established progression routes from GCSE for all candidates.
- All units externally assessed – No coursework.

Support and In-Service Training for Teachers

In support of these specifications, OCR will make the following materials and services available to teachers:

- up-to-date copies of these specifications;
- a full programme of In-Service Training (INSET) meetings;
- specimen question papers and mark schemes;
- past question papers and mark schemes after each examination session;
- a report on the examination, compiled by senior examining personnel, after each examination session.

If you would like further information about the specification, please contact OCR.

Foreword

This booklet contains OCR's specifications for Advanced Subsidiary GCE (AS) and Advanced GCE (A Level) Mathematics, and associated certification titles, for teaching from September 2004. It has been revised to take account of the changes to the Subject Criteria announced by QCA in December 2002 and August 2003.

The revisions to the Subject Criteria mean that there are changes to the way that the content of the Core is assessed, and in combinations of units that can lead to certification at Advanced Subsidiary GCE or Advanced GCE. In particular:

- combinations of units that can lead to certification of Advanced Subsidiary GCE Mathematics require the mandatory 'core' units, *C1* and *C2*, together with a single 'application' unit assessed at AS standard;
- combinations of units that can lead to certification of Advanced Subsidiary GCE Further Mathematics require the mandatory unit *FP1*, together with two other units, the levels of which are unspecified;
- combinations of units that can lead to certification of Advanced GCE Mathematics require the mandatory 'core' units, *C1*, *C2*, *C3*, *C4*, together with two 'application' units, at least one of which must be at AS standard.

Advanced Subsidiary GCE units are assessed at a standard appropriate for candidates who have completed the first year of study of a two-year Advanced GCE course. A2 units are assessed at a standard appropriate for candidates who have completed a two-year Advanced GCE course. Advanced Subsidiary GCE forms the first half of Advanced GCE in terms of teaching time and content. However, the Advanced Subsidiary can be taken as a 'stand-alone' qualification.

Given the above, and the fact that the re-designation of some of the units means that there are now **six** units assessed at AS standard, candidates intending to complete nine or 12 units will now find it possible, even desirable, to take six units all at AS standard during the first year of study.

In these specifications, the term module is used to describe specific teaching and learning requirements. The term unit describes a unit of assessment. Each teaching and learning module is assessed by an associated unit of assessment.

These specifications meet the requirements of the Common Criteria (QCA, 1999), the GCE Advanced Subsidiary and Advanced Level Qualification-Specific Criteria (QCA, 2003) and the relevant Subject Criteria.

CONTENTS

SECTION A: SPECIFICATION SUMMARY

COURSE OUTLINE

These specifications have been developed in line with the requirements of the revised Subject Criteria for Mathematics (QCA, 2003), while continuing to provide a straightforward assessment scheme that allows for a variety of different courses to be followed.

The modules in the scheme cover the following areas:

- Core Mathematics (modules 4721 to 4724);
- Further Pure Mathematics (modules 4725 to 4727);
- Mechanics (modules 4728 to 4731);
- Probability and Statistics (modules 4732 to 4735);
- Decision Mathematics (modules 4736 and 4737).

SPECIFICATION CONTENT

The content defined in the QCA Subject Criteria as 'core' material for Advanced Subsidiary GCE Mathematics is included within Core Mathematics *C1* and *C2* (modules 4721 and 4722), and that for Advanced GCE Mathematics is included within Core Mathematics *C3* and *C4* (modules 4723 and 4724).

SPECIFICATION UNITS

All units are externally assessed by a written examination of duration 1 hour 30 minutes; further details are shown in the table below. The weighting of each unit is $33\frac{1}{3}\%$ if contributing to Advanced Subsidiary GCE certification and $16\frac{2}{3}\%$ if contributing to Advanced GCE certification.

Entry Code	Unit Code	Unit Name	Level	Entry Code	Unit Code	Unit Name	Level
4721	C1*	Core Mathematics 1*	AS	4730	M3	Mechanics 3	A2
4722	C2	Core Mathematics 2	AS	4731	M4	Mechanics 4	A2
4723	C3	Core Mathematics 3	A2	4732	S1	Probability and Statistics 1	AS
4724	C4	Core Mathematics 4	A2	4733	S2	Probability and Statistics 2	A2
4725	FP1	Further Pure Mathematics 1	AS	4734	S3	Probability and Statistics 3	A2
4726	FP2	Further Pure Mathematics 2	A2	4735	S4	Probability and Statistics 4	A2
4727	FP3	Further Pure Mathematics 3	A2	4736	D1	Decision Mathematics 1	AS
4728	M1	Mechanics 1	AS	4737	D2	Decision Mathematics 2	A2
4729	M2	Mechanics 2	A2				

* indicates the unit in which no calculator may be used.

SCHEME OF ASSESSMENT

Units at AS Level have been designed for candidates following the first year of a two-year Advanced GCE course.
Units at A2 Level have been designed for candidates following the second year of a two-year Advanced GCE course.

Assessment is by means of **three units of assessment** for Advanced Subsidiary GCE and **six units of assessment** for Advanced GCE.

Units *C1*, *C2*, *FP1*, *M1*, *S1* and *D1* are designated as AS units, while *C3*, *C4*, *FP2*, *FP3*, *M2*, *M3*, *M4*, *S2*, *S3*, *S4* and *D2* are designated as A2 units.

The following combinations of units, are available for certification.

Certification Title and Number	Units Required
Advanced Subsidiary GCE Mathematics (3890)	*C1* and *C2*, together with one of *M1*, *S1*, *D1*
Advanced Subsidiary GCE Pure Mathematics (3891)	*C1*, *C2* and *FP1*
Advanced Subsidiary GCE Further Mathematics (3892)	*FP1* together with two other units which may not include any of *C1*, *C2*, *C3*, *C4*
Advanced GCE Mathematics (7890)	*C1*, *C2*, *C3* and *C4*, together with **either** two from *M1*, *S1*, *D1* **or** *M1*, *M2* **or** *S1*, *S2* **or** *D1*, *D2*
Advanced GCE Pure Mathematics (7891)	*C1*, *C2*, *C3*, *C4*, *FP1* and either *FP2* or *FP3*
Advanced GCE Further Mathematics (7892)	*FP1* together with *FP2* or *FP3* or both, plus three or four other units, as appropriate

QUESTION PAPER REQUIREMENTS

For each unit in the scheme, the Question Paper consists of a number of questions of different lengths and mark allocations. The total mark for each paper is 72. Candidates attempt all questions. Each question paper has a duration of 1 hour 30 minutes.

Question papers are designed to have a gradient of difficulty, with the more straightforward questions towards the beginning of the paper, and more demanding question towards the end. Where appropriate there is also a gradient of difficulty within individual questions.

No calculators may be used in answering unit *C1*; for all other units candidates may use a graphic calculator if they wish. Computers, and calculators with computer algebra functions, are not permitted in any of the units.

COURSEWORK REQUIREMENTS

There is no requirement for coursework associated with any of the units in these specifications.

UNIT COMBINATIONS

The scheme of assessment has been designed to recognise the different stages of maturity of candidates following a two year GCE course. AS Further Mathematics has been designed to broaden the mathematical experience of candidates and be independent from the A2 units of the Advanced GCE Mathematics course. Nevertheless, it is recognised that the needs of Centres differ and the scheme of assessment also includes other approaches within its structure and rules of combination set out in Sections 4.2 and 4.3.

The illustrations below give a number of the more common assessment patterns. Further examples are given in Section 4.7. However, it should be emphasised that, while the natural order would be to take AS units in Year 12 and A2 units in Year 13, there are no rules concerning either the order in which units should be taken or indeed which units should be taken in Year 12 and which in Year 13.

6 units over two years:
Advanced GCE Mathematics (Decision option)

	Units taken	**Certification**
Year 12	*C1 C2 D1*	Advanced Subsidiary GCE Mathematics
Year 13	*C3 C4 D2*	Advanced GCE Mathematics

6 units over two years:
Advanced GCE Mathematics (mixed Statistics and Mechanics option)

	Units taken	**Certification**
Year 12	*C1 C2 S1*	Advanced Subsidiary GCE Mathematics
Year 13	*C3 C4 M1*	Advanced GCE Mathematics

9 units over two years:
Advanced GCE Mathematics + AS Further Mathematics

	Units taken	**Certification**
Year 12	*C1 C2 S1 M1 D1 FP1*	Advanced Subsidiary GCE Mathematics
Year 13	*C3 C4 M2*	Advanced GCE Mathematics + AS Further Mathematics

Note: Six of the nine units (e.g. *C1 C2 M1 C3 C4 M2*) could be co-taught with candidates who are taking only six units.

12 units over two years:
Advanced GCE Mathematics + Advanced GCE Further Mathematics (balanced option)

	Units taken	Certification
Year 12	*C1 C2 S1 M1 D1 FP1*	Advanced Subsidiary GCE Mathematics
Year 13	*C3 C4 S2 M2 D2 FP3**	Advanced GCE Mathematics + Advanced GCE Further Mathematics

Note 1: Six of the 12 units (e.g. *C1 C2 M1 C3 C4 M2*) could be co-taught with candidates who are taking only six units.

Note 2: * or *FP2*.

12 units over two years:
Advanced GCE Mathematics + Advanced GCE Further Mathematics (Statistics and Mechanics)

	Units taken	Certification
Year 12	*C1 C2 C3 C4 S1 M1*	Advanced Subsidiary GCE Mathematics
Year 13	*FP1 FP2* S2 S3 M2 M3*	Advanced GCE Mathematics + Advanced GCE Further Mathematics

Note 1: This pattern of assessment does not have the advantage of being able to co-teach some of the units with candidates who are taking only six units.

Note 2: * or *FP3*.

MODULE CONTENT SUMMARY

Core Mathematics 1 (C1) (AS Unit 4721)

Indices and surds; Polynomials; Coordinate geometry and graphs; Differentiation.

Core Mathematics 2 (C2) (AS Unit 4722)

Trigonometry; Sequences and series; Algebra; Integration.

Core Mathematics 3 (C3) (A2 Unit 4723)

Algebra and functions; Trigonometry; Differentiation and integration; Numerical methods.

Core Mathematics 4 (C4) (A2 Unit 4724)

Algebra and graphs; Differentiation and integration; Differential equations; Vectors.

Further Pure Mathematics 1 (FP1) (AS Unit 4725)

Summation of series; Mathematical induction; Roots of polynomial equations; Complex numbers; Matrices.

Further Pure Mathematics 2 (FP2) (A2 Unit 4726)

Rational functions and graphs; Polar coordinates; Hyperbolic functions; Differentiation and integration; Numerical methods.

Further Pure Mathematics 3 (FP3) (A2 Unit 4727)

Differential equations; Vectors; Complex numbers; Groups.

Mechanics 1 (M1) (AS Unit 4728)

Force as a vector; Equilibrium of a particle; Kinematics of motion in a straight line; Newton's laws of motion; Linear momentum.

Mechanics 2 (M2) (A2 Unit 4729)

Centre of mass; Equilibrium of a rigid body; Motion of a projectile; Uniform motion in a circle; Coefficient of restitution and impulse; Energy, work and power.

Mechanics 3 (M3) (A2 Unit 4730)

Equilibrium of rigid bodies in contact; Elastic strings and springs; Impulse and momentum in two dimensions; Motion in a vertical circle; Linear motion under a variable force; Simple harmonic motion.

Mechanics 4 (M4) (A2 Unit 4731)

Relative motion; Centre of mass; Moment of Inertia; Rotation of a rigid body; Stability and oscillations.

Probability and Statistics 1 (S1) (AS Unit 4732)

Representation of data; Probability; Discrete random variables; Bivariate data.

Probability and Statistics 2 (S2) (A2 Unit 4733)

Continuous random variables; The normal distribution; The Poisson distribution; Sampling and hypothesis tests.

Probability and Statistics 3 (S3) (A2 Unit 4734)

Continuous random variables; Linear combinations of random variables; Confidence intervals and the t distribution; Difference of population means and proportions; χ^2 tests.

Probability and Statistics 4 (S4) (A2 Unit 4735)

Probability; Non-parametric tests; Probability generating functions; Moment generating functions; Estimators; Discrete bivariate distributions.

Decision Mathematics 1 (D1) (AS Unit 4736)

Algorithms; Graph theory; Networks; Linear programming.

Decision Mathematics 2 (D2) (A2 Unit 4737)

Game theory; Flows in a network; Matching and allocation problems; Critical path analysis; Dynamic programming.

SECTION B: GENERAL INFORMATION

1 Introduction

1.1 RATIONALE

The aim in preparing these Advanced Subsidiary GCE and Advanced GCE specifications has been to promote the teaching and learning of mathematics post GCSE in schools and colleges by providing a scheme that meets all the requirements of recent criteria while at the same time maintaining as far as possible features that have proved popular in recent past specifications.

The broad objectives in designing the scheme have been to include a sufficient range of mathematical topics to allow schools and colleges to devise courses to suit the varied requirements of a broad range of students, while at the same time avoiding administrative complexity and maintaining comparability between different options.

Students who successfully complete courses based on these specifications will have a suitable basis for progression to further study in mathematics or related subjects, or directly into employment.

Overview of Scheme

Mathematics forms part of the suite of OCR Advanced Subsidiary GCE and Advanced GCE modular specifications in which a single Advanced Subsidiary GCE comprises three teaching modules and a single Advanced GCE comprises six teaching modules, with each module representing about 45 hours of contact time. Each teaching module is assessed by an associated unit of assessment.

The 17 units in the scheme cover the areas of Core Mathematics (four units, 4721 to 4724), Further Pure Mathematics (three units, 4725 to 4727), Mechanics (four units, 4728 to 4731), Probability and Statistics (four units, 4732 to 4735), and Decision Mathematics (two units, 4736 and 4737).

The scheme allows for certification of the following:

- Advanced Subsidiary GCE Mathematics (three units);
- Advanced Subsidiary GCE Mathematics and Advanced Subsidiary GCE Further Mathematics (six units);
- Advanced GCE Mathematics (six units);
- Advanced GCE Mathematics and Advanced Subsidiary GCE Further Mathematics (nine units);
- Advanced GCE Mathematics and Advanced GCE Further Mathematics (twelve units);
- Advanced Subsidiary GCE Pure Mathematics (three units);
- Advanced GCE Pure Mathematics (six units).

Titles listed in the bullet points above cover certification in the subjects ‘Mathematics’, ‘Pure Mathematics’ and ‘Further Mathematics’ and consequently the necessary combinations of modules meet in full the relevant requirements of the Subject Criteria for Mathematics (QCA, 2003).

The requirement of the subject criteria for assessments of specifications in Mathematics and Pure Mathematics to include one unit of the assessment which must be answered without the help of a calculator is met by forbidding all calculating aids in the examination on the first Core paper (Unit *C1*) in the scheme. In other units the use of a graphic calculator is permitted. The specifications provide opportunities for the use of graphic calculators and computers to enhance the learning process and to help develop mathematical understanding, and such use is encouraged wherever appropriate.

The specifications are intended to build on the knowledge, understanding and skills established at GCSE. Students embarking on an Advanced Subsidiary GCE or Advanced GCE course of study are expected to have achieved at least grade C in GCSE Mathematics and to have covered all the material in the Intermediate Tier.

1.2 CERTIFICATION TITLE

These specifications will be shown on a certificate as one or more of the following.

- OCR Advanced Subsidiary GCE in Mathematics (3890)
- OCR Advanced Subsidiary GCE in Pure Mathematics (3891)
- OCR Advanced Subsidiary GCE in Further Mathematics (3892)

- OCR Advanced GCE in Mathematics (7890)
- OCR Advanced GCE in Pure Mathematics (7891)
- OCR Advanced GCE in Further Mathematics (7892)

1.3 LANGUAGE

These specifications, and all associated assessment materials, are available only in English. The language used in all question papers will be plain, clear, free from bias and appropriate to the qualification.

1.4 EXCLUSIONS

Candidates for Advanced Subsidiary GCE in Further Mathematics may be expected to have obtained or to be obtaining concurrently an Advanced Subsidiary GCE in Mathematics.

Candidates for Advanced GCE in Further Mathematics may be expected to have obtained or to be obtaining concurrently an Advanced GCE in Mathematics.

No Advanced Subsidiary GCE qualification within these specifications may be taken with any other Advanced Subsidiary GCE having the same title.

Advanced Subsidiary GCE in Pure Mathematics may **not** be taken with any other Advanced Subsidiary GCE qualification within these specifications.

Advanced GCE in Pure Mathematics may **not** be taken with any other Advanced GCE qualification within these specifications.

No Advanced GCE qualification within these specifications may be taken with any other Advanced GCE qualification having the same title.

Candidates may **not** enter a unit from these Mathematics specifications and a unit with the same title from other Mathematics specifications.

Every specification is assigned to a national classification code indicating the subject area to which it belongs.

Centres should be aware that candidates who enter for more than one GCE qualification with the same classification code, will have only one grade (the highest) counted for the purpose of the School and College Performance Tables.

The classification codes for these specifications are:

Mathematics	2210
Further Mathematics	2330
Pure Mathematics	2230

1.5 KEY SKILLS

Key Skills signposting appears in **three** sections of OCR specifications:

- *Key Skills Coverage* – the matrix aids curriculum managers in mapping the potential Key Skills coverage within each OCR Advanced Subsidiary/Advanced GCE specification.
- *Specification Content* (Section 5) – the specific evidence references enable subject teachers to identify opportunities for meeting specific Key Skills evidence requirements within the modules they are delivering.
- *Appendix A* – provides guidance to teachers in trying to identify those parts of their normal teaching programme which might most appropriately be used to develop or provide evidence for the Key Skills signposted.

These specifications provide opportunities for the development of the Key Skills of *Communication*, *Application of Number*, *Information Technology*, *Working With Others*, *Improving Own Learning and Performance* and *Problem Solving* as required by QCA's subject criteria for Mathematics.

Through classwork, coursework and preparation for external assessment, students may produce evidence for Key Skills at Level 3. However, the extent to which this evidence fulfils the requirements of the QCA Key Skills specifications at this level will be dependent on the style of teaching and learning adopted for each module. In some cases, the work produced may meet the evidence requirements of the Key Skills specifications at a higher or lower level.

Throughout Section 5 the symbol is used in the margin to highlight where Key Skills development opportunities are signposted. The following abbreviations are used to represent the above Key Skills:

C = Communication
N = Application of Number
IT = Information Technology
WO = Working with Others
LP = Improving Own Learning and Performance
PS = Problem Solving

These abbreviations are taken from the Key Skills specifications for use in programmes starting from September 2000. References in Section 5 and Appendix A , for example **IT3.1** show the Key Skill (**IT**), the level (**3**) and subsection (**1**).

Centres are encouraged to consider the OCR Key Skills scheme to provide certification of Key Skills for their candidates.

For each module, the following matrix indicates those Key Skills for which opportunities for at least some coverage of the relevant Key Skills unit exist at Level 3.

Module		Communication	Application of Number	Information Technology	Working with Others	Improving Own Learning and Performance	Problem Solving
C1	4721						
C2	4722						
C3	4723	✓		✓			
C4	4724						
FP1	4725						
FP2	4726	✓		✓			
FP3	4727						
M1	4728	✓					
M2	4729	✓					
M3	4730	✓					
M4	4731						
S1	4732	✓	✓	✓	✓		
S2	4733			✓			
S3	4734						
S4	4735						
D1	4736						
D2	4737						

1.6 CODE OF PRACTICE REQUIREMENTS

All qualifications covered by these specifications will comply in all aspects with the revised GCE Code of Practice for courses starting in September 2004.

1.7 SPIRITUAL, MORAL, ETHICAL, SOCIAL AND CULTURAL ISSUES

Students are required to examine arguments critically and so to distinguish between truth and falsehood. They are also expected to interpret the results of modelling exercises and there are times, particularly in statistical work, when this inevitably raises moral, ethical, social and cultural issues. Such issues are not assessed in examination questions.

1.8 ENVIRONMENTAL EDUCATION, EUROPEAN DIMENSION AND HEALTH AND SAFETY ISSUES

OCR has taken account of the 1988 Resolution of the Council of the European Community and the Report *Environmental Responsibility: An Agenda for Further and Higher Education*, 1993 in preparing these specifications and associated specimen assessment materials.

1.9 AVOIDANCE OF BIAS

OCR has taken great care in the preparation of these specifications and assessment materials to avoid bias of any kind.

1.10 CALCULATORS AND COMPUTERS

Candidates are expected to make appropriate use of graphical calculators and computers.

2 Specification Aims

2.1 AIMS

The aims of these Advanced Subsidiary GCE and Advanced GCE specifications are to encourage students to:

- develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment;
- develop abilities to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs;
- extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems;
- develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected;
- recognise how a situation may be represented mathematically and understand the relationship between ‘real world’ problems and standard and other mathematical models and how these can be refined and improved;
- use mathematics as an effective means of communication;
- read and comprehend mathematical arguments and articles concerning applications of mathematics;
- acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations;
- develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general;
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

3 Assessment Objectives

3.1 APPLICATION TO AS/A2

These specifications require candidates to demonstrate the following assessment objectives in the context of the knowledge, understanding and skills prescribed. The Assessment Objectives for Advanced Subsidiary GCE and for Advanced GCE are the same.

AO1

Candidates should be able to:

- recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts.

AO2

Candidates should be able to

- construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.

AO3

Candidates should be able to:

- recall, select and use their knowledge of standard mathematical models to represent situations in the real world;
- recognise and understand given representations involving standard models;
- present and interpret results from such models in terms of the original situation, including discussion of assumptions made and refinement of such models.

AO4

Candidates should be able to:

- comprehend translations of common realistic contexts into mathematics;
- use the results of calculations to make predictions, or comment on the context;
- where appropriate, read critically and comprehend longer mathematical arguments or examples of applications.

AO5

Candidates should be able to:

- use contemporary calculator technology and other permitted resources (such as formula booklets or statistical tables) accurately and efficiently;
- understand when not to use such technology, and its limitations;
- give answers to appropriate accuracy.

3.2 SPECIFICATION GRID

The relationship between Assessment Objectives and units of assessment is shown in the grid below. The figures are the ranges of marks (out of a total of 72 for each unit) within which the number of marks relating to each Objective fall. Each valid combination of units that can lead to certification for Advanced Subsidiary GCE or Advanced GCE Mathematics gives overall percentages that satisfy the requirement of the Subject Criteria that the figures for AO1 to AO5 should be 30%, 30%, 10%, 5%, 5% respectively.

Entry Code	Unit Code	Unit Name	Level	Weighting of Assessment Objective				
				AO1	AO2	AO3	AO4	AO5
4721	C1	Core Mathematics 1	AS	34-37	34-37	0	0-5	0
4722	C2	Core Mathematics 2	AS	26-32	26-32	0	0-5	11-15
4723	C3	Core Mathematics 3	A2	28-34	28-34	0	0-5	6-10
4724	C4	Core Mathematics 4	A2	28-34	28-34	0	0-5	6-10
4725	FP1	Further Pure Mathematics 1	AS	28-37	28-37	0	0-5	0-5
4726	FP2	Further Pure Mathematics 2	A2	26-35	26-35	0	0-5	5-10
4727	FP3	Further Pure Mathematics 3	A2	28-37	28-37	0	0-5	0-5
4728	M1	Mechanics 1	AS	12-17	12-17	22-32	11-15	5-10
4729	M2	Mechanics 2	A2	12-17	12-17	22-32	11-15	5-10
4730	M3	Mechanics 3	A2	12-17	12-17	22-32	11-15	5-10
4731	M4	Mechanics 4	A2	18-25	18-25	18-25	5-10	0-5
4732	S1	Probability and Statistics 1	AS	12-17	12-17	22-32	11-15	5-10
4733	S2	Probability and Statistics 2	A2	12-17	12-17	22-32	11-15	5-10
4734	S3	Probability and Statistics 3	A2	12-17	12-17	22-32	11-15	5-10
4735	S4	Probability and Statistics 4	A2	18-25	18-25	18-25	5-10	0-5
4736	D1	Decision Mathematics 1	AS	12-17	12-17	22-32	11-15	0
4737	D2	Decision Mathematics 2	A2	12-17	12-17	22-32	11-15	0

Units *C1* and *C2* include the content of the Advanced Subsidiary GCE subject criteria for Mathematics. The content of the Advanced GCE subject criteria for Mathematics is included within units *C1*, *C2*, *C3* and *C4*.

The specification for each of the units *M1* to *M4*, *S1* to *S4*, *D1* and *D2* involves the application of mathematics.

All combinations of units that can lead to certification for Advanced Subsidiary GCE Mathematics include one 'applied' unit, and all combinations of units that can lead to certification for Advanced GCE Mathematics include two 'applied' units.

Unit *C1* is the unit in which candidates are not allowed the use of any calculation aids. This ensures that any combination of units that can lead to certification in Mathematics or in Pure Mathematics at either Advanced Subsidiary GCE or Advanced GCE will satisfy the requirement in the QCA Subject Criteria that one assessment unit is calculator-free.

4 Scheme of Assessment

4.1 UNITS OF ASSESSMENT

Candidates take **three** units for Advanced Subsidiary GCE, followed by a further **three** units if they are seeking an Advanced GCE award.

The table below shows all the units in the overall scheme.

Entry Code	Unit Code	Unit Name	Level	Entry Code	Unit Code	Unit Name	Level
4721	C1*	Core Mathematics 1*	AS	4730	M3	Mechanics 3	A2
4722	C2	Core Mathematics 2	AS	4731	M4	Mechanics 4	A2
4723	C3	Core Mathematics 3	A2	4732	S1	Probability and Statistics 1	AS
4724	C4	Core Mathematics 4	A2	4733	S2	Probability and Statistics 2	A2
4725	FP1	Further Pure Mathematics 1	AS	4734	S3	Probability and Statistics 3	A2
4726	FP2	Further Pure Mathematics 2	A2	4735	S4	Probability and Statistics 4	A2
4727	FP3	Further Pure Mathematics 3	A2	4736	D1	Decision Mathematics 1	AS
4728	M1	Mechanics 1	AS	4737	D2	Decision Mathematics 2	A2
4729	M2	Mechanics 2	A2				

* indicates the unit in which no calculator may be used.

4.2 STRUCTURE

4.2.1 Recommended Order

The units of the scheme are arranged into groups as follows:

Core Mathematics (4721 to 4724),
Further Pure Mathematics (4725 to 4727),
Mechanics (4728 to 4731),
Probability and Statistics (4732 to 4735)
Decision Mathematics (4736 and 4737).

Within each group, later units are dependent on earlier ones, except that while Further Pure Mathematics 2 and Further Pure Mathematics 3 both depend on Further Pure Mathematics 1 they do not depend on each other.

'Dependency' of one unit on another means that in the assessment of the later unit the specification content of the earlier unit may be assumed, and that questions, or parts of questions, in the later unit may require knowledge and use of the earlier material. Similarly, 'applied' modules are dependent on certain Core and Further Pure Mathematics modules, as detailed at the start of the content list in each case.

There are no restrictions on the sequence in which units may be taken.

4.2.2 Weighting

For all certifications, the units contributing are equally weighted, i.e. each unit carries $33^1/_3$% of the total marks when contributing to Advanced Subsidiary GCE certification and $16^2/_3$% when contributing to Advanced GCE certification.

4.2.3 Synoptic Assessment

The subject criteria for mathematics require that any combination of units valid for the certification of Advanced GCE Mathematics (7890) or Advanced GCE Pure Mathematics (7891) must include a minimum of 20% synoptic assessment.

Synoptic assessment in mathematics addresses candidates' understanding of the connections between different elements of the subject. It involves the explicit drawing together of knowledge, understanding and skills learned in different parts of the Advanced GCE course through using and applying methods developed at earlier stages of study in solving problems. Making and understanding connections in this way is intrinsic to learning mathematics.

In the Core Modules *C1* to *C4*, much of the content of later modules builds on that of earlier ones, so that the assessment of *C3* and *C4* in particular has a large synoptic element. All 'application' modules make use, to a greater or lesser extent, of core content and/or methods from earlier stages of the relevant application area. In the assessment units for *C3* and *C4*, about 50% of the marks will be of a synoptic nature, and the synoptic assessment that also occurs within Unit *C2* and within two 'application' units (in the case of Advanced GCE Mathematics) or two Further Pure Mathematics units (in th e case of of Advanced GCE Pure Mathematics) will ensure that the 20% requirement of the criteria is satisfied.

There are no requirements concerning synoptic assessment relating to the certification of Advanced Subsidiary GCE or to Advanced GCE Further Mathematics.

4.2.4 Assessment Routes

Students who are intending to complete no more than six units over the course of two years will normally complete three AS units during Year 12 and certificate for Advanced Subsidiary GCE Mathematics. Many of these students will then continue their study of Mathematics in Year 13 and take three A2 units and finally certificate for Advanced GCE Mathematics at the end of Year 13. While this would be the natural pattern for such students, many other patterns exist and it is important to appreciate that there are no rules concerning the order in which units are taken or concerning which units are taken in each year.

Once again, many other patterns exist and some examples of these are given in Section 4.7. However, for the 'natural' pattern described above, there are advantages concerning the efficiency of teaching groups and it should be possible to arrange the teaching of the nine- and twelve-unit students such that as many as six of these units can be taught in groups which also contain students who are following a six-unit programme.

4.3 RULES OF COMBINATION

4.3.1 Advanced Subsidiary GCE Mathematics (3890)

Candidates take **one** of the following combinations:

either *C1*, *C2* and *M1*
or *C1*, *C2* and *S1*
or *C1*, *C2* and *D1*

4.3.2 Advanced GCE Mathematics (7890)

Candidates take:

C1, *C2*, *C3* and *C4*

together with:

either any two from *M1*, *S1*, *D1*
or *M1* and *M2*
or *S1* and *S2*
or *D1* and *D2*

4.3.3 Advanced Subsidiary GCE Pure Mathematics (3891)

Candidates take the following combination:

C1, *C2* and *FP1*

4.3.4 Advanced GCE Pure Mathematics (7891)

Candidates take the **three** units listed above for Advanced Subsidiary GCE Pure Mathematics. They also take **one** of the following combinations:

either *C3*, *C4* and *FP2*
or *C3*, *C4* and *FP3*

4.3.5 Advanced Subsidiary GCE Further Mathematics (3892)

Candidates take:

FP1

together with **two** other units which may **not** include any of *C1*, *C2*, *C3*, *C4*.

(Note that units that have already been included in Advanced GCE Mathematics may **not** also be included in Advanced Subsidiary GCE Further Mathematics.)

Candidates would generally be advised **not** to certificate for Advanced Subsidiary GCE Further Mathematics before certificating for Advanced GCE Mathematics.

In a small number of cases, candidates who were intending eventually to take nine (or even 12) units may decide not to proceed beyond six units. Providing they have valid combinations of units, these candidates may, at this point, certificate simultaneously for Advanced Subsidiary GCE Mathematics and Advanced Subsidiary GCE Further Mathematics.

4.3.6 Advanced GCE Further Mathematics (7892)

Candidates for Advanced GCE Further Mathematics will be expected to have obtained or to be obtaining concurrently an Advanced GCE in Mathematics.

Candidates take **three** units as described above for Advanced Subsidiary GCE Further Mathematics. A further **three** units are required which must include (unless already included in AS Further Mathematics) either *FP2* or *FP3*. The six units must include at least three which are assessed at A2 standard, and may **not** include any of *C1*, *C2*, *C3*, *C4*.

(Note that units that have already been included in Advanced GCE Mathematics may **not** also be included in Advanced GCE Further Mathematics.)

4.4 FINAL CERTIFICATION

Each unit is given a grade and a Uniform Mark, using procedures laid down by QCA in the document 'GCE A and AS Code of Practice'. The relationship between total Uniform Mark and subject grade follows the national scheme.

4.4.1 Certification

To claim an award at the end of the course, candidates' unit results must be aggregated. This does not happen automatically and Centres must make separate 'certification entries'.

4.4.2 Order of Aggregation

Units that contribute to an award in Advanced GCE Mathematics may not also be used for an award in Advanced GCE Further Mathematics. Candidates who are awarded certificates in both Advanced GCE Mathematics and Advanced GCE Further Mathematics must use unit results from 12 different teaching modules. Candidates who are awarded certificates in both Advanced GCE Mathematics and Advanced Subsidiary GCE Further Mathematics must use unit results from nine different teaching modules.

When a candidate has requested awards in both Mathematics and Further Mathematics, OCR will adopt the following procedures.

In the majority of cases, certification for Advanced GCE Mathematics will be made **at the same time** as the request for Further Mathematics certification. In this situation:

- The best Advanced GCE Mathematics grade available to the candidate will be determined.
- The combination of units which allows the least total Uniform Mark to be used in achieving that grade legally is selected.
- The remaining units are then used to grade Further Mathematics (both AS and Advanced GCE level).

Note: In the aggregation process, in order to achieve the best set of grades for a candidate as described above, it is possible that AS Further Mathematics may include some A2 units.

In a small number of cases (described earlier in Section 4.3.5) a candidate, who originally embarked on a nine-unit or twelve-unit course, may decide not to go beyond the six units obtained in Year 12. Providing the candidate has a valid combination for AS Mathematics + AS Further Mathematics (as opposed to Advanced GCE Mathematics), then:

- The best AS Mathematics grade available to the candidate will be determined.
- The combination of units which allows the least total Uniform Mark to be used in achieving that grade legally is selected.
- The remaining units are then used to grade AS Further Mathematics.

4.4.3 Awarding of Grades

The Advanced Subsidiary has a weighting of 50% when used in an Advanced GCE award.

Both Advanced Subsidiary GCE and Advanced GCE qualifications are awarded on the scale A to E or U (unclassified).

4.4.4 Extra Units

A candidate may submit more than the required number of units for a subject award (for example, seven instead of six for an Advanced GCE). In that case the legal combination for that award which allows the least total uniform mark to be used in achieving that grade will normally be chosen.

4.4.5 Enquiries on Results

Candidates will receive their final unit results at the same time as their subject results. In common with other Advanced GCE results, the subject results are at that stage provisional to allow enquiries on results. Enquiries concerning marking are made at the unit level and so only those units taken at the last sitting may be the subject of such appeals. Enquiries are subject to OCR's general regulations.

4.5 AVAILABILITY

4.5.1 Unit Availability

There are **two** examination sessions each year, in January and June.

In January 2005, the following units assessed at AS standard will be available:

C1, C2, M1, S1, D1

From June 2005 onwards, all units that are assessed at AS standard will be available at every examination session.

In June 2005 the following units, assessed at A2 standard, will be available:

C3, C4, M1, M2, S1, S2, D2

From January 2006 onwards, all units that are assessed at A2 standard, with the exception of *M4* and *S4* (which will be available in the June examination session only), will be available at every examination session.

4.5.2 Certification Availability

The first certification session for Advanced Subsidiary GCE qualifications will be June 2005.
The first certification session for Advanced GCE qualifications will be June 2005.

4.5.3 Shelf-life of Units

Individual unit results, prior to certification of the qualification, have a shelf-life limited only by that of the specification.

4.6 RE-SITS

4.6.1 Re-sits of Units

There is no limit to the number of times a candidate may re-sit a unit. The best result will count.

4.6.2 Re-sits of Advanced Subsidiary GCE and Advanced GCE

Candidates may take the whole qualification more than once.

4.7 UNIT COMBINATIONS

The scheme of assessment has been designed to recognise the different stages of maturity of candidates following a two year GCE course. AS Further Mathematics has been designed to broaden the mathematical experience of candidates and be independent from the A2 units of the Advanced GCE Mathematics course. Nevertheless, it is recognised that the needs of Centres differ and the scheme of assessment also includes other approaches within its structure and rules of combination set out in Sections 4.2 and 4.3.

The illustrations below give a number of the more common assessment patterns. Further examples are given in Section 4.7. However, it should be emphasised that, while the natural order would be to take AS units in Year 12 and A2 units in Year 13, there are no rules concerning either the order in which units should be taken or indeed which units should be taken in Year 12 and which in Year 13.

6 units over two years:
Advanced GCE Mathematics (Mechanics option)

	Units taken	Certification
Year 12	*C1 C2 M1*	Advanced Subsidiary GCE Mathematics
Year 13	*C3 C4 M2*	Advanced GCE Mathematics

Note: Statistics and Decision Mathematics options can be constructed in a similar way.

6 units over two years:
Advanced GCE Mathematics (mixed Statistics and Decision option)

	Units taken	Certification
Year 12	*C1 C2 S1*	Advanced Subsidiary GCE Mathematics
Year 13	*C3 C4 D1*	Advanced GCE Mathematics

9 units over two years:
Advanced GCE Mathematics + AS Further Mathematics

	Units taken	Certification
Year 12	*C1 C2 S1 M1 D1 FP1*	Advanced Subsidiary GCE Mathematics
Year 13	*C3 C4 S2*	Advanced GCE Mathematics + AS Further Mathematics

Note: Six or more of the nine units (e.g. *C1 C2 S1 C3 C4 S2*) could be co-taught with candidates who are taking only six units.

12 units over two years:
Advanced GCE Mathematics + Advanced GCE Further Mathematics (balanced option)

	Units taken	Certification
Year 12	*C1 C2 S1 M1 D1 FP1*	Advanced Subsidiary GCE Mathematics
Year 13	*C3 C4 S2 M2 D2 FP2**	Advanced GCE Mathematics + Advanced GCE Further Mathematics

Note 1: Six of the 12 units (e.g. *C1 C2 M1 C3 C4 M2*) could be co-taught with candidates who are taking only six units.

Note 2: * or *FP3*.

The following example is a more traditional pattern which some Centres may find attractive.

12 units over two years:
Advanced GCE Mathematics + Advanced GCE Further Mathematics (Pure and Mechanics)

	Units taken	Certification
Year 12	*C1 C2 C3 C4 M1 S1*	Advanced Subsidiary GCE Mathematics
Year 13	*FP1 FP2 FP3 M2 M3 M4*	Advanced GCE Mathematics + Advanced GCE Further Mathematics

Note: This pattern of assessment does not have the advantage of being able to co-teach some of the units with candidates who are taking only six units.

The following examples illustrate possible options for candidates who decide **not** to continue with Mathematics in Year 13.

6 units over one year:
AS Mathematics + AS Further Mathematics

	Units taken	Certification
Year 12	*C1 C2 S1 M1 D1 FP1*	Advanced Subsidiary GCE Mathematics + Advanced Subsidiary GCE Further Mathematics

6 units over one year:
Advanced GCE Mathematics (Statistics option)

	Units taken	Certification
Year 12	*C1 C2 C3 C4 S1 S2*	Advanced GCE Mathematics

Note: Mechanics and Decision Mathematics options can be constructed in a similar way.

4.8 QUESTION PAPERS

The examination on each unit is by means of a single written paper, of duration 1 hour and 30 minutes, and carrying a total of 72 marks.

Each question paper consists of a number of questions of different lengths and mark allocations. Candidates should attempt all the questions.

Question papers are designed to have a gradient of difficulty, with the more straightforward questions towards the beginning of the paper, and more demanding questions towards the end. Where appropriate there is also a gradient of difficulty within individual longer questions. The order in which the questions are printed in the paper will generally correspond to increasing numbers of marks for questions.

Units *C1, C2, FP1, M1, S1* and *D1* are designated as AS units, while units *C3, C4, FP2, FP3, M2, M3, M4, S2, S3, S4* and *D2* are designated as A2 units.

4.8.1 Use of Language

Candidates are expected to use clear, precise and appropriate mathematical language, as described in Assessment Objective 2.

4.8.2 Standard

Candidates and Centres must note that each A2 unit is assessed at Advanced GCE standard and that no concessions are made to any candidate on the grounds that the examination has been taken early in the course. Centres may disadvantage their candidates by entering them for a unit examination before they are ready.

4.8.3 Thresholds

At the time of setting, each examination paper will be designed so that 50% of the marks are available to grade E candidates, 75% to grade C and 100% to grade A. Typically candidates are expected to achieve about four fifths of the marks available to achieve a grade, giving design grades of : A 80%, B 70 %, C 60%, D 50% and E 40%. The actual grading is carried out by the Awarding Committee. They make allowance for examination performance and for any features of a particular paper that only become apparent after it has been taken. Thus some variation from the design grades can be expected in the Award.

4.8.4 Calculators

No calculating aids may be used in answering Unit *C1*.

For all other units candidates are permitted to use graphic calculators. Computers, and calculators with computer algebra functions, are not permitted in answering any of the units.

4.8.5 Mathematical Formulae and Statistical Tables

A booklet (MF12) containing Mathematical Formulae and Statistical Tables is available for the use of candidates in all unit examinations. Details of the items included in this booklet are contained in Appendix B.

Those formulae which candidates are required to know, and which are not included in the booklet of Mathematical Formulae and Statistical Tables, are listed within the specification of the first module for which they may be required. These formulae include all those specified in the subject criteria together with others of comparable significance relating to non-core modules.

4.9 COURSEWORK

There is no requirement for assessed coursework for any of the units in these specifications.

4.10 SPECIAL ARRANGEMENTS

For candidates who are unable to complete the full assessment or whose performance may be adversely affected through no fault of their own, teachers should consult the Inter-Board Regulations and Guidance Booklet for Special Arrangements and Special Consideration. In such cases advice should be sought from OCR as early as possible during the course.

4.11 DIFFERENTIATION

In the question papers, differentiation is achieved by setting questions which are designed to assess candidates at their appropriate level of ability and which are intended to allow all candidates to demonstrate what they know, understand and can do.

Both Advanced Subsidiary GCE and Advanced GCE qualifications are awarded on the scale A to E or U (unclassified).

4.12 GRADE DESCRIPTIONS

The following grade descriptions indicate the level of attainment characteristic of the given grade at Advanced GCE. They give a general indication of the required learning outcomes at each specified grade. The descriptions should be interpreted in relation to the content outlined in the specification; they are not designed to define that content. The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of the examination may be balanced by better performance in others.

Grade A

Candidates recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill. They use mathematical language correctly and proceed logically and rigorously through extended arguments or proofs. When confronted with unstructured problems they can often devise and implement an effective solution strategy. If errors are made in their calculations or logic, these are sometimes noticed and corrected.

Candidates recall or recognise almost all the standard models that are needed, and select appropriate ones to represent a wide variety of situations in the real world. They correctly refer results from calculations using the model to the original situation; they give sensible interpretations of their results in the context of the original realistic situation. They make intelligent comments on the modelling assumptions and possible refinements to the model.

Candidates comprehend or understand the meaning of almost all translations into mathematics of common realistic contexts. They correctly refer the results of calculations back to the given context and usually make sensible comments or predictions. They can distil the essential mathematical information from extended pieces of prose having mathematical content. They can comment meaningfully on the mathematical information.

Candidates make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are aware of any limitations to their use. They present results to an appropriate degree of accuracy.

Grade C

Candidates recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill. They use mathematical language with some skill and sometimes proceed logically through extended arguments or proofs. When confronted with unstructured problems they sometimes devise and implement an effective and efficient solution strategy. They occasionally notice and correct errors in their calculations.

Candidates recall or recognise most of the standard models that are needed, and usually select appropriate ones to represent a variety of situations in the real world. They often correctly refer results from calculations using the model to the original situation; they sometimes give sensible interpretations of their results in the context of the original realistic situation. They sometimes make intelligent comments on the modelling assumptions and possible refinements to the model.

Candidates comprehend or understand the meaning of most translations into mathematics of common realistic contexts. They often correctly refer the results of calculations back to the given context and sometimes make sensible comments or predictions. They distil much of the essential mathematical information from extended pieces of prose having mathematical content. They give some useful comments on this mathematical information.

Candidates usually make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are sometimes aware of any limitations to their use. They usually present results to an appropriate degree of accuracy.

Grade E

Candidates recall or recognise some of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use in some contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill. They sometimes use mathematical language correctly and occasionally proceed logically through extended arguments or proofs.

Candidates recall or recognise some of the standard models that are needed, and sometimes select appropriate ones to represent a variety of situations in the real world. They sometimes correctly refer results from calculations using the model to the original situation; they try to interpret their results in the context of the original realistic situation.

Candidates sometimes comprehend or understand the meaning of translations into mathematics of common realistic contexts. They sometimes correctly refer the results of calculations back to the given context and attempt to give comments or predictions. They distil some of the essential mathematical information from extended pieces of prose having mathematical content. They attempt to comment on this mathematical information.

Candidates often make appropriate and efficient use of contemporary calculator technology and other permitted resources. They often present results to an appropriate degree of accuracy.

5 Specification Content

It should be noted that individual questions may involve ideas from more than one section of the relevant content, and that topics may be tested in the context of solving problems and in the application of Mathematics.

In all examinations candidates are expected to construct and present clear mathematical arguments, consisting of logical deductions and precise statements involving correct use of symbols and connecting language. In particular, terms such as ‘equals’, ‘identically equals’, ‘therefore’, ‘because’, ‘implies’, ‘is implied by’, ‘necessary’, ‘sufficient’, and notations such as $\Rightarrow$, $\Leftarrow$, $\Leftrightarrow$ and $\therefore$ should be understood and used accurately.

In addition, candidates are expected to understand the nature of a mathematical proof.
In A2 units, questions that require

- proof by contradiction;
- disproof by counter-example

may be set.

5.1 AS MODULE 4721: CORE MATHEMATICS 1 (C1)

Preamble

No calculators are permitted in the assessment of this unit.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Algebra

Solution of $ax^2 + bx + c = 0$ is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant of $ax^2 + bx + c$ is $b^2 - 4ac$

Coordinate Geometry

Equation of the straight line through (x_1, y_1) with gradient m is $y - y_1 = m(x - x_1)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Equation of the circle with centre (a, b) and radius r is $(x-a)^2 + (y-b)^2 = r^2$

Differentiation

If $y = x^n$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = nx^{n-1}$

If $y = \mathrm{f}(x) + \mathrm{g}(x)$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}'(x) + \mathrm{g}'(x)$

Indices and Surds

Candidates should be able to:

(a) understand rational indices (positive, negative and zero), and use laws of indices in the course of algebraic applications;

(b) recognise the equivalence of surd and index notation (e.g. $\sqrt{a} = a^{\frac{1}{2}}$, $\sqrt[3]{a^2} = a^{\frac{2}{3}}$);

(c) use simple properties of surds such as $\sqrt{12} = 2\sqrt{3}$, including rationalising denominators of the form $a + \sqrt{b}$.

Polynomials

Candidates should be able to:

(a) carry out operations of addition, subtraction, and multiplication of polynomials (including expansion of brackets, collection of like terms and simplifying);

(b) carry out the process of completing the square for a quadratic polynomial $ax^2 + bx + c$, and use this form, e.g. to locate the vertex of the graph of $y = ax^2 + bx + c$;

(c) find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant, e.g. to determine the number of real roots of the equation $ax^2 + bx + c = 0$;

(d) solve quadratic equations, and linear and quadratic inequalities, in one unknown;

(e) solve by substitution a pair of simultaneous equations of which one is linear and one is quadratic;

(f) recognise and solve equations in x which are quadratic in some function of x, e.g. $x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 4 = 0$.

Coordinate Geometry and Graphs

Candidates should be able to:

(a) find the length, gradient and mid-point of a line-segment, given the coordinates of its end-points;

(b) find the equation of a straight line given sufficient information (e.g. the coordinates of two points on it, or one point on it and its gradient);

(c) understand and use the relationships between the gradients of parallel and perpendicular lines;

(d) interpret and use linear equations, particularly the forms $y = mx + c$, $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$;

(e) understand that the equation $(x-a)^2 + (y-b)^2 = r^2$ represents the circle with centre (a, b) and radius r;

(f) use algebraic methods to solve problems involving lines and circles, including the use of the equation of a circle in expanded form $x^2 + y^2 + 2gx + 2fy + c = 0$ (knowledge of the following circle properties is included: the angle in a semicircle is a right angle; the perpendicular from the centre to a chord bisects the chord; the perpendicularity of radius and tangent);

(g) understand the relationship between a graph and its associated algebraic equation, use points of intersection of graphs to solve equations, and interpret geometrically the algebraic solution of equations (to include, in simple cases, understanding of the correspondence between a line being tangent to a curve and a repeated root of an equation);

(h) sketch curves with equations of the form

(i) $y = kx^n$, where n is a positive or negative integer and k is a constant,

(ii) $y = k\sqrt{x}$, where k is a constant,

(iii) $y = ax^2 + bx + c$, where a, b and c are constants,

(iv) $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is the product of at most 3 linear factors, not necessarily all distinct;

(i) understand and use the relationships between the graphs of $y = \mathrm{f}(x)$, $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$, where a is a constant, and express the transformations involved in terms of translations, reflections and stretches.

Differentiation

Candidates should be able to:

(a) understand the gradient of a curve at a point as the limit of the gradients of a suitable sequence of chords (an informal understanding only is required, and the technique of differentiation from first principles is not included);

(b) understand the ideas of a derived function and second order derivative, and use the notations $\mathrm{f}'(x)$, $\mathrm{f}''(x)$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$;

(c) use the derivative of x^n (for any rational n), together with constant multiples, sums and differences;

(d) apply differentiation (including applications to practical problems) to gradients, tangents and normals, rates of change, increasing and decreasing functions, and the location of stationary points (the ability to distinguish between maximum points and minimum points is required, but identification of points of inflexion is not included).

5.2 AS MODULE 4722: CORE MATHEMATICS 2 (C2)

Preamble

Knowledge of the specification content of Module *C1* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *C2*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Algebra

The remainder when a polynomial $\mathrm{f}(x)$ is divided by $(x-a)$ is $\mathrm{f}(a)$

$a^b = c \Leftrightarrow b = \log_a c$

Laws of logarithms:

$$\log_a x + \log_a y \equiv \log_a(xy)$$

$$\log_a x - \log_a y \equiv \log_a\left(\frac{x}{y}\right)$$

$$k\log_a x \equiv \log_a(x^k)$$

Trigonometry

In triangle *ABC*:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Area} = \tfrac{1}{2}ab\sin C$$

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$\cos^2\theta + \sin^2\theta \equiv 1$$

π radians is $180°$

For a sector of a circle:

$$s = r\theta$$

$$A = \tfrac{1}{2}r^2\theta$$

Integration

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + c, \ (n \neq -1)$$

$$\int \{f'(x) + g'(x)\} \, dx = f(x) + g(x) + c$$

Area between a curve and the x-axis is $\int_a^b y \, dx$ (for $y \geq 0$)

Area between a curve and the y-axis is $\int_c^d x \, dy$ (for $x \geq 0$)

Trigonometry

Candidates should be able to:

(a) use the sine and cosine rules in the solution of triangles (excluding the ambiguous case of the sine rule);

(b) use the area formula $\Delta = \frac{1}{2} ab \sin C$;

(c) understand the definition of a radian, and use the relationship between degrees and radians;

(d) use the formulae $s = r\theta$ and $A = \frac{1}{2} r^2 \theta$ for the arc length and sector area of a circle;

(e) relate the periodicity and symmetries of the sine, cosine and tangent functions to the form of their graphs;

(f) use the identities $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$ and $\cos^2\theta + \sin^2\theta \equiv 1$;

(g) use the exact values of the sine, cosine and tangent of $30°$, $45°$, $60°$ e.g $\cos 30° = \frac{1}{2}\sqrt{3}$;

(h) find all the solutions, within a specified interval, of the equations $\sin(kx) = c$, $\cos(kx) = c$, $\tan(kx) = c$, and of equations (for example, a quadratic in $\sin x$) which are easily reducible to these forms.

Sequences and Series

Candidates should be able to:

(a) understand the idea of a sequence of terms, and use definitions such as $u_n = n^2$ and relations such as $u_{n+1} = 2u_n$ to calculate successive terms and deduce simple properties;

(b) understand and use Σ notation;

(c) recognise arithmetic and geometric progressions;

(d) use the formulae for the nth term and for the sum of the first n terms to solve problems involving arithmetic or geometric progressions (including the formula $\frac{1}{2} n(n+1)$ for the sum of the first n natural numbers);

(e) use the condition $|r|<1$ for convergence of a geometric series, and the formula for the sum to infinity of a convergent geometric series;

(f) use the expansion of $(a+b)^n$ where n is a positive integer, including the recognition and use of the notations $\binom{n}{r}$ and $n!$ (finding a general term is not included).

Algebra

Candidates should be able to:

(a) use the factor theorem and the remainder theorem;

(b) carry out simple algebraic division (restricted to cases no more complicated than division of a cubic by a linear polynomial);

(c) sketch the graph of $y=a^x$, where $a>0$, and understand how different values of a affect the shape of the graph;

(d) understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base);

(e) use logarithms to solve equations of the form $a^x=b$, and similar inequalities.

Integration

Candidates should be able to:

(a) understand indefinite integration as the reverse process of differentiation, and integrate x^n (for any rational n except -1), together with constant multiples, sums and differences;

(b) solve problems involving the evaluation of a constant of integration, (e.g. to find the equation of the curve through $(-1, 2)$ for which $\frac{\mathrm{d}y}{\mathrm{d}x}=2x+1$;

(c) evaluate definite integrals (including e.g. $\int_0^1 x^{-\frac{1}{2}}\,\mathrm{d}x$ and $\int_1^\infty x^{-2}\,\mathrm{d}x$;

(d) use integration to find the area of a region bounded by a curve and lines parallel to the coordinate axes, or between two curves or between a line and a curve;

(e) use the trapezium rule to estimate the area under a curve, and use sketch graphs, in simple cases, to determine whether the trapezium rule gives an over-estimate or an under-estimate.

5.3 A2 MODULE 4723: CORE MATHEMATICS 3 (C3)

Preamble

Knowledge of the specification content of Modules *C1* and *C2* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *C3*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Trigonometry

$$\sec\theta \equiv \frac{1}{\cos\theta}$$

$$\operatorname{cosec}\theta \equiv \frac{1}{\sin\theta}$$

$$\cot\theta \equiv \frac{1}{\tan\theta}$$

$$\sec^2\theta \equiv 1+\tan^2\theta$$

$$\operatorname{cosec}^2\theta \equiv 1+\cot^2\theta$$

$$\sin 2A \equiv 2\sin A\cos A$$

$$\cos 2A \equiv \cos^2 A-\sin^2 A \equiv 2\cos^2 A-1 \equiv 1-2\sin^2 A$$

$$\tan 2A \equiv \frac{2\tan A}{1-\tan^2 A}$$

Differentiation and Integration

If $y=\mathrm{e}^{kx}$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x}=k\,\mathrm{e}^{kx}$

If $y=\ln x$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{x}$

If $y=\mathrm{f}(x).\mathrm{g}(x)$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{f}'(x)\mathrm{g}(x)+\mathrm{f}(x)\mathrm{g}'(x)$

If $y=\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{f}'(x)\mathrm{g}(x)-\mathrm{f}(x)\mathrm{g}'(x)}{\{\mathrm{g}(x)\}^2}$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=1\div\frac{\mathrm{d}x}{\mathrm{d}y}$$

If $y = \mathrm{f}(\mathrm{g}(x))$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}'(\mathrm{g}(x))\,\mathrm{g}'(x)$

Connected rates of change: $\dfrac{\mathrm{d}y}{\mathrm{d}t} = \dfrac{\mathrm{d}y}{\mathrm{d}x}.\dfrac{\mathrm{d}x}{\mathrm{d}t}$

$\int \mathrm{e}^{kx}\,\mathrm{d}x = \dfrac{1}{k}\mathrm{e}^{kx} + c$

$\int \dfrac{1}{x}\mathrm{d}x = \ln|x| + c$

Volumes of revolution about the axes: $V_x = \pi \int_a^b y^2\,\mathrm{d}x$

$$V_y = \pi \int_c^d x^2\,\mathrm{d}y$$

Algebra and Functions

Candidates should be able to:

(a) understand the terms function, domain, range, one-one function, inverse function and composition of functions;

(b) identify the range of a given function in simple cases, and find the composition of two given functions;

(c) determine whether or not a given function is one-one, and find the inverse of a one-one function in simple cases;

(d) illustrate in graphical terms the relation between a one-one function and its inverse;

(e) use and recognise compositions of transformations of graphs, such as the relationship between the graphs of $y = \mathrm{f}(x)$ and $y = a\,\mathrm{f}(x + b)$, where a and b are constants;

(f) understand the meaning of $|x|$ and use relations such as $|a| = |b| \Leftrightarrow a^2 = b^2$ and $|x - a| < b \Leftrightarrow a - b < x < a + b$ in the course of solving equations and inequalities;

(g) understand the relationship between the graphs of $y = \mathrm{f}(x)$ and $y = |\mathrm{f}(x)|$;

(h) understand the properties of the exponential and logarithmic functions e^x and $\ln x$ and their graphs, including their relationship as inverse functions;

(i) understand exponential growth and decay.

Trigonometry

Candidates should be able to:

(a) use the notations $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ to denote the principal values of the inverse trigonometric relations, and relate their graphs (for the appropriate domains) to those of sine, cosine and tangent;

(b) understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude;

(c) use trigonometrical identities for the simplification and exact evaluation of expressions, and in the course of solving equations within a specified interval, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of

(i) $\sec^2\theta \equiv 1+\tan^2\theta$ and $\operatorname{cosec}^2\theta \equiv 1+\cot^2\theta$,

(ii) the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$,

(iii) the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$,

(iv) the expression of $a\sin\theta + b\cos\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$.

Differentiation and Integration

Candidates should be able to:

(a) use the derivatives of e^x and $\ln x$, together with constant multiples, sums, and differences;

(b) differentiate composite functions using the chain rule;

(c) differentiate products and quotients;

(d) understand and use the relation $\frac{dy}{dx} = 1 \div \frac{dx}{dy}$;

(e) apply differentiation to connected rates of change;

(f) integrate e^x and $\frac{1}{x}$, together with constant multiples, sums, and differences;

(g) integrate expressions involving a linear substitution, e.g. $(2x-1)^8$, e^{3x+2};

(h) use definite integration to find a volume of revolution about one of the coordinate axes (including, for example, the region between the curves $y = x^2$ and $y = \sqrt{x}$, rotated about the x-axis.

Numerical Methods

C3.3 IT3.2 IT3.3

Candidates should be able to:

(a) locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign-change;

(b) understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation;

(c) understand how a given simple iterative formula of the form $x_{n+1} = \mathrm{F}(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy (knowledge of the condition for convergence is not included, but candidates should understand that an iteration may fail to converge);

(d) carry out numerical integration of functions by means of Simpson's rule.

5.4 A2 MODULE 4724: CORE MATHEMATICS 4 (C4)

Preamble

Knowledge of the specification content of Modules *C1*, *C2* and *C3* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *C4*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Differentiation and Integration

If $y = \sin kx$ then $\dfrac{dy}{dx} = k\cos kx$

If $y = \cos kx$ then $\dfrac{dy}{dx} = -k\sin kx$

$\int \cos kx \, dx = \dfrac{1}{k}\sin kx + c$

$\int \sin kx \, dx = -\dfrac{1}{k}\cos kx + c$

$\int f'(g(x))\, g'(x)\, dx = f(g(x)) + c$

Vectors

$|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{x^2 + y^2 + z^2}$

$(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}).(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = ax + by + cz$

$\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$

Equation of a line through **a** parallel to **b** is $\mathbf{r} = \mathbf{a} + t\mathbf{b}$

Algebra and Graphs

Candidates should be able to:

(a) simplify rational expressions, including factorising and cancelling;

(b) divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero);

(c) recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than

(i) $(ax+b)(cx+d)(ex+f)$,

(ii) $(ax+b)(cx+d)^2$,

and where the degree of the numerator is less than that of the denominator;

(d) use the expansion of $(1+x)^n$ where n is a rational number and $|x|<1$ (finding a general term is not included, but adapting the standard series to expand, e.g. $\left(2-\frac{1}{2}x\right)^{-1}$ is included);

(e) understand the use of a pair of parametric equations to define a curve, and use a given parametric representation of a curve in simple cases;

(f) convert the equation of a curve between parametric and cartesian forms.

Differentiation and Integration

Candidates should be able to:

(a) use the derivatives of $\sin x$, $\cos x$ and $\tan x$, together with sums, differences and constant multiples;

(b) find and use the first derivative of a function which is defined parametrically or implicitly;

(c) extend the idea of 'reverse differentiation' to include the integration of trigonometric functions (e.g. $\cos x$ and $\sec^2 2x$);

(d) use trigonometric relations (such as double angle formulae) in order to facilitate the integration of functions such as $\cos^2 x$;

(e) integrate rational functions by means of decomposition into partial fractions (restricted to the types of partial fractions specified in section 5.4.1);

(f) recognise an integrand of the form $\dfrac{k\,\mathrm{f}'(x)}{\mathrm{f}(x)}$, and integrate, for example, $\dfrac{x}{x^2+1}$ or $\tan x$;

(g) recognise when an integrand can usefully be regarded as a product, and use integration by parts to integrate, for example, $x\sin 2x$, $x^2\mathrm{e}^x$, $\ln x$ (the relationship between integration by parts and differentiation of a product should be understood);

(h) use a given substitution to simplify and evaluate either a definite or an indefinite integral (the relationship between integration by substitution and the chain rule should be understood).

First Order Differential Equations

Candidates should be able to:

(a) formulate a simple statement involving a rate of change as a differential equation, including the introduction if necessary of a constant of proportionality;

(b) find by integration a general form of solution for a differential equation in which the variables are separable;

(c) use an initial condition to find a particular solution of a differential equation;

(d) interpret the solution of a differential equation in the context of a problem being modelled by the equation.

Vectors

Candidates should be able to:

(a) use standard notations for vectors, i.e. $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $\overrightarrow{AB}$, $\mathbf{a}$;

(b) carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms;

(c) use unit vectors, position vectors and displacement vectors;

(d) calculate the magnitude of a vector, and identify the magnitude of a displacement vector $\overrightarrow{AB}$ as being the distance between the points A and B;

(e) calculate the scalar product of two vectors (in either two or three dimensions), and use the scalar product to determine the angle between two directions and to solve problems concerning perpendicularity of vectors;

(f) understand the significance of all the symbols used when the equation of a straight line is expressed in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$;

(g) determine whether two lines are parallel, intersect or are skew;

(h) find the angle between two lines, and the point of intersection of two lines when it exists.

5.5 AS MODULE 4725: FURTHER PURE MATHEMATICS 1 (FP1)

Preamble

Knowledge of the specification content of Modules *C1* and *C2* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *FP1*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Algebra

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1)$$

For $ax^2+bx+c=0$: $\quad \alpha+\beta=-\frac{b}{a}, \quad \alpha\beta=\frac{c}{a}$

For $ax^3+bx^2+cx+d=0$: $\quad \Sigma\alpha=-\frac{b}{a}, \quad \Sigma\alpha\beta=\frac{c}{a}, \quad \alpha\beta\gamma=-\frac{d}{a}$

Matrices

$$(\mathbf{AB})^{-1}=\mathbf{B}^{-1}\mathbf{A}^{-1}$$

Summation of Series

Candidates should be able to:

(a) use the standard results for Σr, Σr^2, Σr^3 to find related sums;

(b) use the method of differences to obtain the sum of a finite series;

(c) recognise, by direct consideration of the sum to n terms, when a series is convergent, and find the sum to infinity in such cases.

Proof by Induction

Candidates should be able to:

(a) use the method of mathematical induction to establish a given result (not restricted to summation of series);

(b) recognise situations where conjecture based on a limited trial followed by inductive proof is a useful strategy, and carry this out in simple cases, e.g. to find the nth power of the matrix $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

Roots of Polynomial Equations

Candidates should be able to:

(a) use the relations between the symmetric functions of the roots of polynomial equations and the coefficients (for equations of degree 2 or 3 only);

(b) use a given simple substitution to obtain an equation whose roots are related in a simple way to those of the original equation.

Complex Numbers

Candidates should be able to:

(a) understand the idea of a complex number, recall the meaning of the terms 'real part', 'imaginary part', 'modulus', 'argument', 'conjugate', and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal;

(b) carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in cartesian form $(x + \mathrm{i}\,y)$;

(c) use the result that, for a polynomial equation with real coefficients, any non-real roots occur in conjugate pairs;

(d) represent complex numbers geometrically by means of an Argand diagram, and understand the geometrical effects of conjugating a complex number and of adding and subtracting two complex numbers;

(e) find the two square roots of a complex number;

(f) illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram, e.g. $|z-a| < k$, $|z-a| = |z-b|$, $\arg(z-a) = \alpha$.

Matrices

Candidates should be able to:

(a) carry out operations of matrix addition, subtraction and multiplication, and recognise the terms null (or zero) matrix and identity (or unit) matrix;

(b) recall the meaning of the terms 'singular' and 'non-singular' as applied to square matrices, and, for 2×2 and 3×3 matrices, evaluate determinants and find inverses of non-singular matrices;

(c) understand and use the result, for non-singular matrices, that $(\mathbf{AB})^{-1}=\mathbf{B}^{-1}\mathbf{A}^{-1}$;

(d) understand the use of 2×2 matrices to represent certain geometrical transformations in the *x*-*y* plane, and in particular

(i) recognise that the matrix product **AB** represents the transformation that results from the transformation represented by **B** followed by the transformation represented by **A**,

(ii) recall how the area scale-factor of a transformation is related to the determinant of the corresponding matrix,

(iii) find the matrix that represents a given transformation or sequence of transformations (understanding of the terms 'rotation', 'reflection', 'enlargement', 'stretch' and 'shear' will be required);

(e) formulate a problem involving the solution of 2 linear simultaneous equations in 2 unknowns, or 3 equations in 3 unknowns, as a problem involving the solution of a matrix equation, and vice-versa;

(f) understand the cases that may arise concerning the consistency or inconsistency of 2 or 3 linear simultaneous equations, relate them to the singularity or otherwise of the corresponding square matrix, and solve consistent systems.

5.6 A2 MODULE 4726: FURTHER PURE MATHEMATICS 2 (FP2)

Preamble

Knowledge of the specification content of Modules *C1, C2, C3, C4* and *FP1* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *FP2*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Hyperbolic Functions

$\cosh x \equiv \frac{1}{2}(e^x + e^{-x})$

$\sinh x \equiv \frac{1}{2}(e^x - e^{-x})$

$\tanh x \equiv \dfrac{\sinh x}{\cosh x}$

Rational Functions and Graphs

Candidates should be able to:

(a) express in partial fractions a rational function in which the denominator may include a factor of the form $(x^2 + a^2)$ in addition to linear factors as specified in section 5.4.1, and in which the degree of the numerator may exceed the degree of the denominator;

(b) determine the salient features of the graph of a rational function for which the numerator and denominator are of degree at most 2, including in particular

 (i) asymptotic behaviour (understanding of oblique asymptotes, as well as asymptotes parallel to the axes, is expected),

 (ii) any restrictions on the values taken by the function;

(c) understand and use the relationship between the graphs of $y = f(x)$ and $y^2 = f(x)$.

Polar Coordinates

Candidates should be able to:

(a) understand the relations between cartesian and polar coordinates (using the convention $r \geq 0$), and convert equations of curves from cartesian to polar form and *vice versa*;

(b) sketch simple polar curves, for $0 \leq \theta < 2\pi$ or $-\pi < \theta \leq \pi$ or a subset of either of these intervals, and identify significant features of polar curves such as symmetry, least/greatest values of *r*, and the form of the curve at the pole (knowledge that any values of θ for which $r = 0$ give directions of tangents at the pole is included);

(c) use the formula $\frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$ for the area of a sector in simple cases.

Hyperbolic Functions

Candidates should be able to:

(a) recall definitions of the six hyperbolic functions in terms of exponentials, and sketch the graphs of simple hyperbolic functions;

(b) derive and use identities such as $\cosh^2 x - \sinh^2 x \equiv 1$ and $\sinh 2x \equiv 2\sinh x \cosh x$;

(c) use the notations $\sinh^{-1} x$, $\cosh^{-1} x$, $\tanh^{-1} x$ to denote the principal values of the inverse hyperbolic relations, and derive and use expressions in terms of logarithms for these.

Differentiation and Integration

Candidates should be able to:

(a) derive and use the derivatives of $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$;

(b) derive and use the derivatives of $\sinh x$, $\cosh x$, $\tanh x$, $\sinh^{-1} x$, $\cosh^{-1} x$, $\tanh^{-1} x$;

(c) use the first few terms of the Maclaurin series of e^x, $\sin x$, $\cos x$ and $\ln(1+x)$;

(d) derive and use the first few terms of the Maclaurin series of simple functions, e.g. $\sin x$, $\cos 3x$, $e^x \sin x$, $\ln(3+2x)$ (derivation of a general term is not included);

(e) integrate $\dfrac{1}{\sqrt{a^2 - x^2}}$, $\dfrac{1}{a^2 + x^2}$, $\dfrac{1}{\sqrt{x^2 - a^2}}$ and $\dfrac{1}{\sqrt{x^2 + a^2}}$, and use appropriate trigonometric or hyperbolic substitutions for the evaluation of definite or indefinite integrals (the substitution $t = \tan\frac{1}{2}x$ is included);

(f) derive and use reduction formulae for the evaluation of definite integrals in simple cases;

(g) understand how the area under a curve may be approximated by areas of rectangles, and use rectangles to estimate or set bounds for the area under a curve or to derive inequalities concerning sums.

Numerical Methods

C3.3 IT3.2 IT3.3

Candidates should be able to:

(a) understand, in geometrical terms involving ‘staircase’ and ‘cobweb’ diagrams, the convergence (or not) of an iteration of the form $x_{n+1} = \mathrm{F}(x_n)$ to a root of the equation $x = \mathrm{F}(x)$;

(b) use the facts that, for an iteration $x_{n+1} = \mathrm{F}(x_n)$ which converges to α, successive (small) errors e_n are such that:

(i) $\mathrm{e}_{n+1} \approx \mathrm{F}'(\alpha)\mathrm{e}_n$, if $\mathrm{F}'(\alpha) \neq 0$,

(ii) e_{n+1} is approximately proportional to e_n^2 (in general) if $\mathrm{F}'(\alpha) = 0$;

(c) understand, in geometrical terms, the working of the Newton-Raphson method, and appreciate conditions under which the method may fail to converge to the desired root;

(d) derive and use iterations based on the Newton-Raphson method, and understand that this method is an example of an iteration of the form $x_{n+1} = \mathrm{F}(x_n)$ with $\mathrm{F}'(\alpha) = 0$.

5.7 A2 MODULE 4727: FURTHER PURE MATHEMATICS 3 (FP3)

Preamble

Knowledge of the specification content of Modules *C1, C2, C3, C4* and *FP1* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *FP3*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Differential Equations

An integrating factor for $\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{P}(x)y = \mathrm{Q}(x)$ is $\mathrm{e}^{\int \mathrm{P}(x)\,\mathrm{d}x}$

Vectors

The plane through **a** with normal vector **n** is $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$

Complex Numbers

If $z = \mathrm{e}^{\mathrm{i}\theta}$ then: $\cos\theta = \frac{1}{2}\left(z + \frac{1}{z}\right)$

$$\sin\theta = \frac{1}{2\mathrm{i}}\left(z - \frac{1}{z}\right)$$

Differential Equations

Candidates should be able to:

(a) find an integrating factor for a first-order linear differential equation, and use an integrating factor to find the general solution;

(b) use a given substitution to reduce a first-order differential equation to linear form or to a form in which the variables are separable;

(c) recall the meaning of the terms ‘complementary function’ and ‘particular integral’ in the context of linear differential equations, and use the fact that the general solution is the sum of the complementary function and a particular integral;

(d) find the complementary function for a first or second order linear differential equation with constant coefficients;

(e) recall the form of, and find, a particular integral for a first or second order linear differential equation in the cases where $ax+b$ or $a\,\mathrm{e}^{bx}$ or $a\cos px + b\sin px$ is a suitable form, and in other cases find the appropriate coefficient(s) given a suitable form of particular integral;

(f) use initial conditions to find a particular solution to a differential equation, and interpret the solution in the context of a problem modelled by the differential equation.

Vectors

Candidates should be able to:

(a) understand the significance of all the symbols used when the equation of a line is expressed in the form $\dfrac{x-a}{p} = \dfrac{y-b}{q} = \dfrac{z-c}{r}$;

(b) understand the significance of all the symbols used when the equation of a plane is expressed in any of the forms $ax+by+cz=d$ or $(\mathbf{r}-\mathbf{a}).\mathbf{n}=0$ or $\mathbf{r}=\mathbf{a}+\lambda\mathbf{b}+\mu\mathbf{c}$;

(c) recall the definition, in geometrical terms, of the vector product of two vectors, and, in cases where **a** and **b** are expressed in component form, calculate $\mathbf{a}\times\mathbf{b}$ in component form;

(d) use equations of lines and planes to solve problems concerning distances, angles and intersections, and in particular

  (i) determine whether a line lies in a plane, is parallel to a plane, or intersects a plane, and find the point of intersection of a line and a plane when it exists,

  (ii) find the line of intersection of two non-parallel planes,

  (iii) find the perpendicular distance from a point to a plane, and from a point to a line,

  (iv) find the angle between a line and a plane, and the angle between two planes,

  (v) find the shortest distance between two skew lines.

Complex Numbers

Candidates should be able to:

(a) carry out operations of multiplication and division of two complex numbers expressed in polar form $\left(r(\cos\theta + \mathrm{i}\sin\theta) \equiv r\,\mathrm{e}^{\mathrm{i}\theta}\right)$, and interpret these operations in geometrical terms;

(b) understand de Moivre's theorem, for positive and negative integer exponent, in terms of the geometrical effect of multiplication and division of complex numbers;

(c) use de Moivre's theorem to express trigonometrical ratios of multiple angles in terms of powers of trigonometrical ratios of the fundamental angle;

(d) use expressions for $\sin\theta$ and $\cos\theta$ in terms of $\mathrm{e}^{\mathrm{i}\theta}$, e.g. in expressing powers of $\sin\theta$ and $\cos\theta$ in terms of multiple angles or in summing series;

(e) find and use the *n*th roots of unity, e.g. to solve an equation of the form $z^n = a + \mathrm{i}b$.

Groups

Candidates should be able to:

(a) recall that a group consists of a set of elements together with a binary operation which is closed and associative, for which an identity exists in the set, and for which every element has an inverse in the set;

(b) use the basic group properties to show that a given structure is, or is not, a group (questions may be set on, for example, groups of matrices, transformations, integers modulo n);

(c) use algebraic methods to establish properties in abstract groups in easy cases, e.g. to show that any group in which every element is self-inverse is commutative;

(d) recall the meaning of the term 'order', as applied both to groups and to elements of a group, and determine the order of elements in a given group;

(e) understand the idea of a subgroup of a group, find subgroups in simple cases, and show that given subsets are, or are not, (proper) subgroups;

(f) recall and apply Lagrange's theorem concerning the order of a subgroup of a finite group (the proof of the theorem is not required);

(g) recall the meaning of the term 'cyclic' as applied to groups, and show familiarity with the structure of finite groups up to order 7 (questions on groups of higher order are not excluded, but no particular prior knowledge of such groups is expected);

(h) understand the idea of isomorphism between groups, and determine whether given finite groups are, or are not, isomorphic.

5.8 AS MODULE 4728: MECHANICS 1 (M1)

Preamble

Knowledge of the specification content of Modules *C1* and *C2* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *M1*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Forces and Equilibrium

Weight and mass: $\text{Weight} = \text{mass} \times g$

Limiting friction: $F = \mu R$

Newton's second law: $F = ma$

Kinematics

For linear motion with constant acceleration:

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = \tfrac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \tfrac{1}{2}at^2$$

For general linear motion: $v = \dfrac{\mathrm{d}s}{\mathrm{d}t}$, $a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$

Linear Momentum

Momentum of a particle: mv

Force as a Vector

Candidates should be able to:

(a) understand the vector nature of force, and use directed line segments to represent forces (acting in at most two dimensions);

(b) understand the term ‘resultant’ as applied to two or more forces acting at a point, and use vector addition in solving problems involving resultants and components of forces (solutions involving calculation, rather than scale drawing, will be expected);

(c) find and use perpendicular components of a force, e.g. in finding the resultant of a system of forces, or to calculate the magnitude and direction of a force (knowledge of column vector or **i**, **j** notation is not required, though candidates are free to use any such notation in answering questions if they wish).

Equilibrium of a Particle

C3.1a C3.1b C3.3

Candidates should be able to:

(a) identify the forces acting in a given situation, and use the relationship between mass and weight;

(b) understand and use the principle that a particle is in equilibrium if and only if the vector sum of the forces acting is zero, or equivalently if and only if the sum of the resolved parts in any given direction is zero (problems may involve resolution of forces in direction(s) to be chosen by the candidate, or the use of a ‘triangle of forces’);

(c) use the model of a ‘smooth’ contact and understand the limitations of the model;

(d) represent the contact force between two rough surfaces by two components, the ‘normal force’ and the ‘frictional force’, understand the concept of limiting friction and limiting equilibrium, recall the definition of coefficient of friction, and use the relationship $F \le \mu R$ or $F = \mu R$ as appropriate;

(e) use Newton’s third law.

Kinematics of Motion in a Straight Line

Candidates should be able to:

(a) understand the concepts of distance and speed as scalar quantities, and of displacement, velocity and acceleration as vector quantities (in one dimension only);

(b) sketch and interpret (t, x) and (t, v) graphs, and in particular understand and use the facts that

  (i) the area under a (t, v) graph represents displacement,

  (ii) the gradient of a (t, x) graph represents velocity,

  (iii) the gradient of a (t, v) graph represents acceleration;

(c) use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration;

(d) use appropriate formulae for motion with constant acceleration.

Newton's Laws of Motion

Candidates should be able to:

(a) apply Newton's laws of motion to the linear motion of bodies of constant mass moving under the action of constant forces (which may include friction); for example, a car pulling a caravan;

(b) model, in suitable circumstances, the motion of a body moving vertically or on an inclined plane, as motion with constant acceleration and understand any limitations of this model;

(c) solve simple problems which may be modelled as the motion of two particles, connected by a light inextensible string which may pass over a fixed smooth peg or light pulley (including, for example, situations in which a pulley is placed at the top of an inclined plane).

Linear Momentum

Candidates should be able to:

(a) recall and use the definition of linear momentum and show understanding of its vector nature (in one dimension only);

(b) understand and use conservation of linear momentum in simple applications involving the direct collision of two bodies moving in the same straight line before and after impact, including the case where the bodies coalesce (knowledge of impulse and of the coefficient of restitution is not required).

5.9 A2 MODULE 4729: MECHANICS 2 (M2)

Preamble

Knowledge of the specification content of Modules *C1* to *C4* and *M1* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *M2*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Projectile Motion

Equation of trajectory: $y = x\tan\theta - \dfrac{gx^2}{2V^2\cos^2\theta}$

Work, Energy and Power

Gravitational potential energy:	mgh
Kinetic energy of a particle:	$\frac{1}{2}mv^2$
Work done by a force:	$Fd\cos\alpha$
Power of a moving force:	$P = Fv$

Collisions

Newton’s experimental law: separation speed $= e \times$ approach speed

Centre of Mass

C3.3

Candidates should be able to:

(a) use the result that the effect of gravity on a rigid body is equivalent to a single force acting at the centre of mass of the body;

(b) identify the position of the centre of mass of a uniform body using considerations of symmetry;

(c) use given information about the position of the centre of mass of a triangular lamina and other simple shapes (including those listed in the List of Formulae);

(d) determine the position of the centre of mass of a composite rigid body by considering an equivalent system of particles (in simple cases only, e.g. a uniform L-shaped lamina or a hemisphere abutting a cylinder).

Equilibrium of a Rigid Body

C3.3

Candidates should be able to:

(a) calculate the moment of a force about a point in two dimensional situations only (understanding of the vector nature of moments is not required);

(b) use the principle that, under the action of coplanar forces, a rigid body is in equilibrium if and only if (i) the vector sum of the forces is zero, and (ii) the sum of the moments of the forces about any point is zero;

(c) solve problems involving the equilibrium of a single rigid body under the action of coplanar forces, including those involving toppling or sliding (problems set will not involve complicated trigonometry).

Motion of a Projectile

Candidates should be able to:

(a) model the motion of a projectile as a particle moving with constant acceleration and understand any limitations of this model;

(b) use horizontal and vertical equations of motion to solve problems on the motion of projectiles, including finding the magnitude and direction of the velocity at a given time or position, the range on a horizontal plane and the greatest height reached;

(c) derive and use the cartesian equation of the trajectory of a projectile, including problems in which the initial speed and/or angle of projection may be unknown.

Uniform Motion in a Circle

Candidates should be able to:

(a) understand the concept of angular speed for a particle moving in a circle, and use the relation $v = r\omega$;

(b) understand that the acceleration of a particle moving in a circle with constant speed is directed towards the centre of the circle, and use the formulae $r\omega^2$ and $\frac{v^2}{r}$;

(c) solve problems which can be modelled by the motion of a particle moving in a horizontal circle with constant speed.

Coefficient of Restitution; Impulse

Candidates should be able to:

(a) recall and use Newton's experimental law and the definition of coefficient of restitution, the property $0 \le e \le 1$, and the meaning of the terms 'perfectly elastic' ($e = 1$) and 'inelastic' ($e = 0$);

(b) use Newton's experimental law in the course of solving problems that may be modelled as the direct impact of two smooth spheres or as the direct impact of a smooth sphere with a fixed plane surface;

(c) recall and use the definition of impulse as change of momentum (in one dimension only, restricted to 'instantaneous' events, so that calculations involving force and time are not included).

Energy, Work and Power

Candidates should be able to:

(a) understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force (use of the scalar product is not required);

(b) understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae;

(c) understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of conservation of energy;

(d) use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion;

(e) solve problems involving, for example, the instantaneous acceleration of a car moving on a hill with resistance.

5.10 A2 MODULE 4730: MECHANICS 3 (M3)

Preamble

Knowledge of the specification content of Modules *C1* to *C4*, *M1* and *M2* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *M3*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Kinematics

$$a = v\frac{\mathrm{d}v}{\mathrm{d}s}$$

Simple Harmonic Motion

$$\ddot{x} = -\omega^2 x$$

$$v^2 = \omega^2(a^2 - x^2)$$

$$T = \frac{2\pi}{\omega}$$

$x = a\sin(\omega t)$ or $a\cos(\omega t)$ or, in general, $a\cos(\omega t + \varepsilon)$

Elastic Strings and Springs

Hooke's law: $T = \dfrac{\lambda x}{l}$

Elastic potential energy: $\dfrac{\lambda x^2}{2l}$

Small-angle Approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$

Equilibrium of Rigid Bodies in Contact

Candidates should be able to:

(a) understand and use Newton's third law in the context of problems involving the equilibrium of two or more rigid bodies in contact, including examples with two freely jointed rods.

Elastic Strings and Springs

C3.3

Candidates should be able to:

(a) use Hooke's law as a model relating the force in an elastic string or spring to the extension or compression, and understand the term 'modulus of elasticity';

(b) use the formula for the elastic potential energy stored in a string or spring;

(c) solve problems involving forces due to elastic strings or springs, including those where considerations of work and energy are needed.

Impulse and Momentum in Two Dimensions

Candidates should be able to:

(a) understand the vector nature of impulse and momentum, and solve problems concerning impulse and momentum for motion in two dimensions;

(b) solve problems that may be modelled as the oblique impact of two smooth spheres or as the oblique impact of a smooth sphere with a fixed surface (the appropriate use of Newton's experimental law is included).

Motion in a Vertical Circle

Candidates should be able to:

(a) use formulae for the radial and transverse components of acceleration for a particle moving in a circle with variable speed;

(b) solve problems which can be modelled as that of a particle, or a pair of connected particles, moving without loss of energy in a vertical circle (including the determination of points where circular motion breaks down, e.g. through loss of contact with a surface or a string becoming slack).

Linear Motion under a Variable Force

Candidates should be able to:

(a) use $\frac{\mathrm{d}x}{\mathrm{d}t}$ for velocity, and $\frac{\mathrm{d}v}{\mathrm{d}t}$ or $v\frac{\mathrm{d}v}{\mathrm{d}x}$ for acceleration, as appropriate;

(b) solve problems which can be modelled as the linear motion of a particle under the action of a variable force, by setting up and solving an appropriate differential equation (restricted to equations in which the variables are separable).

Simple Harmonic Motion

Candidates should be able to:

(a) recall a definition of SHM, understand the concepts of period and amplitude, and use standard SHM formulae in solving problems;

(b) set up the differential equation of motion in problems leading to SHM, quote appropriate forms of solution, and identify the period and amplitude of the motion;

(c) use Newton's second law, together with the approximation $\sin\theta \approx \theta$, to show that small oscillations of a simple pendulum may be modelled as SHM, and understand the limitations of this model;

(d) solve problems involving small oscillations of a simple pendulum.

5.11 A2 MODULE 4731: MECHANICS 4 (M4)

Preamble

Knowledge of the specification content of Modules *C1* to *C4* and *M1* to *M3* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *M4*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Relative Velocity

$${}_P\mathbf{v}_Q + {}_Q\mathbf{v}_R = {}_P\mathbf{v}_R$$

Angular Motion

For constant angular acceleration: $\omega_1 = \omega_0 + \alpha t$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta = \frac{1}{2}(\omega_0 + \omega_1)t$

$\omega_1^2 = \omega_0^2 + 2\alpha\theta$

$\theta = \omega_1 t - \frac{1}{2}\alpha t^2$

Angular momentum: $I\omega$

Kinetic energy: $\frac{1}{2}I\omega^2$

Equation of motion: $C = I\ddot{\theta}$

Work done by a constant couple: $W = C\theta$

Relative Motion

Candidates should be able to:

(a) understand the concept of the displacement or velocity of one point relative to another, and the representation of these quantities by vectors (including the use of ${}_P\mathbf{v}_Q + {}_Q\mathbf{v}_R = {}_P\mathbf{v}_R$, where ${}_P\mathbf{v}_Q$ denotes the velocity of P relative to Q, etc);

(b) use graphical and/or calculation methods to solve problems involving relative displacements and velocities, including interception or the determination of the course for closest approach.

Centre of Mass

Candidates should be able to:

(a) use integration to determine the position of the centre of mass of

  (i) a straight rod of variable density,

  (ii) a uniform lamina,

  (iii) a uniform solid of revolution.

Moment of Inertia

Candidates should be able to:

(a) understand and use the definition of the moment of inertia of a system of particles about a fixed axis as Σmr^2, and the additive property of moment of inertia for a rigid body composed of several parts (standard moments of inertia may be quoted from the List of Formulae unless a proof by integration is specifically asked for);

(b) use integration to find the moment of inertia of

  (i) a uniform rod about a perpendicular axis,

  (ii) a uniform lamina bounded by one of the axes about that axis,

  (iii) a uniform solid of revolution about the axis of revolution;

(c) use the parallel and perpendicular axes theorems (including cases where integration is also required, e.g. to find the moment of inertia of a uniform cylinder about the diameter of one of its ends).

Rotation of a Rigid Body

Candidates should be able to:

(a) use the equation of angular motion $C = I\ddot{\theta}$ for the motion of a rigid body about a fixed axis;

(b) understand and use the angular equivalents of the formulae for linear motion with constant acceleration;

(c) understand the concept of angular momentum, and use conservation of angular momentum (about a fixed axis only) to solve problems;

(d) use the formulae $I\omega$ and $\frac{1}{2}I\omega^2$ for the angular momentum and kinetic energy, respectively, of a rigid body rotating about a fixed axis;

(e) understand the nature of a couple, and use the formula $C\theta$ to calculate the work done by a couple of constant moment C in rotating through an angle θ;

(f) use considerations of work and energy in solving problems concerning mechanical systems where motion of a rigid body about a fixed axis is involved;

(g) show that small oscillations of a compound pendulum may be modelled as SHM, and determine the period of such oscillations (the formula $T = 2\pi\sqrt{\dfrac{I}{mgh}}$ may be quoted without proof in answering questions unless a derivation of the SHM model is specifically requested);

(h) calculate the force acting at the axis when a rigid body rotates freely about a fixed axis.

Stability and Oscillations

Candidates should be able to:

(a) locate an equilibrium position for a simple mechanical system by considering positions where the potential energy has a stationary value;

(b) determine whether the equilibrium is stable or unstable (problems in which the second derivative of the potential energy is zero are excluded);

(c) differentiate an energy equation to obtain an equation of motion, in simple cases;

(d) find the (approximate) period of small oscillations of a mechanical system about a position of stable equilibrium.

5.12 AS MODULE 4732: PROBABILITY AND STATISTICS 1 (S1)

Preamble

Knowledge of the specification content of Modules *C1* and *C2* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *S1*.

The specification content of this module include some items that are part of GCSE Mathematics at Intermediate Tier (e.g. mean, mode, median, interquartile range, stem-and-leaf diagrams, box-and-whisker plots). These are included for completeness; where any such items are involved in examination questions, the main focus will be on interpretation rather than on elementary calculations.

The specification content of this module is be understood in the context of modelling real-life situations, and examination questions may ask for comment and interpretation, including where appropriate, cross-checking between a model and reality.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Representation of Data

Mean: $\dfrac{\Sigma x}{n}$ or $\dfrac{\Sigma xf}{\Sigma f}$

Standard deviation: $\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ or $\sqrt{\dfrac{\Sigma(x-\bar{x})^2 f}{\Sigma f}} = \sqrt{\dfrac{\Sigma x^2 f}{\Sigma f} - \bar{x}^2}$

Representation of Data

C3.1a C3.1b C3.2 C3.3 N3.1 N3.2 N3.3 IT3.3 WO3.1

Candidates should be able to:

(a) select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations of data may have (in addition to the representations in (c) below, candidates should be familiar with pie charts, bar charts and frequency polygons);

(b) extract from a table or statistical diagram salient features of the data, and express conclusions verbally;

(c) construct and interpret stem-and-leaf diagrams (including ordered and back-to-back stem-and-leaf diagrams), box-and-whisker plots, histograms and cumulative frequency graphs;

(d) understand, use and interpret different measures of central tendency (mean, median, mode) and variation (range, interquartile range, standard deviation), e.g. in comparing and contrasting sets of data;

(e) calculate the mean and standard deviation of a set of data (including grouped data) either from the data itself or from given totals such as Σx and Σx^2, or $\Sigma(x-a)$ and $\Sigma(x-a)^2$.

Probability

Candidates should be able to:

(a) understand the terms permutation and combination;

(b) solve problems about selections, e.g. finding the number of ways in which a team of 3 men and 2 women can be selected from a group of 6 men and 5 women;

(c) solve problems about arrangements of objects in a line, including those involving

  (i) repetition (e.g. the number of ways of arranging the letters of the word 'NEEDLESS'),

  (ii) restriction (e.g. the number of ways several people can stand in a line if 2 particular people must — or must not — stand next to each other);

(d) evaluate probabilities in simple cases by means of enumeration of elementary events (e.g. for the total score when two fair dice are thrown) or by calculation using permutations and combinations;

(e) use addition and multiplication of probabilities, as appropriate, in simple cases;

(f) understand informally the meaning of exclusive and independent events, and calculate and use conditional probabilities in simple cases, e.g. situations that can be represented by means of a tree diagram.

Discrete Random Variables

Candidates should be able to:

(a) construct a probability distribution table relating to a given situation involving a discrete random variable X, and calculate the expectation, variance and standard deviation of X (the notations $\mathrm{E}(X)$ for expectation (also referred to as expected value or mean) and $\mathrm{Var}(X)$ for variance, are included);

(b) use formulae for probabilities for the binomial and geometric distributions, and model given situations by one of these, as appropriate (the notations $\mathrm{B}(n, p)$ and $\mathrm{Geo}(p)$ are included);

(c) use tables of cumulative binomial probabilities (or equivalent calculator functions);

(d) use formulae for the expectation and variance of the binomial distribution, and for the expectation of the geometric distribution.

Bivariate Data

C3.1a C3.3 IT3.3

Candidates should be able to:

(a) calculate, both from simple raw data and from summarised data, the product moment correlation coefficient for a set of bivariate data;

(b) understand the basis of Spearman's coefficient of rank correlation, and calculate its value (questions set will not involve tied ranks);

(c) interpret the value of a product moment correlation coefficient or of Spearman's rank correlation coefficient in relation to the appearance of a scatter diagram, with particular reference to values close to $-1, 0, 1$;

(d) understand that the value of a correlation coefficient is unaffected by linear transformations (coding) of the variables;

(e) understand the difference between an independent (or controlled) variable and a dependent variable;

(f) understand the concepts of least squares and regression lines in the context of a scatter diagram;

(g) calculate, both from simple raw data and from summarised data, the equation of a regression line, understand the distinction between the regression line of y on x and that of x on y, and use the fact that both regression lines pass through the mean centre $(\bar{x}, \bar{y})$;

(h) select and use, in the context of a problem, the appropriate regression line to estimate a value, and be able to interpret in context the uncertainties of such estimations.

5.13 A2 MODULE 4733: PROBABILITY AND STATISTICS 2 (S2)

Preamble

Knowledge of the specification content of Modules *C1* to *C4* and *S1* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *S2*.

Continuous Random Variables

Candidates should be able to:

(a) understand the concept of a continuous random variable, and recall and use properties of a probability density function (restricted to functions defined over a single interval);

(b) use a probability density function to solve problems involving probabilities, and to calculate the mean and variance of a distribution (explicit knowledge of the cumulative distribution function is not included, but location of the median, for example, in simple cases by direct consideration of an area may be required).

The Normal Distribution

Candidates should be able to:

(a) understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables, or equivalent calculator functions (knowledge of the density function is not expected);

(b) solve problems concerning a variable *X*, where $X \sim \mathrm{N}(\mu, \sigma^2)$, including

  (i) finding the value of $\mathrm{P}(X > x_1)$, or a related probability, given the values of x_1, μ, σ,

  (ii) finding a relationship between x_1, μ and σ given the value of $\mathrm{P}(X > x_1)$ or a related probability;

(c) recall conditions under which the normal distribution can be used as an approximation to the binomial distribution (*n* large enough to ensure that $np > 5$ and $nq > 5$), and use this approximation, with a continuity correction, in solving problems.

The Poisson Distribution

Candidates should be able to:

(a) calculate probabilities for the distribution $\mathrm{Po}(\mu)$, both directly from the formula and also by using tables of cumulative Poisson probabilities (or equivalent calculator functions);

(b) use the result that if $X \sim \mathrm{Po}(\mu)$ then the mean and variance of *X* are each equal to μ;

(c) understand informally the relevance of the Poisson distribution to the distribution of random events, and use the Poisson distribution as a model;

(d) use the Poisson distribution as an approximation to the binomial distribution where appropriate ($n > 50$ and $np < 5$, approximately);

(e) use the normal distribution, with continuity correction, as an approximation to the Poisson distribution where appropriate ($\mu > 15$, approximately).

Sampling and Hypothesis Tests

IT3.1

Candidates should be able to:

(a) understand the distinction between a sample and a population, and appreciate the benefits of randomness in choosing samples;

(b) explain in simple terms why a given sampling method may be unsatisfactory and suggest possible improvements (knowledge of particular methods of sampling, such as quota or stratified sampling, is not required, but candidates should have an elementary understanding of the use of random numbers in producing random samples);

(c) recognise that a sample mean can be regarded as a random variable, and use the facts that $\mathrm{E}(\bar{X}) = \mu$ and that $\mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{n}$;

(d) use the fact that $\bar{X}$ has a normal distribution if X has a normal distribution;

(e) use the Central Limit Theorem where appropriate;

(f) calculate unbiased estimates of the population mean and variance from a sample, using either raw or summarised data (only a simple understanding of the term 'unbiased' is required);

(g) understand the nature of a hypothesis test, the difference between one-tail and two-tail tests, and the terms 'null hypothesis', 'alternative hypothesis', 'significance level', 'rejection region' (or 'critical region'), 'acceptance region' and 'test statistic';

(h) formulate hypotheses and carry out a hypothesis test of a population proportion in the context of a single observation from a binomial distribution, using either direct evaluation of binomial probabilities or a normal approximation with continuity correction;

(i) formulate hypotheses and carry out a hypothesis test of a population mean in the following cases:

   (i) a sample drawn from a normal distribution of known variance,

   (ii) a large sample, using the Central Limit Theorem and an unbiased variance estimate derived from the sample,

   (iii) a single observation drawn from a Poisson distribution, using direct evaluation of Poisson probabilities;

(j) understand the terms 'Type I error' and 'Type II error' in relation to hypothesis tests;

(k) calculate the probabilities of making Type I and Type II errors in specific situations involving tests based on a normal distribution or approximation, or on direct evaluation of binomial or Poisson probabilities.

5.14 A2 MODULE 4734: PROBABILITY AND STATISTICS 3 (S3)

Preamble

Knowledge of the specification content of Modules *C1* to *C4*, *S1* and *S2* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *S3*.

Candidates should know the following formulae, none of which is included in the List of Formulae made available for use in the examination.

Expectation Algebra

$\mathrm{E}(aX+b)=a\,\mathrm{E}(X)+b$

$\mathrm{Var}(aX+b)=a^2\,\mathrm{Var}(X)$

$\mathrm{E}(aX+bY)=a\,\mathrm{E}(X)+b\,\mathrm{E}(Y)$

Continuous Random Variables

Candidates should be able to:

(a) use probability density functions which may be defined 'piecewise';

(b) use, in simple cases, the general result $\mathrm{E}(\mathrm{g}(X))=\int \mathrm{g}(x)\,\mathrm{f}(x)\,\mathrm{d}x$, where $\mathrm{f}(x)$ is the probability density function of the continuous random variable *X* and $\mathrm{g}(X)$ is a function of *X*;

(c) understand and use the relationship between the probability density function and the (cumulative) distribution function, and use either to evaluate the median, quartiles and other percentiles;

(d) use (cumulative) distribution functions of related variables in simple cases, e.g. given the c.d.f. of a variable *X*, to find the c.d.f. and hence the p.d.f. of *Y*, where $Y=X^3$.

Linear Combinations of Random Variables

Candidates should be able to:

(a) use, in the course of solving problems, the results that

(i) $\mathrm{E}(aX+b)=a\,\mathrm{E}(X)+b$ and $\mathrm{Var}(aX+b)=a^2\,\mathrm{Var}(X)$,

(ii) $\mathrm{E}(aX+bY)=a\,\mathrm{E}(X)+b\,\mathrm{E}(Y)$,

(iii) $\mathrm{Var}(aX+bY)=a^2\,\mathrm{Var}(X)+b^2\,\mathrm{Var}(Y)$ for independent *X* and *Y*,

(iv) if *X* has a normal distribution then so does $aX+b$,

(v) if *X* and *Y* have independent normal distributions then $aX+bY$ has a normal distribution,

(vi) if *X* and *Y* have independent Poisson distributions then $X+Y$ has a Poisson distribution.

Confidence Intervals; the *t* Distribution

Candidates should be able to:

(a) determine a confidence interval for a population mean, using a normal distribution, in the context of

- (i) a sample drawn from a normal population of known variance,
- (ii) a large sample, using the Central Limit Theorem and an unbiased variance estimate derived from the sample;

(b) determine, from a large sample, an approximate confidence interval for a population proportion;

(c) use a *t* distribution, with the appropriate number of degrees of freedom, in the context of a small sample drawn from a normal population of unknown variance

- (i) to determine a confidence interval for the population mean,
- (ii) to carry out a hypothesis test of the population mean.

Difference of Population Means and Proportions

Candidates should be able to:

(a) understand the difference between a two-sample test and a paired-sample test, and select the appropriate form of test in solving problems;

(b) formulate hypotheses and carry out a test for a difference of population means or population proportions using a normal distribution, and appreciate the conditions necessary for the test to be valid;

(c) calculate a pooled estimate of a population variance based on the data from two samples;

(d) formulate hypotheses and carry out either a two-sample *t*-test or a paired-sample *t*-test, as appropriate, for a difference of population means, and appreciate the conditions necessary for these tests to be valid;

(e) calculate a confidence interval for a difference of population means, using a normal distribution or a *t* distribution, as appropriate.

χ^2 tests

Candidates should be able to:

(a) fit a theoretical distribution, as prescribed by a given hypothesis, to given data (questions set will not involve lengthy calculations);

(b) use a χ^2 test with the appropriate number of degrees of freedom to carry out the corresponding goodness of fit test (classes should be combined so that each expected frequency is at least 5);

(c) use a χ^2 test with the appropriate number of degrees of freedom to test for independence in a contingency table (rows or columns, as appropriate, should be combined so that each expected frequency is at least 5, and Yates' correction should be used in the special case of a 2×2 table).

5.15 A2 MODULE 4735: PROBABILITY AND STATISTICS 4 (S4)

Preamble

Knowledge of the specification content of Modules *C1* to *C4* and *S1* to *S3* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *S4*.

Candidates will need to be familiar with, at least, the section on using the first few terms of the Maclaurin series from *FP2* and, in particular, be familiar with the series expansion of e^x.

Probability

Candidates should be able to:

(a) use the notation $\mathrm{P}(A)$ for the probability of the event A, and the notations $\mathrm{P}(A \cup B)$, $\mathrm{P}(A \cap B)$, $\mathrm{P}(A \mid B)$ relating to probabilities involving two events;

(b) understand and use the result $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$, and extend it to deal with the union of three events;

(c) use the result $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ and the ideas underlying Bayes' theorem to solve problems involving conditional probability (the formal statement of Bayes' theorem itself is not required).

Non-parametric Tests

Candidates should be able to:

(a) understand what is meant by a non-parametric significance test, appreciate situations where such tests are useful, and select an appropriate test;

(b) understand, in simple terms, the basis of sign tests, Wilcoxon signed-rank tests and the Wilcoxon rank-sum test, and use normal approximations where appropriate in these tests;

(c) test a hypothesis concerning a population median using a single-sample sign test and a single-sample Wilcoxon signed-rank test (problems in which observations coincide with the hypothetical population median will not be set);

(d) test for identity of populations using a paired-sample sign test, a Wilcoxon matched-pairs signed-rank test and (for unpaired samples) a Wilcoxon rank-sum test (problems involving tied ranks will not be set).

Probability Generating Functions

Candidates should be able to:

(a) understand the concept of a probability generating function and construct and use the probability generating function for given distributions (including the discrete uniform, binomial, geometric and Poisson);

(b) use formulae for the mean and variance of a discrete random variable in terms of its probability generating function, and use these formulae to calculate the mean and variance of probability distributions;

(c) use the result that the probability generating function of the sum of independent random variables is the product of the probability generating functions of those random variables.

Moment Generating Functions

Candidates should be able to:

(a) understand the concept of a moment generating function for both discrete and continuous random variables, construct and use the moment generating function for given distributions;

(b) use the moment generating function of a given distribution to find the mean and variance;

(c) use the result that the moment generating function of the sum of independent random variables is the product of the moment generating functions of those random variables.

Estimators

Candidates should be able to:

(a) understand that an estimator of a parameter of a distribution is a random variable, and understand the meaning of 'biased' and 'unbiased' as applied to estimators (both continuous and discrete distributions are included);

(b) determine in simple cases whether a given estimator is biased or unbiased, and find the variance of an estimator;

(c) determine which of two unbiased estimators of a parameter is more efficient, by comparing their variances, and understand why the more efficient estimator is generally to be preferred.

Discrete Bivariate Distributions

Candidates should be able to:

(a) understand the joint probability distribution of a pair of discrete random variables, and find and use marginal distributions;

(b) determine and use the distribution of one variable conditional on a particular value of the other;

(c) determine whether or not two random variables are independent;

(d) calculate the covariance for a pair of discrete random variables, and understand the relationship between zero covariance and independence;

(e) use $\mathrm{Var}(aX \pm bY) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y) \pm 2ab\,\mathrm{Cov}(X, Y)$.

5.16 AS MODULE 4736: DECISION MATHEMATICS 1 (D1)

Preamble

Knowledge of the specification content of Modules *C1* and *C2* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *D1*.

The specification content of this module is to be understood in the context of modelling real-life situations, and examination questions may ask for comment and interpretation, including where appropriate cross-checking between a model and reality.

Algorithms

Candidates should be able to:

(a) understand the definition of an algorithm;

(b) appreciate why an algorithmic approach to problem-solving is generally preferable to *ad hoc* methods, and understand the limitations of algorithmic methods;

(c) understand the meaning of the order of an algorithm, and determine the order of a given algorithm in simple cases, including the algorithms for standard network problems;

(d) interpret and apply simple algorithms defined by flow diagrams or given as a listing in words;

(e) show familiarity with simple algorithms concerning sorting and packing, including

  (i) bubble and shuttle sorts,

  (ii) first-fit methods (first-fit and first-fit decreasing).

Graph Theory

Candidates should be able to:

(a) understand the meaning of the terms 'arc' (or 'edge'), 'node' (or 'vertex'), 'path', 'tree' and 'cycle';

(b) use the orders of the nodes in a graph to determine whether the graph is Eulerian or semi-Eulerian or neither;

(c) solve simple problems involving planar graphs, both directed and undirected.

Networks

Candidates should be able to:

(a) recall that a network is a graph in which each arc is assigned a ‘weight’, and use networks as mathematical models;

(b) apply Prim’s and Kruskal’s algorithms in solving the minimum connector problem to find a minimum spanning tree (including the use of a matrix representation for Prim’s algorithm);

(c) find a solution to the travelling salesperson problem in simple cases, and in other cases

  (i) determine an upper bound by using the nearest neighbour method,

  (ii) use short-cuts where possible to improve on an upper bound,

  (iii) use minimum connector methods on a reduced network to determine a lower bound;

(d) use Dijkstra’s algorithm to determine the shortest path between two nodes;

(e) solve simple cases of the route inspection problem for at most six odd nodes by consideration of all possible pairings of the odd nodes.

Linear Programming

Candidates should be able to:

(a) formulate in algebraic terms a real-world problem posed in words, including the identification of relevant variables, constraints and objective function;

(b) set up a linear programming formulation in the form ‘maximise (or minimise) objective, subject to inequality constraints and trivial constraints of the form $variable \geq 0$’, and use slack variables to convert inequality constraints into equations together with trivial constraints;

(c) carry out a graphical solution for 2-variable problems, including cases where integer solutions are required;

(d) use the Simplex method for maximising an objective function, interpret the values of the variables and the objective function at any stage in the Simplex method.

5.17 A2 MODULE 4737: DECISION MATHEMATICS 2 (D2)

Preamble

Knowledge of the specification content of Modules *C1* to *C4* and *D1* is assumed, and candidates may be required to demonstrate such knowledge in answering questions in Unit *D2*.

The specification content of this module is to be understood in the context of modelling real-life situations, and examination questions may ask for comment and interpretation, including where appropriate cross-checking between a model and reality.

Game Theory

Candidates should be able to:

(a) understand the idea of a zero-sum game and its representation by means of a pay-off matrix;

(b) identify play-safe strategies and stable solutions;

(c) reduce a matrix by using a dominance argument;

(d) determine an optimal mixed strategy for a game with no stable solution

  (i) by using a graphical method for $2 \times n$ or $n \times 2$ games, where $n = 1, 2$ or 3,

  (ii) by converting higher order games to linear programming problems that could then be solved using the Simplex method.

Flows in a Network

Candidates should be able to:

(a) represent flow problems by means of a network of directed arcs, and interpret network diagrams;

(b) find the optimum flow rate in a network, subject to given constraints (problems may include both upper and lower capacities);

(c) understand the meaning of the value of a cut, use the maximum flow-minimum cut theorem and explain why it works;

(d) introduce a supersource or supersink in networks with more than one source or sink, and replace a vertex of restricted capacity by a pair of unrestricted vertices connected by a suitable flow;

(e) use a labelling procedure, with arrows showing how much less could flow in each direction, to augment a flow and hence determine the maximum flow in a network.

Matching and Allocation Problems

Candidates should be able to:

(a) represent a matching problem by means of a bipartite graph;

(b) use an algorithm to find a maximal matching by the construction of an alternating path;

(c) interpret allocation problems as minimum-cost matching problems;

(d) use the Hungarian algorithm to find a solution to an allocation problem, including the use of a dummy row or column, use the covering method to check whether a matching is maximal, and augment and interpret a revised cost matrix.

Critical Path Analysis

Candidates should be able to:

(e) construct and interpret activity networks, using activity on arc;

(f) carry out forward and reverse passes to determine earliest and latest start times and finish times, or early and late event times;

(g) identify critical activities and find a critical path;

(h) construct and interpret cascade charts and resource histograms, and carry out resource levelling.

Dynamic Programming

Candidates should be able to:

(i) understand the concept of dynamic programming, working backwards with sub-optimisation;

(j) use stage and state variables, actions and costs;

(k) set up a dynamic programming tabulation and use it to solve a problem involving finding a minimum, maximum, minimax or maximin.

6 Further Information and Training for Teachers

In support of these specifications, OCR will make the following materials and services available to teachers:

- up-to-date copies of these specifications;
- a full programme of In-Service Training (INSET) meetings;
- specimen question papers and mark schemes;
- past question papers and mark schemes after each examination session;
- a Report on the Examination, compiled by senior examining personnel, after each examination session.

If you would like further information about the specification, please contact OCR.

Appendix A: Key Skills

This Appendix offers detailed guidance on the Key Skills evidence that a candidate might produce during their programme. It focuses on the evidence required to meet the criteria for the internally assessed Key Skills portfolio. For example, in producing work for assessment of C3.2 (Read and synthesise information from two extended documents about a complex subject. One of these documents should include at least one image.) a candidate is required to:

- select and read material that contains the information you need;
- identify accurately, and compare, the lines of reasoning and main points from texts and images; and
- synthesise the key information in a form that is relevant to your purpose.

The Key Skills and Evidence Requirements below are quoted from Part B of the QCA Key Skills specifications and, as such, are addressed to the candidate. The text below the Evidence Requirements is guidance for teachers about how the specifications might be used to provide teaching and learning opportunities and/or assessment opportunities for the Key Skill.

For further information, teachers should refer to QCA's Key Skills specifications (for use in programmes starting from September 2000).

For further information about the assessment and certification of Key Skills, teachers should contact OCR.

C3 COMMUNICATION (LEVEL 3)

C3.1a Contribute to a group discussion about a complex subject

Evidence requirements

- Make clear and relevant contributions in a way that suits your purpose and situation.
- Listen and respond appropriately to others, and develop points and ideas.
- Create opportunities for others to contribute when appropriate.

Possible opportunities

The extent to which the work produced by candidates can provide evidence for C3.1a depends upon the individual approach to teaching and learning. In each of the module specifications there is very little which addresses C3.1a directly. However, some parts of the specifications, particularly in the application areas, lend themselves to an investigative approach which could generate evidence.

Module M1: For example, in 5.8.2 (Equilibrium of a particle), candidates could use a mechanics kit to explore friction. This work could be organised to produce evidence from discussions between groups of candidates.

Module S1: Similarly, in 5.12.1 (Representation of data) or 5.12.4 (Bivariate data), candidates who had collected their own data as part of the learning process could discuss with each other and with other members of a class details of the methodology and any problems that they had encountered.

C3.1b Make a presentation about a complex subject, using at least one image to illustrate complex points

Evidence requirements

- Speak clearly and adapt your style of presentation to suit your purpose, audience and situation.
- Structure what you say so that the sequence of information and ideas may be easily followed.
- Use a range of techniques to engage the audience, including effective use of images.

Possible opportunities

Module M1: A candidate who had carried out experimental work to investigate the modelling of friction in the course of studying the material of 5.8 2 (Equilibrium of a particle) could present the conclusions to other members of a class.

Module S1: In studying 5.12.1 (Representation of data), a candidate could analyse a set of raw data and present his or her conclusions to other members of a class.

C3.2 Read and synthesise information from two extended documents that deal with a complex subject. One of these documents should include at least one image

Evidence requirements

- Select and read material that contains the information you need.
- Identify accurately, and compare, the lines of reasoning and main points from texts and images.
- Synthesise the key information in a form that is relevant to your purpose.

Possible opportunities

It is expected that all candidates will be able to demonstrate some competence in this area. All candidates could use text-books to obtain information and will practise the skill of extracting relevant information from various sources to provide data for calculations. Such sources could be in a variety of forms including tables, charts, diagrams and graphs, and could include redundant information, requiring a selection to be made.

Module S1: For example, in 5.12.1 (Representation of data), a candidate is required to be able to extract important features from a table or statistical diagram and to summarise his or her conclusions in words.

C3.3 Write two different types of documents about complex subjects. One piece of writing should be an extended document and include at least one image

Evidence requirements

- Select and use a form and style of writing that is appropriate to your purpose and complex subject matter.
- Organise relevant information clearly and coherently, using specialist vocabulary when appropriate.
- Ensure your text is legible and your spelling, grammar and punctuation are accurate so your meaning is clear.

Possible opportunities

It is expected that a candidate is able to use mathematical expressions, graphs, sketches and diagrams with accuracy and skill and to use mathematical language correctly to proceed logically through extended arguments. These skills will enable the candidate to demonstrate evidence of an appropriate form of written presentation and appropriate use of specialist vocabulary. The extent to which this evidence can contribute towards meeting the requirements of C3.3 depends upon the individual approach to teaching and learning. There are opportunities throughout these specifications to organise work on particular topics so that evidence could be generated.

Modules C3, FP2: In studying approximate numerical methods for solution of equations in 5.3.4 (Numerical methods in module C3) or 5.6.5 (Numerical methods in module FP2), a candidate could compare different methods for a variety of equations, including cases where a method failed, and then present the findings as an extended document.

Modules M1, M2, M3: A number of Mechanics topics might lend themselves to experimental work, e.g. using a mechanics kit, leading to a written report in which conclusions are presented, and in which appropriate images and diagrams will naturally occur. Examples are Friction (5.8.2 in module M1), Centre of mass and Moments (5.9.1 and 5.9.2 in module M2), and Elasticity (5.10.2 in module M3).

Module S1: Data collected and/or analysed by a candidate in the course of studying 5.12.1 (Representation of data) or 5.12.4 (Bivariate data) might form the basis of a written report which would include relevant graphs and diagrams.

N3 APPLICATION OF NUMBER (LEVEL 3)

You must:

Plan and carry through at least **one** substantial and complex activity that includes tasks for N3.1, N3.2 and N3.3

Note: There are many opportunities within all branches of mathematics for providing evidence for some aspects of this Key Skill, but references to 'working with a large data set' and 'handling statistics' indicate that a task that is largely statistics-based is most likely to be satisfactory.

N3.1 Plan, and interpret information from two different types of sources, including a large data set

Evidence requirements

- Plan how to obtain and use the information required to meet the purpose of your activity.
- Obtain the relevant information.
- Choose appropriate methods for obtaining the results you need and justify your choice.

Possible opportunities

Some aspects of the statistics covered in these specifications lend themselves to the possibility of investigative work, in which the use of a large data set could play a part.

Module S1: In 5.12.1 (Representation of data) an investigative task that involved the collection of a substantial amount of data, and its analysis and subsequent presentation in readily comprehensible form, could be capable of providing evidence relating to N3.1.

N3.2 Carry out multi-stage calculations to do with amounts and sizes; scales and proportion; handling statistics; rearranging and using formulae

You should work with a large data set on at least **one** occasion.

Evidence requirements

- Carry out calculations to appropriate levels of accuracy, clearly showing your methods.
- Check methods and results to help ensure errors are found and corrected.

Possible opportunities

Any substantial task of a statistical nature should provide many opportunities for carrying out and checking calculations, so a task chosen to satisfy the requirement of N3.1 should also enable evidence for N3.2 to be generated.

Module S1: In 5.12.1 (Representation of data) an investigative task that involved the collection of a substantial amount of data, and its analysis and subsequent presentation in readily comprehensible form, could be capable of providing evidence relating to N3.2.

N3.3 Interpret results of your calculations, present your findings and justify your methods. You must use at least one graph, one chart and one diagram

Evidence requirements

- Select appropriate methods of presentation and justify your choice.
- Present your findings effectively.
- Explain how the results of your calculations relate to the purpose of your activity.

Possible opportunities

Any substantial task of a statistical nature should provide many opportunities for presentations involving graphs, charts and diagrams, and matters of interpretation and methodology will usually arise also. Thus a task chosen to satisfy the requirement of N3.1 and N3.2 above should also enable evidence for N3.3 to be generated.

Module S1: In 5.12.1 (Representation of data) an investigative task that involved the collection of a substantial amount of data, and its analysis and subsequent presentation in readily comprehensible form, could be capable of providing evidence relating to N3.3.

IT3 IT (LEVEL 3)

You must:

Plan and carry through at least **one** substantial activity that includes tasks for IT3.1, IT3.2 and IT3.3.

IT3.1 Plan, and use different sources to search for, and select, information required for two different purposes

Evidence requirements

- Plan how to obtain and use the information required to meet the purpose of your activity.
- Choose appropriate sources and techniques for finding information and carry out effective searches.
- Make selections based on judgements of relevance and quality.

Possible opportunities

Although these specifications provide many opportunities for the use of different aspects of IT, the particular requirements concerning searching and selecting information are of very limited relevance in most of the areas of mathematics than candidates will be studying, and opportunities for producing evidence are correspondingly unlikely to arise naturally.

Module S2: In the course of studying 5.13.4 (Sampling and hypothesis tests) candidates might carry out practical work involving selecting samples from a database.

IT3.2 Explore, develop, and exchange information and derive new information to meet two different purposes

Evidence requirements

- Enter and bring together information in a consistent form, using automated routines where appropriate.
- Create and use appropriate structures and procedures to explore and develop information and derive new information.
- Use effective methods of exchanging information to support your purpose.

Possible opportunities

The most relevant areas of mathematics for the use of IT in a way relevant to generating evidence for IT3.2 are likely to be those in which numerical processes dominate (e.g. approximate solution of equations) or in which graphical output is useful. There may be some additional opportunities in the teaching and learning of statistical topics in which simulation is used, and in those parts of decision mathematics involving algorithms, e.g. the Simplex method.

Module C3: A candidate could choose to use a graph-plotting package to locate approximately the roots of an equation and then progress to use a spreadsheet to locate each root to a prescribed degree of accuracy.

IT3.3 Present information from different sources for two different purposes and audiences. Your work must include at least one example of text, one example of images and one example of numbers

Evidence requirements

- Develop the structure and content of your presentation using the views of others, where appropriate, to guide refinements.
- Present information effectively, using a format and style that suits your purpose and audience.
- Ensure your work is accurate and makes sense.

Possible opportunities

Candidates can be encouraged, in many areas of the specifications, to make use of available technology to present mathematical information effectively. Where candidates produce written reports they can be encouraged to do so via software that allows them to combine text, graphics and numerical information in one document, for example by importing numerical work and graphics into a word-processed text. It is probable that candidates will be able to show proficiency in using word-processing packages to produce correctly formatted mathematical expressions.

Modules C2, C3 and FP2: In 5.2.2 (Sequences and series in module C2), a candidate could use a spreadsheet to investigate the behaviour of a sequence and then embed a scatter-graph on the spreadsheet to show the behaviour visually. Similarly, in the sections on Numerical methods (5.3.5 in module C3 and 5.6.5 in module FP2) a candidate who has carried out computer-based investigative work on approximate solutions of equations would have the material available to prepare a report which would naturally include text, numbers and graphics.

Module S1: Any investigation involving the content of this module that a candidate had carried out (e.g. in accordance with the possibilities noted above under C3) should be suitable for presenting as a document including text and images.

WO3 WORKING WITH OTHERS (LEVEL 3)

You must:

Provide at least **one** substantial example of meeting the standard for WO3.1, WO3.2 and WO3.3 (you must show you can work in both one-to-one and group situations).

WO3.1 Plan the activity with others, agreeing objectives, responsibilities and working arrangements

Evidence requirements

- Agree realistic objectives for working together and what needs to be done to achieve them.
- Exchange information, based on appropriate evidence, to help agree responsibilities.
- Agree suitable working arrangements with those involved.

Possible opportunities

Opportunities within these specifications for generating evidence relevant to WO3.1 are extremely limited. There may be a few instances where investigative work on a topic within one of the modules can be adapted to involve a collaborative approach by a number of candidates, but such tasks are not likely to be sufficiently extensive to be capable of generating useful evidence for WO3.1.

Module S1: There may be some limited opportunity to organise investigative work on 5.12.1 (Representation of data) on a group basis, so that different members of a group work on different aspects of data collection or analysis.

WO3.2 Seek to establish and maintain co-operative working relationships over an extended period of time, agreeing changes to achieve agreed objectives

Evidence requirements

- Organise and carry out tasks so that you can be effective and efficient in meeting your responsibilities and produce the quality of work required.
- Seek to establish and maintain co-operative working relationships, agreeing ways to overcome any difficulties.
- Exchange accurate information on progress of work, agreeing changes where necessary to achieve objectives.

Possible opportunities

Opportunities within these specifications for generating evidence relevant to WO3.2 are extremely limited. There may be a few instances where investigative work on a topic within one of the modules can be adapted to involve a collaborative approach by a number of candidates, but such tasks are not likely to be sufficiently extensive to be capable of generating useful evidence for WO3.2.

WO3.3 Review work with others and agree ways of improving collaborative work in the future

Evidence requirements

- Agree the extent to which work with others has been successful and the objectives have been met.
- Identify factors that have influenced the outcome.
- Agree ways of improving work with others in the future.

Possible opportunities

Not applicable to these specifications.

LP3 IMPROVING OWN LEARNING AND PERFORMANCE (LEVEL 3)

You must:

Provide at least **one** substantial example of meeting the standard for LP3.1, LP3.2 and LP3.3.

LP3.1 Agree targets and plan how these will be met over an extended period of time, using support from appropriate people

Evidence requirements

- Seek information on ways to achieve what you want to do, including factors that might affect your plans.
- Use this information to agree realistic targets with appropriate people.
- Plan how you will effectively manage your time and use of support to meet targets, including alternative action for overcoming possible difficulties.

Possible opportunities

All modules in these specifications give some opportunities for generating evidence for aspects of LP3, particularly study-based learning, but the subject matter of individual modules is not of particular relevance.

LP3.2 Take responsibility for your learning by using your plan, and seeking feedback and support from relevant sources to help meet targets

Improve your performance by:

- studying a complex subject;
- learning through a complex practical activity;
- further study or practical activity that involves independent learning.

Evidence requirements

- Manage your time effectively to complete tasks, revising your plan as necessary.
- Seek and actively use feedback and support from relevant sources to help you meet your targets.
- Select and use different ways of learning to improve your performance, adapting approaches to meet new demands.

Possible opportunities

All modules in these specifications give some opportunities for generating evidence for aspects of LP3, particularly study-based learning, but the subject matter of individual modules is not of particular relevance.

LP3.3 Review progress on two occasions and establish evidence of achievements, including hoe you have used learning from other tasks to meet new demands.

Evidence requirements

- Provide information on the quality of your learning and performance, including factors that have affected the outcome.
- Identify targets you have met, seeking information from relevant sources to establish evidence of your achievements.
- Exchange views with appropriate people to agree ways to further improve your performance.

Possible opportunities

All modules in these specifications give some opportunities for generating evidence for aspects of LP3, particularly study-based learning, but the subject matter of individual modules is not of particular relevance.

PS3 Problem Solving (Level 3)

You must:

Provide at least **one** substantial example of meeting the standard for PS3.1, PS3.2 and PS3.3.

PS3.1 Explore a complex problem, come up with three options for solving it and justify the option for taking further

Evidence requirements

- Explore the problem, accurately analysing its features, and agree with others on how to show success in solving it.
- Select and use a variety of methods to come up with different ways of tackling the problem.
- Compare the main features of each possible option, including risk factors. and justify the option you select to take forward.

Possible opportunities

Not applicable to these specifications.

PS3.2 Plan and implement at least one option for solving the problem, review progress and revise your approach as necessary

Evidence requirements

- Plan how to carry out your chosen option and obtain agreement to go ahead from an appropriate person.
- Implement your plan effectively, using support and feedback from others.
- Review progress towards solving the problem and revise your approach as necessary.

Possible opportunities

Not applicable to these specifications.

PS3.3 Apply agreed methods to check if the problem has been solved, describe the results and review your approach to problem solving

Evidence requirements

- Agree, with an appropriate person, methods to check if the problem has been solved.
- Apply these methods accurately, draw conclusions and fully describe the results.
- Review your approach to problem solving, including whether alternative methods and options might have proved more effective.

Possible opportunities

Not applicable to these specifications.

Appendix B: Mathematical Formulae and Statistical Tables

Pure Mathematics

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Trigonometry

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Arithmetic Series

$$u_n = a + (n-1)d$$

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a + (n-1)d\}$$

Geometric Series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Binomial Series

$$\binom{n}{r}+\binom{n}{r+1}=\binom{n+1}{r+1}$$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ \ldots \ + \binom{n}{r} a^{n-r}b^r + \ \ldots \ + b^n \qquad (n \in \mathbb{N}),$$

where $\binom{n}{r} = {}^n\mathrm{C}_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ \ldots \ + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}x^r + \ \ldots \qquad (|x|<1,\ \ n \in \mathbb{R})$$

Logarithms and Exponentials

$$\mathrm{e}^{x\ln a} = a^x$$

Complex Numbers

$$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$$

$$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$$

The roots of $z^n = 1$ are given by $z = \mathrm{e}^{\frac{2\pi k \mathrm{i}}{n}}$, for $k = 0, 1, 2, \ldots, n-1$

Maclaurin's Series

$$\mathrm{f}(x) = \mathrm{f}(0) + x\,\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(0) + \ldots$$

$$\mathrm{e}^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \ (-1 < x \le 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \quad \text{for all } x$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots \quad (-1 \le x \le 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2r+1}}{(2r+1)!} + \ldots \text{ for all } x$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{2r}}{(2r)!} + \ldots \text{ for all } x$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots + \frac{x^{2r+1}}{2r+1} + \ldots (-1 < x < 1)$$

Hyperbolic Functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln\left\{x + \sqrt{x^2 - 1}\right\} \quad (x \ge 1)$$

$$\sinh^{-1} x = \ln\left\{x + \sqrt{x^2 + 1}\right\}$$

$$\tanh^{-1} x = \tfrac{1}{2} \ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$$

Coordinate Geometry

The perpendicular distance from (h, k) to $ax + by + c = 0$ is $\dfrac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$

The acute angle between lines with gradients m_1 and m_2 is $\tan^{-1}\left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

For $t = \tan \frac{1}{2} A$: $\sin A = \dfrac{2t}{1+t^2}$, $\cos A = \dfrac{1-t^2}{1+t^2}$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Vectors

The resolved part of $\mathbf{a}$ in the direction of $\mathbf{b}$ is $\dfrac{\mathbf{a}.\mathbf{b}}{|\mathbf{b}|}$

The point dividing AB in the ratio $\lambda : \mu$ is $\dfrac{\mu \mathbf{a} + \lambda \mathbf{b}}{\lambda + \mu}$

Vector product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \sin\theta\ \hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

If A is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector $\mathbf{b}$ is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through A with direction vector $\mathbf{b}$ has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \ (= \lambda)$$

The plane through A with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation

$n_1x + n_2y + n_3z + d = 0$ where $d = -\mathbf{a.n}$

The plane through non-collinear points A, B and C has vector equation

$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

The plane through the point with position vector $\mathbf{a}$ and parallel to $\mathbf{b}$ and $\mathbf{c}$ has equation

$\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$ is $\dfrac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

Matrix Transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\dfrac{1}{1-x^2}$

Integration (+ constant; $a > 0$ where relevant)

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$
$\sec^2 kx$	$\dfrac{1}{k}\tan kx$
$\tan x$	$\ln\lvert \sec x \rvert$
$\cot x$	$\ln\lvert \sin x \rvert$
$\operatorname{cosec} x$	$-\ln\lvert \operatorname{cosec} x + \cot x \rvert = \ln\lvert \tan(\tfrac{1}{2}x) \rvert$
$\sec x$	$\ln\lvert \sec x + \tan x \rvert = \ln\lvert \tan(\tfrac{1}{2}x + \tfrac{1}{4}\pi) \rvert$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln\cosh x$
$\dfrac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right) \quad (\lvert x \rvert < a)$
$\dfrac{1}{a^2 + x^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{x^2 - a^2}}$	$\cosh^{-1}\left(\dfrac{x}{a}\right)$ or $\ln\left\{x + \sqrt{x^2 - a^2}\right\} \quad (x > a)$
$\dfrac{1}{\sqrt{a^2 + x^2}}$	$\sinh^{-1}\left(\dfrac{x}{a}\right)$ or $\ln\left\{x + \sqrt{x^2 + a^2}\right\}$
$\dfrac{1}{a^2 - x^2}$	$\dfrac{1}{2a}\ln\left\lvert \dfrac{a+x}{a-x} \right\rvert = \dfrac{1}{a}\tanh^{-1}\left(\dfrac{x}{a}\right) \quad (\lvert x \rvert < a)$
$\dfrac{1}{x^2 - a^2}$	$\dfrac{1}{2a}\ln\left\lvert \dfrac{x-a}{x+a} \right\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Area of a Sector

$A = \frac{1}{2}\int r^2\,\mathrm{d}\theta$ (polar coordinates)

$A = \frac{1}{2}\int\left(x\frac{\mathrm{d}y}{\mathrm{d}t} - y\frac{\mathrm{d}x}{\mathrm{d}t}\right)\mathrm{d}t$ (parametric form)

Numerical Mathematics

Numerical Integration

The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Simpson's rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{3}h\{(y_0 + y_n) + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\}$,

where $h = \frac{b-a}{n}$ and n is even

Numerical Solution of Equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Mechanics

Motion in a Circle

Transverse velocity: $v = r\dot{\theta}$

Transverse acceleration: $\dot{v} = r\ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\frac{v^2}{r}$

Centres of Mass

For uniform bodies

Triangular lamina: $\frac{2}{3}$ along median from vertex

Solid hemisphere, radius $r : \frac{3}{8} r$ from centre

Hemispherical shell, radius $r : \frac{1}{2} r$ from centre

Circular arc, radius r, angle at centre $2\alpha : \dfrac{r \sin\alpha}{\alpha}$ from centre

Sector of circle, radius r, angle at centre $2\alpha : \dfrac{2r \sin\alpha}{3\alpha}$ from centre

Solid cone or pyramid of height $h : \frac{1}{4} h$ above the base on the line from centre of base to vertex

Conical shell of height $h : \frac{1}{3} h$ above the base on the line from centre of base to vertex

Moments of Inertia

For uniform bodies of mass m

Thin rod, length $2l$, about perpendicular axis through centre: $\frac{1}{3} ml^2$

Rectangular lamina about axis in plane bisecting edges of length $2l$: $\frac{1}{3} ml^2$

Thin rod, length $2l$, about perpendicular axis through end: $\frac{4}{3} ml^2$

Rectangular lamina about edge perpendicular to edges of length $2l$: $\frac{4}{3} ml^2$

Rectangular lamina, sides $2a$ and $2b$, about perpendicular axis through centre: $\frac{1}{3} m(a^2 + b^2)$

Hoop or cylindrical shell of radius r about axis: mr^2

Hoop of radius r about a diameter: $\frac{1}{2} mr^2$

Disc or solid cylinder of radius r about axis: $\frac{1}{2} mr^2$

Disc of radius r about a diameter: $\frac{1}{4} mr^2$

Solid sphere, radius r, about a diameter: $\frac{2}{5}mr^2$

Spherical shell of radius r about a diameter: $\frac{2}{3}mr^2$

Parallel axes theorem: $I_A = I_G + m(AG)^2$

Perpendicular axes theorem: $I_z = I_x + I_y$ (for a lamina in the x-y plane)

Probability and Statistics

Probability

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$

$$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B \mid A)$$

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(B \mid A)\,\mathrm{P}(A)}{\mathrm{P}(B \mid A)\,\mathrm{P}(A) + \mathrm{P}(B \mid A')\,\mathrm{P}(A')}$$

Bayes' Theorem: $\mathrm{P}(A_j \mid B) = \dfrac{\mathrm{P}(A_j)\,\mathrm{P}(B \mid A_j)}{\Sigma\,\mathrm{P}(A_i)\,\mathrm{P}(B \mid A_i)}$

Discrete Distributions

For a discrete random variable X taking values x_i with probabilities p_i

Expectation (mean): $\mathrm{E}(X) = \mu = \Sigma x_i p_i$

Variance: $\mathrm{Var}(X) = \sigma^2 = \Sigma(x_i - \mu)^2 p_i = \Sigma x_i^2 p_i - \mu^2$

For a function $\mathrm{g}(X)$: $\mathrm{E}(\mathrm{g}(X)) = \Sigma\,\mathrm{g}(x_i)\,p_i$

The probability generating function of X is $\mathrm{G}_X(t) = \mathrm{E}(t^X)$, and

$\mathrm{E}(X) = \mathrm{G}'_X(1)$,

$\mathrm{Var}(X) = \mathrm{G}''_X(1) + \mathrm{G}'_X(1) - \{\mathrm{G}'_X(1)\}^2$

For $Z = X + Y$, where X and Y are independent: $\mathrm{G}_Z(t) = \mathrm{G}_X(t)\,\mathrm{G}_Y(t)$

Standard Discrete Distributions:

Distribution of X	$P(X = x)$	Mean	Variance	P.G.F.
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$	$(1-p+pt)^n$
Poisson $\mathrm{Po}(\lambda)$	$e^{-\lambda} \frac{\lambda^x}{x!}$	λ	λ	$e^{\lambda(t-1)}$
Geometric Geo (p) on 1, 2, …	$p(1-p)^{x-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pt}{1-(1-p)t}$

Continuous Distributions

For a continuous random variable X having probability density function f

Expectation (mean): $\mathrm{E}(X) = \mu = \int x \, \mathrm{f}(x) \, \mathrm{d}x$

Variance: $\mathrm{Var}(X) = \sigma^2 = \int (x-\mu)^2 \, \mathrm{f}(x) \, \mathrm{d}x = \int x^2 \, \mathrm{f}(x) \, \mathrm{d}x - \mu^2$

For a function $\mathrm{g}(X)$: $\mathrm{E}(\mathrm{g}(X)) = \int \mathrm{g}(x) \mathrm{f}(x) \, \mathrm{d}x$

Cumulative distribution function: $\mathrm{F}(x) = \mathrm{P}(X \le x) = \int_{-\infty}^{x} \mathrm{f}(t) \, \mathrm{d}t$

The moment generating function of X is $\mathrm{M}_X(t) = \mathrm{E}(e^{tX})$ and

$\mathrm{E}(X) = \mathrm{M}'_X(0)$,

$\mathrm{E}(X^n) = \mathrm{M}_X^{(n)}(0)$,

$\mathrm{Var}(X) = \mathrm{M}''_X(0) - \{\mathrm{M}'_X(0)\}^2$

For $Z = X + Y$, where X and Y are independent: $\mathrm{M}_Z(t) = \mathrm{M}_X(t)\mathrm{M}_Y(t)$

Standard Continuous Distributions:

Distribution of *X*	**P.D.F.**	**Mean**	**Variance**	**M.G.F.**
Uniform (Rectangular) on $[a,\ b]$	$\dfrac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$	$\dfrac{e^{bt}-e^{at}}{(b-a)t}$
Exponential	$\lambda e^{-\lambda x}$	$\dfrac{1}{\lambda}$	$\dfrac{1}{\lambda^2}$	$\dfrac{\lambda}{\lambda-t}$
Normal $N(\mu,\ \sigma^2)$	$\dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2	$e^{\mu t+\frac{1}{2}\sigma^2 t^2}$

Expectation Algebra

Covariance: $\mathrm{Cov}(X,\ Y)=\mathrm{E}((X-\mu_X)(Y-\mu_Y))=\mathrm{E}(XY)-\mu_X\mu_Y$

$\mathrm{Var}(aX \pm bY)=a^2\,\mathrm{Var}(X)+b^2\,\mathrm{Var}(Y)\pm 2ab\,\mathrm{Cov}(X,\ Y)$

Product moment correlation coefficient: $\rho=\dfrac{\mathrm{Cov}(X,\ Y)}{\sigma_X\sigma_Y}$

If $X=aX'+b$ and $Y=cY'+d$, then $\mathrm{Cov}(X,\ Y)=ac\,\mathrm{Cov}(X',\ Y')$

For independent random variables X and Y

$\mathrm{E}(XY)=\mathrm{E}(X)\mathrm{E}(Y)$

$\mathrm{Var}(aX \pm bY)=a^2\,\mathrm{Var}(X)+b^2\,\mathrm{Var}(Y)$

Sampling Distributions

For a random sample $X_1, X_2, \ldots, X_n$ of n independent observations from a distribution having mean μ and variance σ^2

$\overline{X}$ is an unbiased estimator of μ, with $\mathrm{Var}(\overline{X})=\dfrac{\sigma^2}{n}$

S^2 is an unbiased estimator of σ^2, where $S^2=\dfrac{\Sigma(X_i-\overline{X})^2}{n-1}$

For a random sample of n observations from $\mathrm{N}(\mu,\ \sigma^2)$

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,\ 1)$$

$$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1} \quad \text{(also valid in matched-pairs situations)}$$

If X is the observed number of successes in n independent Bernoulli trials in each of which the probability of success is p, and $Y = \frac{X}{n}$, then

$$\mathrm{E}(Y) = p \ \text{ and } \ \mathrm{Var}(Y) = \frac{p(1-p)}{n}$$

For a random sample of n_x observations from $\mathrm{N}(\mu_x,\ \sigma_x^2)$ and, independently, a random sample of n_y observations from $\mathrm{N}(\mu_y,\ \sigma_y^2)$

$$\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{\dfrac{\sigma_x^2}{n_x}+\dfrac{\sigma_y^2}{n_y}}} \sim \mathrm{N}(0,\ 1)$$

If $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (unknown) then

$$\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{S_p^2\left(\dfrac{1}{n_x}+\dfrac{1}{n_y}\right)}} \sim t_{n_x+n_y-2}, \ \text{ where } \ S_p^2 = \frac{(n_x-1)S_x^2+(n_y-1)S_y^2}{n_x+n_y-2}$$

Correlation and Regression

For a set of n pairs of values $(x_i,\ y_i)$

$$S_{xx} = \Sigma(x_i-\bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i-\bar{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i-\bar{x})(y_i-\bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The product moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\{\Sigma(x_i - \bar{x})^2\}\{\Sigma(y_i - \bar{y})^2\}}} = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}}$$

Spearman's rank correlation coefficient is $r_s = 1 - \dfrac{6\Sigma d^2}{n(n^2-1)}$

The regression coefficient of y on x is $b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \bar{y} - b\bar{x}$

Distribution-free (Non-parametric) Tests

Goodness-of-fit test and contingency tables: $\displaystyle\sum \frac{(O_i - E_i)^2}{E_i} \sim \chi_\nu^2$

Approximate distributions for large samples:

Wilcoxon Signed Rank test: $T \sim \mathrm{N}\left(\frac{1}{4}n(n+1),\ \frac{1}{24}n(n+1)(2n+1)\right)$

Wilcoxon Rank Sum test (samples of sizes m and n, with $m \le n$):

$W \sim \mathrm{N}\left(\frac{1}{2}m(m+n+1),\ \frac{1}{12}mn(m+n+1)\right)$

CUMULATIVE BINOMIAL PROBABILITIES

$n = 5$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.7738	0.5905	0.4437	0.4019	0.3277	0.2373	0.1681	0.1317	0.1160	0.0778	0.0503	0.0313	0.0185	0.0102	0.0053	0.0041	0.0024	0.0010	0.0003	0.0001	0.0001	0.0000	0.0000
1	0.9774	0.9185	0.8352	0.8038	0.7373	0.6328	0.5282	0.4609	0.4284	0.3370	0.2562	0.1875	0.1312	0.0870	0.0540	0.0453	0.0308	0.0156	0.0067	0.0033	0.0022	0.0005	0.0000
2	0.9988	0.9914	0.9734	0.9645	0.9421	0.8965	0.8369	0.7901	0.7648	0.6826	0.5931	0.5000	0.4069	0.3174	0.2352	0.2099	0.1631	0.1035	0.0579	0.0355	0.0266	0.0086	0.0012
3	1.0000	0.9995	0.9978	0.9967	0.9933	0.9844	0.9692	0.9547	0.9460	0.9130	0.8688	0.8125	0.7438	0.6630	0.5716	0.5391	0.4718	0.3672	0.2627	0.1962	0.1648	0.0815	0.0226
4	1.0000	1.0000	0.9999	0.9999	0.9997	0.9990	0.9976	0.9959	0.9947	0.9898	0.9815	0.9688	0.9497	0.9222	0.8840	0.8683	0.8319	0.7627	0.6723	0.5981	0.5563	0.4095	0.2262
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 6$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.7351	0.5314	0.3771	0.3349	0.2621	0.1780	0.1176	0.0878	0.0754	0.0467	0.0277	0.0156	0.0083	0.0041	0.0018	0.0014	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
1	0.9672	0.8857	0.7765	0.7368	0.6554	0.5339	0.4202	0.3512	0.3191	0.2333	0.1636	0.1094	0.0692	0.0410	0.0223	0.0178	0.0109	0.0046	0.0016	0.0007	0.0004	0.0001	0.0000
2	0.9978	0.9842	0.9527	0.9377	0.9011	0.8306	0.7443	0.6804	0.6471	0.5443	0.4415	0.3438	0.2553	0.1792	0.1174	0.1001	0.0705	0.0376	0.0170	0.0087	0.0059	0.0013	0.0001
3	0.9999	0.9987	0.9941	0.9913	0.9830	0.9624	0.9295	0.8999	0.8826	0.8208	0.7447	0.6563	0.5585	0.4557	0.3529	0.3196	0.2557	0.1694	0.0989	0.0623	0.0473	0.0159	0.0022
4	1.0000	0.9999	0.9996	0.9993	0.9984	0.9954	0.9891	0.9822	0.9777	0.9590	0.9308	0.8906	0.8364	0.7667	0.6809	0.6488	0.5798	0.4661	0.3446	0.2632	0.2235	0.1143	0.0328
5	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9986	0.9982	0.9959	0.9917	0.9844	0.9723	0.9533	0.9246	0.9122	0.8824	0.8220	0.7379	0.6651	0.6229	0.4686	0.2649
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 7$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.6983	0.4783	0.3206	0.2791	0.2097	0.1335	0.0824	0.0585	0.0490	0.0280	0.0152	0.0078	0.0037	0.0016	0.0006	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9556	0.8503	0.7166	0.6698	0.5767	0.4449	0.3294	0.2634	0.2338	0.1586	0.1024	0.0625	0.0357	0.0188	0.0090	0.0069	0.0038	0.0013	0.0004	0.0001	0.0001	0.0000	0.0000
2	0.9962	0.9743	0.9262	0.9042	0.8520	0.7564	0.6471	0.5706	0.5323	0.4199	0.3164	0.2266	0.1529	0.0963	0.0556	0.0453	0.0288	0.0129	0.0047	0.0020	0.0012	0.0002	0.0000
3	0.9998	0.9973	0.9879	0.9824	0.9667	0.9294	0.8740	0.8267	0.8002	0.7102	0.6083	0.5000	0.3917	0.2898	0.1998	0.1733	0.1260	0.0706	0.0333	0.0176	0.0121	0.0027	0.0002
4	1.0000	0.9998	0.9988	0.9980	0.9953	0.9871	0.9712	0.9547	0.9444	0.9037	0.8471	0.7734	0.6836	0.5801	0.4677	0.4294	0.3529	0.2436	0.1480	0.0958	0.0738	0.0257	0.0038
5	1.0000	1.0000	0.9999	0.9999	0.9996	0.9987	0.9962	0.9931	0.9910	0.9812	0.9643	0.9375	0.8976	0.8414	0.7662	0.7366	0.6706	0.5551	0.4233	0.3302	0.2834	0.1497	0.0444
6	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9994	0.9984	0.9963	0.9922	0.9848	0.9720	0.9510	0.9415	0.9176	0.8665	0.7903	0.7209	0.6794	0.5217	0.3017
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 8$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.6634	0.4305	0.2725	0.2326	0.1678	0.1001	0.0576	0.0390	0.0319	0.0168	0.0084	0.0039	0.0017	0.0007	0.0002	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9428	0.8131	0.6572	0.6047	0.5033	0.3671	0.2553	0.1951	0.1691	0.1064	0.0632	0.0352	0.0181	0.0085	0.0036	0.0026	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
2	0.9942	0.9619	0.8948	0.8652	0.7969	0.6785	0.5518	0.4682	0.4278	0.3154	0.2201	0.1445	0.0885	0.0498	0.0253	0.0197	0.0113	0.0042	0.0012	0.0004	0.0002	0.0000	0.0000
3	0.9996	0.9950	0.9786	0.9693	0.9437	0.8862	0.8059	0.7414	0.7064	0.5941	0.4770	0.3633	0.2604	0.1737	0.1061	0.0879	0.0580	0.0273	0.0104	0.0046	0.0029	0.0004	0.0000
4	1.0000	0.9996	0.9971	0.9954	0.9896	0.9727	0.9420	0.9121	0.8939	0.8263	0.7396	0.6367	0.5230	0.4059	0.2936	0.2586	0.1941	0.1138	0.0563	0.0307	0.0214	0.0050	0.0004
5	1.0000	1.0000	0.9998	0.9996	0.9988	0.9958	0.9887	0.9803	0.9747	0.9502	0.9115	0.8555	0.7799	0.6846	0.5722	0.5318	0.4482	0.3215	0.2031	0.1348	0.1052	0.0381	0.0058
6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9974	0.9964	0.9915	0.9819	0.9648	0.9368	0.8936	0.8309	0.8049	0.7447	0.6329	0.4967	0.3953	0.3428	0.1869	0.0572
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9998	0.9993	0.9983	0.9961	0.9916	0.9832	0.9681	0.9610	0.9424	0.8999	0.8322	0.7674	0.7275	0.5695	0.3366
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 9$ p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.6302	0.3874	0.2316	0.1938	0.1342	0.0751	0.0404	0.0260	0.0207	0.0101	0.0046	0.0020	0.0008	0.0003	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9288	0.7748	0.5995	0.5427	0.4362	0.3003	0.1960	0.1431	0.1211	0.0705	0.0385	0.0195	0.0091	0.0038	0.0014	0.0010	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9916	0.9470	0.8591	0.8217	0.7382	0.6007	0.4628	0.3772	0.3373	0.2318	0.1495	0.0898	0.0498	0.0250	0.0112	0.0083	0.0043	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000
3	0.9994	0.9917	0.9661	0.9520	0.9144	0.8343	0.7297	0.6503	0.6089	0.4826	0.3614	0.2539	0.1658	0.0994	0.0536	0.0424	0.0253	0.0100	0.0031	0.0011	0.0006	0.0001	0.0000
4	1.0000	0.9991	0.9944	0.9910	0.9804	0.9511	0.9012	0.8552	0.8283	0.7334	0.6214	0.5000	0.3786	0.2666	0.1717	0.1448	0.0988	0.0489	0.0196	0.0090	0.0056	0.0009	0.0000
5	1.0000	0.9999	0.9994	0.9989	0.9969	0.9900	0.9747	0.9576	0.9464	0.9006	0.8342	0.7461	0.6386	0.5174	0.3911	0.3497	0.2703	0.1657	0.0856	0.0480	0.0339	0.0083	0.0006
6	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9957	0.9917	0.9888	0.9750	0.9502	0.9102	0.8505	0.7682	0.6627	0.6228	0.5372	0.3993	0.2618	0.1783	0.1409	0.0530	0.0084
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.9986	0.9962	0.9909	0.9805	0.9615	0.9295	0.8789	0.8569	0.8040	0.6997	0.5638	0.4573	0.4005	0.2252	0.0712
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9992	0.9980	0.9954	0.9899	0.9793	0.9740	0.9596	0.9249	0.8658	0.8062	0.7684	0.6126	0.3698
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 10$ p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.5987	0.3487	0.1969	0.1615	0.1074	0.0563	0.0282	0.0173	0.0135	0.0060	0.0025	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9139	0.7361	0.5443	0.4845	0.3758	0.2440	0.1493	0.1040	0.0860	0.0464	0.0233	0.0107	0.0045	0.0017	0.0005	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9885	0.9298	0.8202	0.7752	0.6778	0.5256	0.3828	0.2991	0.2616	0.1673	0.0996	0.0547	0.0274	0.0123	0.0048	0.0034	0.0016	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.9990	0.9872	0.9500	0.9303	0.8791	0.7759	0.6496	0.5593	0.5138	0.3823	0.2660	0.1719	0.1020	0.0548	0.0260	0.0197	0.0106	0.0035	0.0009	0.0003	0.0001	0.0000	0.0000
4	0.9999	0.9984	0.9901	0.9845	0.9672	0.9219	0.8497	0.7869	0.7515	0.6331	0.5044	0.3770	0.2616	0.1662	0.0949	0.0766	0.0473	0.0197	0.0064	0.0024	0.0014	0.0001	0.0000
5	1.0000	0.9999	0.9986	0.9976	0.9936	0.9803	0.9527	0.9234	0.9051	0.8338	0.7384	0.6230	0.4956	0.3669	0.2485	0.2131	0.1503	0.0781	0.0328	0.0155	0.0099	0.0016	0.0001
6	1.0000	1.0000	0.9999	0.9997	0.9991	0.9965	0.9894	0.9803	0.9740	0.9452	0.8980	0.8281	0.7340	0.6177	0.4862	0.4407	0.3504	0.2241	0.1209	0.0697	0.0500	0.0128	0.0010
7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9966	0.9952	0.9877	0.9726	0.9453	0.9004	0.8327	0.7384	0.7009	0.6172	0.4744	0.3222	0.2248	0.1798	0.0702	0.0115
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9995	0.9983	0.9955	0.9893	0.9767	0.9536	0.9140	0.8960	0.8507	0.7560	0.6242	0.5155	0.4557	0.2639	0.0861
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9975	0.9940	0.9865	0.9827	0.9718	0.9437	0.8926	0.8385	0.8031	0.6513	0.4013
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 12$ p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.5404	0.2824	0.1422	0.1122	0.0687	0.0317	0.0138	0.0077	0.0057	0.0022	0.0008	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8816	0.6590	0.4435	0.3813	0.2749	0.1584	0.0850	0.0540	0.0424	0.0196	0.0083	0.0032	0.0011	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9804	0.8891	0.7358	0.6774	0.5583	0.3907	0.2528	0.1811	0.1513	0.0834	0.0421	0.0193	0.0079	0.0028	0.0008	0.0005	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9978	0.9744	0.9078	0.8748	0.7946	0.6488	0.4925	0.3931	0.3467	0.2253	0.1345	0.0730	0.0356	0.0153	0.0056	0.0039	0.0017	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
4	0.9998	0.9957	0.9761	0.9636	0.9274	0.8424	0.7237	0.6315	0.5833	0.4382	0.3044	0.1938	0.1117	0.0573	0.0255	0.0188	0.0095	0.0028	0.0006	0.0002	0.0001	0.0000	0.0000
5	1.0000	0.9995	0.9954	0.9921	0.9806	0.9456	0.8822	0.8223	0.7873	0.6652	0.5269	0.3872	0.2607	0.1582	0.0846	0.0664	0.0386	0.0143	0.0039	0.0013	0.0007	0.0001	0.0000
6	1.0000	0.9999	0.9993	0.9987	0.9961	0.9857	0.9614	0.9336	0.9154	0.8418	0.7393	0.6128	0.4731	0.3348	0.2127	0.1777	0.1178	0.0544	0.0194	0.0079	0.0046	0.0005	0.0000
7	1.0000	1.0000	0.9999	0.9998	0.9994	0.9972	0.9905	0.9812	0.9745	0.9427	0.8883	0.8062	0.6956	0.5618	0.4167	0.3685	0.2763	0.1576	0.0726	0.0364	0.0239	0.0043	0.0002
8	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9961	0.9944	0.9847	0.9644	0.9270	0.8655	0.7747	0.6533	0.6069	0.5075	0.3512	0.2054	0.1252	0.0922	0.0256	0.0022
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995	0.9992	0.9972	0.9921	0.9807	0.9579	0.9166	0.8487	0.8189	0.7472	0.6093	0.4417	0.3226	0.2642	0.1109	0.0196
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968	0.9917	0.9804	0.9576	0.9460	0.9150	0.8416	0.7251	0.6187	0.5565	0.3410	0.1184
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9978	0.9943	0.9923	0.9862	0.9683	0.9313	0.8878	0.8578	0.7176	0.4596
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 14$ p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.4877	0.2288	0.1028	0.0779	0.0440	0.0178	0.0068	0.0034	0.0024	0.0008	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8470	0.5846	0.3567	0.2960	0.1979	0.1010	0.0475	0.0274	0.0205	0.0081	0.0029	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9699	0.8416	0.6479	0.5795	0.4481	0.2811	0.1608	0.1053	0.0839	0.0398	0.0170	0.0065	0.0022	0.0006	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9958	0.9559	0.8535	0.8063	0.6982	0.5213	0.3552	0.2612	0.2205	0.1243	0.0632	0.0287	0.0114	0.0039	0.0011	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9996	0.9908	0.9533	0.9310	0.8702	0.7415	0.5842	0.4755	0.4227	0.2793	0.1672	0.0898	0.0426	0.0175	0.0060	0.0040	0.0017	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
5	1.0000	0.9985	0.9885	0.9809	0.9561	0.8883	0.7805	0.6898	0.6405	0.4859	0.3373	0.2120	0.1189	0.0583	0.0243	0.0174	0.0083	0.0022	0.0004	0.0001	0.0000	0.0000	0.0000
6	1.0000	0.9998	0.9978	0.9959	0.9884	0.9617	0.9067	0.8505	0.8164	0.6925	0.5461	0.3953	0.2586	0.1501	0.0753	0.0576	0.0315	0.0103	0.0024	0.0007	0.0003	0.0000	0.0000
7	1.0000	1.0000	0.9997	0.9993	0.9976	0.9897	0.9685	0.9424	0.9247	0.8499	0.7414	0.6047	0.4539	0.3075	0.1836	0.1495	0.0933	0.0383	0.0116	0.0041	0.0022	0.0002	0.0000
8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9978	0.9917	0.9826	0.9757	0.9417	0.8811	0.7880	0.6627	0.5141	0.3595	0.3102	0.2195	0.1117	0.0439	0.0191	0.0115	0.0015	0.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9983	0.9960	0.9940	0.9825	0.9574	0.9102	0.8328	0.7207	0.5773	0.5245	0.4158	0.2585	0.1298	0.0690	0.0467	0.0092	0.0004
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9989	0.9961	0.9886	0.9713	0.9368	0.8757	0.7795	0.7388	0.6448	0.4787	0.3018	0.1937	0.1465	0.0441	0.0042
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9994	0.9978	0.9935	0.9830	0.9602	0.9161	0.8947	0.8392	0.7189	0.5519	0.4205	0.3521	0.1584	0.0301
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9971	0.9919	0.9795	0.9726	0.9525	0.8990	0.8021	0.7040	0.6433	0.4154	0.1530
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9976	0.9966	0.9932	0.9822	0.9560	0.9221	0.8972	0.7712	0.5123
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 16$ p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.4401	0.1853	0.0743	0.0541	0.0281	0.0100	0.0033	0.0015	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8108	0.5147	0.2839	0.2272	0.1407	0.0635	0.0261	0.0137	0.0098	0.0033	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9571	0.7892	0.5614	0.4868	0.3518	0.1971	0.0994	0.0594	0.0451	0.0183	0.0066	0.0021	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9930	0.9316	0.7899	0.7291	0.5981	0.4050	0.2459	0.1659	0.1339	0.0651	0.0281	0.0106	0.0035	0.0009	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9991	0.9830	0.9209	0.8866	0.7982	0.6302	0.4499	0.3391	0.2892	0.1666	0.0853	0.0384	0.0149	0.0049	0.0013	0.0008	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9999	0.9967	0.9765	0.9622	0.9183	0.8103	0.6598	0.5469	0.4900	0.3288	0.1976	0.1051	0.0486	0.0191	0.0062	0.0040	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.9995	0.9944	0.9899	0.9733	0.9204	0.8247	0.7374	0.6881	0.5272	0.3660	0.2272	0.1241	0.0583	0.0229	0.0159	0.0071	0.0016	0.0002	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9999	0.9989	0.9979	0.9930	0.9729	0.9256	0.8735	0.8406	0.7161	0.5629	0.4018	0.2559	0.1423	0.0671	0.0500	0.0257	0.0075	0.0015	0.0004	0.0002	0.0000	0.0000
8	1.0000	1.0000	0.9998	0.9996	0.9985	0.9925	0.9743	0.9500	0.9329	0.8577	0.7441	0.5982	0.4371	0.2839	0.1594	0.1265	0.0744	0.0271	0.0070	0.0021	0.0011	0.0001	0.0000
9	1.0000	1.0000	1.0000	1.0000	0.9998	0.9984	0.9929	0.9841	0.9771	0.9417	0.8759	0.7728	0.6340	0.4728	0.3119	0.2626	0.1753	0.0796	0.0267	0.0101	0.0056	0.0005	0.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9960	0.9938	0.9809	0.9514	0.8949	0.8024	0.6712	0.5100	0.4531	0.3402	0.1897	0.0817	0.0378	0.0235	0.0033	0.0001
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9992	0.9987	0.9951	0.9851	0.9616	0.9147	0.8334	0.7108	0.6609	0.5501	0.3698	0.2018	0.1134	0.0791	0.0170	0.0009
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9991	0.9965	0.9894	0.9719	0.9349	0.8661	0.8341	0.7541	0.5950	0.4019	0.2709	0.2101	0.0684	0.0070
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9979	0.9934	0.9817	0.9549	0.9406	0.9006	0.8029	0.6482	0.5132	0.4386	0.2108	0.0429
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9967	0.9902	0.9863	0.9739	0.9365	0.8593	0.7728	0.7161	0.4853	0.1892
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9985	0.9967	0.9900	0.9719	0.9459	0.9257	0.8147	0.5599
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 18$ p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.3972	0.1501	0.0536	0.0376	0.0180	0.0056	0.0016	0.0007	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.7735	0.4503	0.2241	0.1728	0.0991	0.0395	0.0142	0.0068	0.0046	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9419	0.7338	0.4797	0.4027	0.2713	0.1353	0.0600	0.0326	0.0236	0.0082	0.0025	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9891	0.9018	0.7202	0.6479	0.5010	0.3057	0.1646	0.1017	0.0783	0.0328	0.0120	0.0038	0.0010	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9985	0.9718	0.8794	0.8318	0.7164	0.5187	0.3327	0.2311	0.1886	0.0942	0.0411	0.0154	0.0049	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9998	0.9936	0.9581	0.9347	0.8671	0.7175	0.5344	0.4122	0.3550	0.2088	0.1077	0.0481	0.0183	0.0058	0.0014	0.0009	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.9988	0.9882	0.9794	0.9487	0.8610	0.7217	0.6085	0.5491	0.3743	0.2258	0.1189	0.0537	0.0203	0.0062	0.0039	0.0014	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9998	0.9973	0.9947	0.9837	0.9431	0.8593	0.7767	0.7283	0.5634	0.3915	0.2403	0.1280	0.0576	0.0212	0.0144	0.0061	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000
8	1.0000	1.0000	0.9995	0.9989	0.9957	0.9807	0.9404	0.8924	0.8609	0.7368	0.5778	0.4073	0.2527	0.1347	0.0597	0.0433	0.0210	0.0054	0.0009	0.0002	0.0001	0.0000	0.0000
9	1.0000	1.0000	0.9999	0.9998	0.9991	0.9946	0.9790	0.9567	0.9403	0.8653	0.7473	0.5927	0.4222	0.2632	0.1391	0.1076	0.0596	0.0193	0.0043	0.0011	0.0005	0.0000	0.0000
10	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9939	0.9856	0.9788	0.9424	0.8720	0.7597	0.6085	0.4366	0.2717	0.2233	0.1407	0.0569	0.0163	0.0053	0.0027	0.0002	0.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9986	0.9961	0.9938	0.9797	0.9463	0.8811	0.7742	0.6257	0.4509	0.3915	0.2783	0.1390	0.0513	0.0206	0.0118	0.0012	0.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9991	0.9986	0.9942	0.9817	0.9519	0.8923	0.7912	0.6450	0.5878	0.4656	0.2825	0.1329	0.0653	0.0419	0.0064	0.0002
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9951	0.9846	0.9589	0.9058	0.8114	0.7689	0.6673	0.4813	0.2836	0.1682	0.1206	0.0282	0.0015
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9990	0.9962	0.9880	0.9672	0.9217	0.8983	0.8354	0.6943	0.4990	0.3521	0.2798	0.0982	0.0109
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9975	0.9918	0.9764	0.9674	0.9400	0.8647	0.7287	0.5973	0.5203	0.2662	0.0581
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9954	0.9932	0.9858	0.9605	0.9009	0.8272	0.7759	0.5497	0.2265
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9993	0.9984	0.9944	0.9820	0.9624	0.9464	0.8499	0.6028
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 20$ p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.3585	0.1216	0.0388	0.0261	0.0115	0.0032	0.0008	0.0003	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.7358	0.3917	0.1756	0.1304	0.0692	0.0243	0.0076	0.0033	0.0021	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9245	0.6769	0.4049	0.3287	0.2061	0.0913	0.0355	0.0176	0.0121	0.0036	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9841	0.8670	0.6477	0.5665	0.4114	0.2252	0.1071	0.0604	0.0444	0.0160	0.0049	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9974	0.9568	0.8298	0.7687	0.6296	0.4148	0.2375	0.1515	0.1182	0.0510	0.0189	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9997	0.9887	0.9327	0.8982	0.8042	0.6172	0.4164	0.2972	0.2454	0.1256	0.0553	0.0207	0.0064	0.0016	0.0003	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.9976	0.9781	0.9629	0.9133	0.7858	0.6080	0.4793	0.4166	0.2500	0.1299	0.0577	0.0214	0.0065	0.0015	0.0009	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9996	0.9941	0.9887	0.9679	0.8982	0.7723	0.6615	0.6010	0.4159	0.2520	0.1316	0.0580	0.0210	0.0060	0.0037	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.9999	0.9987	0.9972	0.9900	0.9591	0.8867	0.8095	0.7624	0.5956	0.4143	0.2517	0.1308	0.0565	0.0196	0.0130	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
9	1.0000	1.0000	0.9998	0.9994	0.9974	0.9861	0.9520	0.9081	0.8782	0.7553	0.5914	0.4119	0.2493	0.1275	0.0532	0.0376	0.0171	0.0039	0.0006	0.0001	0.0000	0.0000	0.0000
10	1.0000	1.0000	1.0000	0.9999	0.9994	0.9961	0.9829	0.9624	0.9468	0.8725	0.7507	0.5881	0.4086	0.2447	0.1218	0.0919	0.0480	0.0139	0.0026	0.0006	0.0002	0.0000	0.0000
11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9870	0.9804	0.9435	0.8692	0.7483	0.5857	0.4044	0.2376	0.1905	0.1133	0.0409	0.0100	0.0028	0.0013	0.0001	0.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9963	0.9940	0.9790	0.9420	0.8684	0.7480	0.5841	0.3990	0.3385	0.2277	0.1018	0.0321	0.0113	0.0059	0.0004	0.0000
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9991	0.9985	0.9935	0.9786	0.9423	0.8701	0.7500	0.5834	0.5207	0.3920	0.2142	0.0867	0.0371	0.0219	0.0024	0.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9997	0.9984	0.9936	0.9793	0.9447	0.8744	0.7546	0.7028	0.5836	0.3828	0.1958	0.1018	0.0673	0.0113	0.0003
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9941	0.9811	0.9490	0.8818	0.8485	0.7625	0.5852	0.3704	0.2313	0.1702	0.0432	0.0026
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9840	0.9556	0.9396	0.8929	0.7748	0.5886	0.4335	0.3523	0.1330	0.0159
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9991	0.9964	0.9879	0.9824	0.9645	0.9087	0.7939	0.6713	0.5951	0.3231	0.0755
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9979	0.9967	0.9924	0.9757	0.9308	0.8696	0.8244	0.6083	0.2642
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9997	0.9992	0.9968	0.9885	0.9739	0.9612	0.8784	0.6415
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 25$ p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.2774	0.0718	0.0172	0.0105	0.0038	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.6424	0.2712	0.0931	0.0629	0.0274	0.0070	0.0016	0.0005	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.8729	0.5371	0.2537	0.1887	0.0982	0.0321	0.0090	0.0035	0.0021	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9659	0.7636	0.4711	0.3816	0.2340	0.0962	0.0332	0.0149	0.0097	0.0024	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9928	0.9020	0.6821	0.5937	0.4207	0.2137	0.0905	0.0462	0.0320	0.0095	0.0023	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9988	0.9666	0.8385	0.7720	0.6167	0.3783	0.1935	0.1120	0.0826	0.0294	0.0086	0.0020	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.9998	0.9905	0.9305	0.8908	0.7800	0.5611	0.3407	0.2215	0.1734	0.0736	0.0258	0.0073	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9977	0.9745	0.9553	0.8909	0.7265	0.5118	0.3703	0.3061	0.1536	0.0639	0.0216	0.0058	0.0012	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.9995	0.9920	0.9843	0.9532	0.8506	0.6769	0.5376	0.4668	0.2735	0.1340	0.0539	0.0174	0.0043	0.0008	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	1.0000	0.9999	0.9979	0.9953	0.9827	0.9287	0.8106	0.6956	0.6303	0.4246	0.2424	0.1148	0.0440	0.0132	0.0029	0.0016	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	1.0000	1.0000	0.9995	0.9988	0.9944	0.9703	0.9022	0.8220	0.7712	0.5858	0.3843	0.2122	0.0960	0.0344	0.0093	0.0056	0.0018	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
11	1.0000	1.0000	0.9999	0.9997	0.9985	0.9893	0.9558	0.9082	0.8746	0.7323	0.5426	0.3450	0.1827	0.0778	0.0255	0.0164	0.0060	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
12	1.0000	1.0000	1.0000	0.9999	0.9996	0.9966	0.9825	0.9585	0.9396	0.8462	0.6937	0.5000	0.3063	0.1538	0.0604	0.0415	0.0175	0.0034	0.0004	0.0001	0.0000	0.0000	0.0000
13	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9940	0.9836	0.9745	0.9222	0.8173	0.6550	0.4574	0.2677	0.1254	0.0918	0.0442	0.0107	0.0015	0.0003	0.0001	0.0000	0.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9982	0.9944	0.9907	0.9656	0.9040	0.7878	0.6157	0.4142	0.2288	0.1780	0.0978	0.0297	0.0056	0.0012	0.0005	0.0000	0.0000
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9984	0.9971	0.9868	0.9560	0.8852	0.7576	0.5754	0.3697	0.3044	0.1894	0.0713	0.0173	0.0047	0.0021	0.0001	0.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9992	0.9957	0.9826	0.9461	0.8660	0.7265	0.5332	0.4624	0.3231	0.1494	0.0468	0.0157	0.0080	0.0005	0.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9988	0.9942	0.9784	0.9361	0.8464	0.6939	0.6297	0.4882	0.2735	0.1091	0.0447	0.0255	0.0023	0.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9927	0.9742	0.9264	0.8266	0.7785	0.6593	0.4389	0.2200	0.1092	0.0695	0.0095	0.0002
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9980	0.9914	0.9706	0.9174	0.8880	0.8065	0.6217	0.3833	0.2280	0.1615	0.0334	0.0012
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9977	0.9905	0.9680	0.9538	0.9095	0.7863	0.5793	0.4063	0.3179	0.0980	0.0072
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9976	0.9903	0.9851	0.9668	0.9038	0.7660	0.6184	0.5289	0.2364	0.0341
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9979	0.9965	0.9910	0.9679	0.9018	0.8113	0.7463	0.4629	0.1271
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9984	0.9930	0.9726	0.9371	0.9069	0.7288	0.3576
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9992	0.9962	0.9895	0.9828	0.9282	0.7226
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 30$ p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.2146	0.0424	0.0076	0.0042	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.5535	0.1837	0.0480	0.0295	0.0105	0.0020	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.8122	0.4114	0.1514	0.1028	0.0442	0.0106	0.0021	0.0007	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9392	0.6474	0.3217	0.2396	0.1227	0.0374	0.0093	0.0033	0.0019	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9844	0.8245	0.5245	0.4243	0.2552	0.0979	0.0302	0.0122	0.0075	0.0015	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9967	0.9268	0.7106	0.6164	0.4275	0.2026	0.0766	0.0355	0.0233	0.0057	0.0011	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.9994	0.9742	0.8474	0.7765	0.6070	0.3481	0.1595	0.0838	0.0586	0.0172	0.0040	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.9999	0.9922	0.9302	0.8863	0.7608	0.5143	0.2814	0.1668	0.1238	0.0435	0.0121	0.0026	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.9980	0.9722	0.9494	0.8713	0.6736	0.4315	0.2860	0.2247	0.0940	0.0312	0.0081	0.0016	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	1.0000	0.9995	0.9903	0.9803	0.9389	0.8034	0.5888	0.4317	0.3575	0.1763	0.0694	0.0214	0.0050	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	1.0000	0.9999	0.9971	0.9933	0.9744	0.8943	0.7304	0.5848	0.5078	0.2915	0.1350	0.0494	0.0138	0.0029	0.0004	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
11	1.0000	1.0000	0.9992	0.9980	0.9905	0.9493	0.8407	0.7239	0.6548	0.4311	0.2327	0.1002	0.0334	0.0083	0.0014	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	1.0000	1.0000	0.9998	0.9995	0.9969	0.9784	0.9155	0.8340	0.7802	0.5785	0.3592	0.1808	0.0714	0.0212	0.0045	0.0025	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
13	1.0000	1.0000	1.0000	0.9999	0.9991	0.9918	0.9599	0.9102	0.8737	0.7145	0.5025	0.2923	0.1356	0.0481	0.0124	0.0072	0.0021	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
14	1.0000	1.0000	1.0000	1.0000	0.9998	0.9973	0.9831	0.9565	0.9348	0.8246	0.6448	0.4278	0.2309	0.0971	0.0301	0.0188	0.0064	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000
15	1.0000	1.0000	1.0000	1.0000	0.9999	0.9992	0.9936	0.9812	0.9699	0.9029	0.7691	0.5722	0.3552	0.1754	0.0652	0.0435	0.0169	0.0027	0.0002	0.0000	0.0000	0.0000	0.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9979	0.9928	0.9876	0.9519	0.8644	0.7077	0.4975	0.2855	0.1263	0.0898	0.0401	0.0082	0.0009	0.0001	0.0000	0.0000	0.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9975	0.9955	0.9788	0.9286	0.8192	0.6408	0.4215	0.2198	0.1660	0.0845	0.0216	0.0031	0.0005	0.0002	0.0000	0.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9986	0.9917	0.9666	0.8998	0.7673	0.5689	0.3452	0.2761	0.1593	0.0507	0.0095	0.0020	0.0008	0.0000	0.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9996	0.9971	0.9862	0.9506	0.8650	0.7085	0.4922	0.4152	0.2696	0.1057	0.0256	0.0067	0.0029	0.0001	0.0000
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9950	0.9786	0.9306	0.8237	0.6425	0.5683	0.4112	0.1966	0.0611	0.0197	0.0097	0.0005	0.0000
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9984	0.9919	0.9688	0.9060	0.7753	0.7140	0.5685	0.3264	0.1287	0.0506	0.0278	0.0020	0.0000
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9974	0.9879	0.9565	0.8762	0.8332	0.7186	0.4857	0.2392	0.1137	0.0698	0.0078	0.0001
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9960	0.9828	0.9414	0.9162	0.8405	0.6519	0.3930	0.2235	0.1526	0.0258	0.0006
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9989	0.9943	0.9767	0.9645	0.9234	0.7974	0.5725	0.3836	0.2894	0.0732	0.0033
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9985	0.9925	0.9878	0.9698	0.9021	0.7448	0.5757	0.4755	0.1755	0.0156
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9981	0.9967	0.9907	0.9626	0.8773	0.7604	0.6783	0.3526	0.0608
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9993	0.9979	0.9894	0.9558	0.8972	0.8486	0.5886	0.1878
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9980	0.9895	0.9705	0.9520	0.8163	0.4465
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9958	0.9924	0.9576	0.7854
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE POISSON PROBABILITIES

λ	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$x = 0$	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
1	1.0000	0.9998	0.9996	0.9992	0.9988	0.9983	0.9977	0.9970	0.9962
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
$x = 0$	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90
$x = 0$	0.3679	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496
1	0.7358	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337
2	0.9197	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037
3	0.9810	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747
4	0.9963	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559
5	0.9994	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868
6	0.9999	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966
7	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	2.00	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90
$x = 0$	0.1353	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550
1	0.4060	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146
2	0.6767	0.6496	0.6227	0.5960	0.5697	0.5438	0.5184	0.4936	0.4695	0.4460
3	0.8571	0.8386	0.8194	0.7993	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696
4	0.9473	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8629	0.8477	0.8318
5	0.9834	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258
6	0.9955	0.9941	0.9925	0.9906	0.9884	0.9858	0.9828	0.9794	0.9756	0.9713
7	0.9989	0.9985	0.9980	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901
8	0.9998	0.9997	0.9995	0.9994	0.9991	0.9989	0.9985	0.9981	0.9976	0.9969
9	1.0000	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9993	0.9991
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9998
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE POISSON PROBABILITIES

λ	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
$x = 0$	0.0498	0.0450	0.0408	0.0369	0.0334	0.0302	0.0273	0.0247	0.0224	0.0202
1	0.1991	0.1847	0.1712	0.1586	0.1468	0.1359	0.1257	0.1162	0.1074	0.0992
2	0.4232	0.4012	0.3799	0.3594	0.3397	0.3208	0.3027	0.2854	0.2689	0.2531
3	0.6472	0.6248	0.6025	0.5803	0.5584	0.5366	0.5152	0.4942	0.4735	0.4532
4	0.8153	0.7982	0.7806	0.7626	0.7442	0.7254	0.7064	0.6872	0.6678	0.6484
5	0.9161	0.9057	0.8946	0.8829	0.8705	0.8576	0.8441	0.8301	0.8156	0.8006
6	0.9665	0.9612	0.9554	0.9490	0.9421	0.9347	0.9267	0.9182	0.9091	0.8995
7	0.9881	0.9858	0.9832	0.9802	0.9769	0.9733	0.9692	0.9648	0.9599	0.9546
8	0.9962	0.9953	0.9943	0.9931	0.9917	0.9901	0.9883	0.9863	0.9840	0.9815
9	0.9989	0.9986	0.9982	0.9978	0.9973	0.9967	0.9960	0.9952	0.9942	0.9931
10	0.9997	0.9996	0.9995	0.9994	0.9992	0.9990	0.9987	0.9984	0.9981	0.9977
11	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993
12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	4.00	4.10	4.20	4.30	4.40	4.50	4.60	4.70	4.80	4.90
$x = 0$	0.0183	0.0166	0.0150	0.0136	0.0123	0.0111	0.0101	0.0091	0.0082	0.0074
1	0.0916	0.0845	0.0780	0.0719	0.0663	0.0611	0.0563	0.0518	0.0477	0.0439
2	0.2381	0.2238	0.2102	0.1974	0.1851	0.1736	0.1626	0.1523	0.1425	0.1333
3	0.4335	0.4142	0.3954	0.3772	0.3594	0.3423	0.3257	0.3097	0.2942	0.2793
4	0.6288	0.6093	0.5898	0.5704	0.5512	0.5321	0.5132	0.4946	0.4763	0.4582
5	0.7851	0.7693	0.7531	0.7367	0.7199	0.7029	0.6858	0.6684	0.6510	0.6335
6	0.8893	0.8786	0.8675	0.8558	0.8436	0.8311	0.8180	0.8046	0.7908	0.7767
7	0.9489	0.9427	0.9361	0.9290	0.9214	0.9134	0.9049	0.8960	0.8867	0.8769
8	0.9786	0.9755	0.9721	0.9683	0.9642	0.9597	0.9549	0.9497	0.9442	0.9382
9	0.9919	0.9905	0.9889	0.9871	0.9851	0.9829	0.9805	0.9778	0.9749	0.9717
10	0.9972	0.9966	0.9959	0.9952	0.9943	0.9933	0.9922	0.9910	0.9896	0.9880
11	0.9991	0.9989	0.9986	0.9983	0.9980	0.9976	0.9971	0.9966	0.9960	0.9953
12	0.9997	0.9997	0.9996	0.9995	0.9993	0.9992	0.9990	0.9988	0.9986	0.9983
13	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	0.9996	0.9995	0.9994
14	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE POISSON PROBABILITIES

λ	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50
$x = 0$	0.0067	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001
1	0.0404	0.0266	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008
2	0.1247	0.0884	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042
3	0.2650	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149
4	0.4405	0.3575	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403
5	0.6160	0.5289	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885
6	0.7622	0.6860	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649
7	0.8666	0.8095	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687
8	0.9319	0.8944	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918
9	0.9682	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218
10	0.9863	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453
11	0.9945	0.9890	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520
12	0.9980	0.9955	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364
13	0.9993	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981
14	0.9998	0.9994	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400
15	0.9999	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665
16	1.0000	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823
17	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911
18	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957
19	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

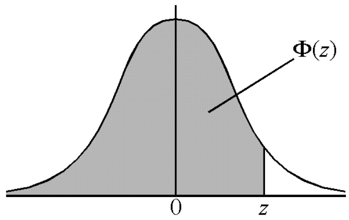
CUMULATIVE POISSON PROBABILITIES

λ	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00
x = 0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000
4	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002	0.0001	0.0000
5	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007	0.0003	0.0002
6	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021	0.0010	0.0005
7	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054	0.0029	0.0015
8	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126	0.0071	0.0039
9	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261	0.0154	0.0089
10	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491	0.0304	0.0183
11	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847	0.0549	0.0347
12	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350	0.0917	0.0606
13	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009	0.1426	0.0984
14	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808	0.2081	0.1497
15	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715	0.2867	0.2148
16	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	0.5660	0.4677	0.3751	0.2920
17	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	0.6593	0.5640	0.4686	0.3784
18	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	0.7423	0.6550	0.5622	0.4695
19	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	0.8122	0.7363	0.6509	0.5606
20	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	0.8682	0.8055	0.7307	0.6472
21	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	0.9108	0.8615	0.7991	0.7255
22	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	0.9418	0.9047	0.8551	0.7931
23	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	0.9633	0.9367	0.8989	0.8490
24	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	0.9777	0.9594	0.9317	0.8933
25	1.0000	0.9999	0.9997	0.9990	0.9974	0.9938	0.9869	0.9748	0.9554	0.9269
26	1.0000	1.0000	0.9999	0.9995	0.9987	0.9967	0.9925	0.9848	0.9718	0.9514
27	1.0000	1.0000	0.9999	0.9998	0.9994	0.9983	0.9959	0.9912	0.9827	0.9687
28	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9978	0.9950	0.9897	0.9805
29	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9973	0.9941	0.9882
30	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9986	0.9967	0.9930
31	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9982	0.9960
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.9978
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9988
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1 then, for each value of z, the table gives the value of $\Phi(z)$, where:

$$\Phi(z) = \mathrm{P}(Z \leq z)$$



For negative values of z use $\Phi(-z) = 1 - \Phi(z)$.

z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															ADD				
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that:

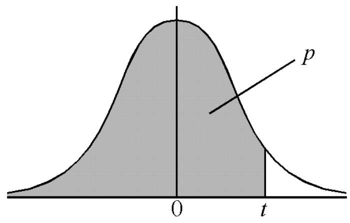
$$\mathrm{P}(Z \leq z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE *t*-DISTRIBUTION

If T has a t-distribution with ν degrees of freedom then, for each pair of values of p and ν, the table gives the value of t such that:

$$P(T \le t) = p \,.$$

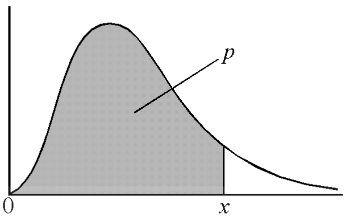


p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$\nu = 1$	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE χ^2-DISTRIBUTION

If X has a χ^2-distribution with ν degrees of freedom then, for each pair of values of p and ν, the table gives the value of x such that:

$$\mathrm{P}(X \le x) = p$$



p	0.01	0.025	0.05	0.9	0.95	0.975	0.99	0.995	0.999
$\nu = 1$	$0.0^3 1571$	$0.0^3 9821$	$0.0^2 3932$	2.706	3.841	5.024	6.635	7.8794	10.83
2	0.02010	0.05064	0.1026	4.605	5.991	7.378	9.210	10.60	13.82
3	0.1148	0.2158	0.3518	6.251	7.815	9.348	11.34	12.84	16.27
4	0.2971	0.4844	0.7107	7.779	9.488	11.14	13.28	14.86	18.47
5	0.5543	0.8312	1.145	9.236	11.07	12.83	15.09	16.75	20.51
6	0.8721	1.237	1.635	10.64	12.59	14.45	16.81	18.55	22.46
7	1.239	1.690	2.167	12.02	14.07	16.01	18.48	20.28	24.32
8	1.647	2.180	2.733	13.36	15.51	17.53	20.09	21.95	26.12
9	2.088	2.700	3.325	14.68	16.92	19.02	21.67	23.59	27.88
10	2.558	3.247	3.940	15.99	18.31	20.48	23.21	25.19	29.59
11	3.053	3.816	4.575	17.28	19.68	21.92	24.73	26.76	31.26
12	3.571	4.404	5.226	18.55	21.03	23.34	26.22	28.30	32.91
13	4.107	5.009	5.892	19.81	22.36	24.74	27.69	29.82	34.53
14	4.660	5.629	6.571	21.06	23.68	26.12	29.14	31.32	36.12
15	5.229	6.262	7.261	22.31	25.00	27.49	30.58	32.80	37.70
16	5.812	6.908	7.962	23.54	26.30	28.85	32.00	34.27	39.25
17	6.408	7.564	8.672	24.77	27.59	30.19	33.41	35.72	40.79
18	7.015	8.231	9.390	25.99	28.87	31.53	34.81	37.16	42.31
19	7.633	8.907	10.12	27.20	30.14	32.85	36.19	38.58	43.82
20	8.260	9.591	10.85	28.41	31.41	34.17	37.57	40.00	45.31
21	8.897	10.28	11.59	29.62	32.67	35.48	38.93	41.40	46.80
22	9.542	10.98	12.34	30.81	33.92	36.78	40.29	42.80	48.27
23	10.20	11.69	13.09	32.01	35.17	38.08	41.64	44.18	49.73
24	10.86	12.40	13.85	33.20	36.42	39.36	42.98	45.56	51.18
25	11.52	13.12	14.61	34.38	37.65	40.65	44.31	46.93	52.62
30	14.95	16.79	18.49	40.26	43.77	46.98	50.89	53.67	59.70
40	22.16	24.43	26.51	51.81	55.76	59.34	63.69	66.77	73.40
50	29.71	32.36	34.76	63.17	67.50	71.42	76.15	79.49	86.66
60	37.48	40.48	43.19	74.40	79.08	83.30	88.38	91.95	99.61
70	45.44	48.76	51.74	85.53	90.53	95.02	100.4	104.2	112.3
80	53.54	57.15	60.39	96.58	101.9	106.6	112.3	116.3	124.8
90	61.75	65.65	69.13	107.6	113.1	118.1	124.1	128.3	137.2
100	70.06	74.22	77.93	118.5	124.3	129.6	135.8	140.2	149.4

WILCOXON SIGNED RANK TEST

P is the sum of the ranks corresponding to the positive differences,
Q is the sum of the ranks corresponding to the negative differences,
T is the smaller of P and Q.

For each value of n the table gives the **largest** value of T which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of *T*

	Level of significance			
One Tail	0.05	0.025	0.01	0.005
Two Tail	0.1	0.05	0.02	0.01
$n = 6$	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37

For larger values of n, each of P and Q can be approximated by the normal distribution with mean $\frac{1}{4}n(n+1)$ and variance $\frac{1}{24}n(n+1)(2n+1)$.

WILCOXON RANK SUM TEST

The two samples have sizes m and n, where $m \le n$.
R_m is the sum of the ranks of the items in the sample of size m.
W is the smaller of R_m and $m(n+m+1) - R_m$.

For each pair of values of m and n, the table gives the **largest** value of W which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of *W*

	Level of significance											
One Tail	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two Tail	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 3$			$m = 4$			$m = 5$			$m = 6$		
3	6	–	–									
4	6	–	–	11	10	–						
5	7	6	–	12	11	10	19	17	16			
6	8	7	–	13	12	11	20	18	17	28	26	24
7	8	7	6	14	13	11	21	20	18	29	27	25
8	9	8	6	15	14	12	23	21	19	31	29	27
9	10	8	7	16	14	13	24	22	20	33	31	28
10	10	9	7	17	15	13	26	23	21	35	32	29

	Level of significance											
One Tail	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01	0.05	0.025	0.01
Two Tail	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02	0.1	0.05	0.02
n	$m = 7$			$m = 8$			$m = 9$			$m = 10$		
7	39	36	34									
8	41	38	35	51	49	45						
9	43	40	37	54	51	47	66	62	59			
10	45	42	39	56	53	49	69	65	61	82	78	74

For larger values of m and n, the normal distribution with mean $\frac{1}{2}m(m+n+1)$ and variance $\frac{1}{12}mn(m+n+1)$ should be used as an approximation to the distribution of R_m.

Appendix C: Mathematical Notation

1 Set Notation

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that $\ldots$
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
E	the universal set
A'	the complement of the set A
$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Z}_n$	the set of integers modulo n, $\{0, 1, 2, \ldots, n-1\}$
$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z},\ q \in \mathbb{Z}^+\right\}$
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
$\mathbb{Q}_0^+$	set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geq 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R} : x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geq 0\}$
$\mathbb{C}$	the set of complex numbers
(x, y)	the ordered pair x, y
$A \times B$	the cartesian product of sets A and B, i.e. $A \times B = \{(a, b) : a \in A,\ b \in B\}$
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$
$[a, b)$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$

$(a, b]$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$y R x$	y is related to x by the relation R
$y \sim x$	y is equivalent to x, in the context of some equivalence relation

2 Miscellaneous Symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is less than
$\leqslant$	is less than or equal to, is not greater than
$>$	is greater than
$\geqslant$	is greater than or equal to, is not less than
∞	infinity
$p \wedge q$	p and q
$p \vee q$	p or q (or both)
$\sim p$	not p
$p \Rightarrow q$	p implies q (if p then q)
$p \Leftarrow q$	p is implied by q (if q then p)
$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
$\exists$	there exists
$\forall$	for all

3 Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b,\ ab,\ a.b$	a multiplied by b
$a \div b,\ \frac{a}{b},\ a/b$	a divided by b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$

$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
$\sqrt{a}$	the positive square root of a
$\lvert a \rvert$	the modulus of a
$n!$	n factorial
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n \in \mathbb{Z}^+$ or $\frac{n(n-1)...(n-r+1)}{r!}$ for $n \in \mathbb{Q}$

4 Functions

$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f} : A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f} : x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse function of the function f
gf	the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
Δx, δx	an increment of x
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x
$\mathrm{f}'(x)$, $\mathrm{f}''(x)$, …, $\mathrm{f}^{(n)}(x)$	the first, second, ..., nth derivatives of $\mathrm{f}(x)$ with respect to x
$\int y \, \mathrm{d}x$	the indefinite integral of y with respect to x
$\int_a^b y \, \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
$\frac{\partial V}{\partial x}$	the partial derivative of V with respect to x
$\dot{x}$, $\ddot{x}$, …	the first, second, ... derivatives of x with respect to t

5 Exponential and Logarithmic Functions

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$, $\log_{\mathrm{e}} x$	natural logarithm of x
$\lg x$, $\log_{10} x$	logarithm of x to base 10

6 Circular and Hyperbolic Functions

sin, cos, tan cosec, sec, cot	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ $\operatorname{cosec}^{-1}$, $\sec^{-1}$, $\cot^{-1}$	the inverse circular functions
sinh, cosh, tanh cosech, sech, coth	the hyperbolic functions
$\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$ $\operatorname{cosech}^{-1}$, sech^{-1}, $\coth^{-1}$	the inverse hyperbolic functions

7 Complex Numbers

i	square root of -1
z	a complex number, $z = x + \mathrm{i}\,y = r(\cos\theta + \mathrm{i}\sin\theta)$
$\operatorname{Re} z$	the real part of z, $\operatorname{Re} z = x$
$\operatorname{Im} z$	the imaginary part of z, $\operatorname{Im} z = y$
$\lvert z \rvert$	the modulus of z, $\lvert z \rvert = \sqrt{x^2 + y^2}$
$\arg z$	the argument of z, $\arg z = \theta$, $-\pi < \theta \le \pi$
z^*	the complex conjugate of z, $x - \mathrm{i}\,y$

8 Matrices

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
$\det \mathbf{M}$ or $\lvert \mathbf{M} \rvert$	the determinant of the square matrix $\mathbf{M}$

9 Vectors

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
$\mathbf{i},\ \mathbf{j},\ \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$\lvert\mathbf{a}\rvert,\ a$	the magnitude of $\mathbf{a}$
$\lvert\overrightarrow{AB}\rvert,\ AB$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a.b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a}\times\mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10 Probability and Statistics

$A,\ B,\ C,$ etc.	events
$A \cup B$	union of the events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A
$\mathrm{P}(A \mid B)$	probability of the event A conditional on the event B
$X,\ Y,\ R,$ etc.	random variables
$x,\ y,\ r,$ etc.	values of the random variables $X,\ Y,\ R$ etc
$x_1,\ x_2,\ \ldots$	observations
$f_1,\ f_2,\ \ldots$	frequencies with which the observations $x_1,\ x_2,\ \ldots$ occur
$\mathrm{p}(x)$	probability function $\mathrm{P}(X = x)$ of the discrete random variable X
$p_1,\ p_2,\ \ldots$	probabilities of the values $x_1,\ x_2,\ \ldots$ of the discrete random variable X
$\mathrm{f}(x),\ \mathrm{g}(x),\ \ldots$	the value of the probability density function of a continuous random variable X
$\mathrm{F}(x),\ \mathrm{G}(x),\ \ldots$	the value of the (cumulative) distribution function $\mathrm{P}(X \leqslant x)$ of a continuous random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}(\mathrm{g}(X))$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{G}(t)$	probability generating function for a random variable which takes the values 0, 1, 2, …

$\mathrm{B}(n,\ p)$	binomial distribution with parameters n and p
$\mathrm{N}(\mu,\ \sigma^2)$	normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x},\ m$	sample mean
$s^2,\ \hat{\sigma}^2$	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $\mathrm{N}(0,\ 1)$
Φ	corresponding cumulative distribution function
ρ	product moment correlation coefficient for a population
r	product moment correlation coefficient for a sample
$\mathrm{Cov}(X,\ Y)$	covariance of X and Y