"Applications, applications, applications"

Lecturers' perceptions of students' mathematical preparedness for STEMM and Social Science degrees

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Executive Summary

A level Mathematics and Further Mathematics are being reformed for first teaching in 2017. These subjects are unique amongst A levels in that they act as 'service subjects' for a wide range of undergraduate degrees. Consequently, any changes to these qualifications will inevitably impact students' transitions to university study in STEMM (Science, Technology, Engineering, Mathematics and Medicine) and the social sciences. The findings of this study build on existing literature that suggests that lecturers perceive new undergraduates to struggle mathematically during the transition to higher education. Whilst there has been recent work in this area (e.g. see ACME, 2011, and the Higher Education Academy's *Tackling Transition* series), this study covers the broadest range of university subjects since the seminal London Mathematical Society report, *Tackling the Mathematics Problem* (1995).

As part of a larger study investigating students' transitions to higher education in mathematical subjects (see Darlington & Bowyer, 2016), we undertook interviews with 30 admissions tutors and lecturers with responsibility for first year teaching in a variety of subjects. The interviews investigated academics' perceptions of undergraduates' mathematical capabilities and difficulties, their perceptions of existing A level Mathematics and Further Mathematics, as well as any areas of mathematics that would provide good preparation for a degree in that subject.

Despite the wide range of subjects and the differing mathematical backgrounds of participants' teaching cohorts, there were several common findings regarding perceptions of A level Mathematics and Further Mathematics and the mathematical preparedness of undergraduates. The primary issue is one of mathematical skills, rather than content: nearly all participants reported that, whilst the content of A level qualifications was broadly appropriate, new undergraduates' main difficulty is applying familiar mathematical concepts in unfamiliar contexts. This manifests itself as a difficulty with 'translating' between mathematics and their particular discipline, such as when creating a mathematical model for a practical engineering or economics problem. Additionally, in some subjects that are traditionally considered to be less mathematical, such as biology and psychology, interviewees reported that students fail to see the relevance of mathematics to their degree, and thus struggle to see where and how to apply their mathematical knowledge. Interviewees also reported that some undergraduates lack basic mathematical skills such as algebraic manipulation, arithmetic and the use of units. Mathematics and computer science undergraduates particularly struggle with proof and formal mathematics.

Qualification reform is intended to rectify existing problems with A level Mathematics, particularly with the transition to a linear system and the introduction of prescribed content. Nevertheless, the reforms will not necessarily improve students' ability to *apply* mathematics in different contexts. To a certain extent, this is a problem that emerges prior to A level. Mathematics teachers, especially those who are non-specialists, should be made fully aware of the broad range of resources that exist, as well as the range of professional development courses available – for example, those run by Mathematics in Education and Industry (MEI) and the Further Mathematics Support Programme (FMSP). Additionally, universities should ensure they are aware of the full-range of post-compulsory mathematics qualifications to guarantee that prospective students are given the most appropriate advice about how to prepare for the mathematical demands of their course.

1. Introduction

1.1 A level reform

A levels in England are currently undergoing significant reform. All A levels are moving to a linear system, meaning that students will take all of their examinations at the end of a two-year period, rather than throughout, as is currently the case. Additionally, the AS and A level have been 'decoupled'. This means that the AS level is now a stand-alone qualification, and no longer contributes to a student's performance in the overall A level. Whilst most subjects have already been reformed, A levels in Mathematics and Further Mathematics are currently scheduled to be developed for first teaching in September 2017.

Currently, students have two mathematics A levels available to them: Mathematics and Further Mathematics[1]. Further Mathematics can only be taken in addition to A level Mathematics. Despite well-documented concerns about the proportion of students studying post-compulsory mathematics, Mathematics is the most popular subject at A level, with 88,816 candidates in 2015, accounting for 10.9% of all A levels taken that year (Joint Council for Qualifications, 2015). Additionally, whilst uptake of A level Further Mathematics is much lower, with 14,993 candidates in 2015, it is one of the fastest growing A level qualifications, with uptake more than doubling since 2006. Consequently, reforms of these subjects stand to impact a large number of students.

The current structure of A level Mathematics and Further Mathematics is complicated, and promotes variation, due to the range of optional units available. Currently, in A level Mathematics, students take four 'Core Pure Mathematics' units and two applied mathematics units (2 Core Pure Mathematics units and one applied unit for AS level). Applied mathematics units come from the three strands currently available: Statistics, Mechanics and Decision Mathematics. Students can take a mixture of applied units, or specialise in one particular strand, although in reality such choices are usually determined by school mathematics departments, depending on teaching expertise, resources and class scheduling. Units in the same applied strand must also be taken consecutively: for example, Mechanics 1 (commonly referred to as 'M1') must be taken before Mechanics 2. In A level Further Mathematics, students must take at least two 'Further Pure Mathematics' units (at least one at AS level), which build on content in the Core Pure Mathematics. The remaining four units can be a mixture of additional Further Pure Mathematics and/or applied units. The inherent flexibility in both qualifications means that students who have taken ostensibly the same A level will often have studied very different material, especially in relation to the applied mathematics strands. For example, whilst a prospective undergraduate physicist may wish to specialise in mechanics, their ability to do so is dependent on their school. Consequently, even where a university requires students to have taken A level Further Mathematics, students in the new cohort could have studied anywhere between 0-5 Mechanics units. This lack of certainty inevitably causes difficulties when planning undergraduate teaching, and lecturers will often assume their students have no, or very limited, applied mathematics knowledge to account for this.

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[1] A level qualifications in Statistics and Use of Mathematics currently exist, but are taken by very few students.

Although some aspects of A level reform, such as decoupling, are applicable to all subjects, some specific changes will be made to Mathematics and Further Mathematics. The most significant of these is the introduction of prescribed content. All of the reformed AS and A level Mathematics content will be centrally prescribed by the Department for Education (DfE). Importantly, this will incorporate both mechanics and statistics content, meaning that all students will gain experience with both areas. There will be no decision mathematics content, due to the A level Content Advisory Board's recommendation that this be removed due to the perception that these units are a 'soft' option and are not universally valued by universities (ALCAB, 2014). For Further Mathematics, 50% of the content will be centrally prescribed. This will be entirely core mathematics content. The remaining 50% will be determined by the awarding bodies, and can include a mixture of applied and core mathematical content. Although the specifications are yet to be accredited, the three main awarding bodies (OCR, AQA and Pearson) have included a mixture of decision/discrete mathematics, statistics and mechanics, as well as additional pure mathematics topics. Consequently, although a degree of flexibility will remain in Further Mathematics, the reforms mean that universities can ensure that any applicants who have taken A level Mathematics have covered particular topics, including calculus, logarithms, vectors, statistical sampling, hypothesis testing and kinematics.

1.2 Undergraduates' mathematical capabilities

Any reforms to A level Mathematics and Further Mathematics will have an inevitable impact on students' mathematical readiness for university study. These A levels act as 'service' subjects for a wide range of undergraduate degrees: subjects in Science, Technology, Engineering, Mathematics and Medicine (STEMM) fields, as well as some social sciences, often require or recommend new undergraduates have studied A level Mathematics. Table 1 shows the top 10 undergraduate subject areas that students who had taken A level Mathematics were accepted to in 2011.

Table 1: Undergraduate subject areas that students with an A level in Mathematics were accepted onto in 2011 (Vidal Rodeiro, 2012)

Subject Area	Number of candidates	Percent (of candidates)
Engineering	10,889	14.4
Mathematics and Computer Science	10,014	13.2
Physical Sciences	7,718	10.2
Business and Admin Studies	7,418	9.8
Social Studies	5,900	7.8
Biological Sciences	5,650	7.5
Subjects allied to Medicine	5,509	7.3
Medicine and Dentistry	3,958	5.2
Sciences combined with social sciences or arts	2,213	2.9
Law	2,072	2.7

Explicit entry requirements for AS or A level Further Mathematics are still rare, and currently only exist for admission to undergraduate mathematics. Although all universities are willing to make exceptions for students whose schools do not offer Further Mathematics, the following universities have specific requirements for mathematics applicants for 2016 admissions:

- Imperial College London, University College London, the University of Cambridge and the University of Oxford all ask for an A* in A level Further Mathematics.

- The universities of Bath, Durham, Nottingham, St Andrew's, Warwick and King's College London all require an A-grade in A level Further Mathematics.
- The universities of Bristol, Loughborough, York, Manchester, Lancaster, Leeds, East Anglia, Southampton and Surrey will make lower overall offers for those who have Further Mathematics (i.e. AAA/AAB rather than A*AA).

Nevertheless, despite so few universities making specific requirements, the proportion of new STEMM undergraduates that have taken A level Further Mathematics has substantially increased since 2005 (see Table 2). The biggest increase has been in mathematics courses, where the majority of students have now taken Further Mathematics.

Table 2: Proportion of accepted applicants with A level Further Mathematics (Baldwin & Lee, 2014, p. 303)

Degree	% of those accepted onto courses who had taken A level Further Mathematics	
	2005/06	2013/14
Statistics	34.1	61.9
Mathematics	35.6	59.6
Physics	15.7	36.1
General Engineering	8.8	25.9
Aerospace Engineering	13.7	23.4
Electronic and Electrical Engineering	10.8	22.6
Mechanical Engineering	11.1	22.5
Civil Engineering	8.1	18.7
Economics	6.2	14.8
Computer science	3.6	11.6
Finance	7.3	21.3

Nevertheless, despite increasing numbers of students taking post-compulsory mathematics qualifications, lecturers have expressed concerns regarding their perceptions of undergraduates' mathematical capabilities. These concerns are long-standing and have been well documented since at least the 1970s. Lecturers and admissions tutors have reported that mathematics undergraduates struggle with proof (Bell, 1976; Harel & Sowder, 1998; Selden & Selden, 2003; Tall & Vinner, 1981; Weber, 2001), especially real analysis (Selden, Selden, & Mason, 1989). Research suggests that the problem is widespread across STEMM: a seminal report, *Tackling the Mathematics Problem* (London Mathematical Society, 1995), documented concerns from lecturers across STEMM subjects that students lacked mathematical fluency, analytical ability and an appreciation of the importance of proof. The report suggested that the content and standards of both GCSE and A level were to blame, arguing that these qualifications had reduced the emphasis on acquisition of skills involving "arithmetic, fractions, ratios, algebraic technique, and the basic geometry of triangles, lines and circles" (p. 15).

These concerns have been echoed more recently by Porkess (2006) and ACME (2011). Porkess (2006) argued that any perceived problems with STEMM undergraduates' mathematical capabilities were a result of the falling number of students taking A level Mathematics following the introduction of Curriculum 2000, rather than the qualification's content. He claims that the A level was sufficiently rigorous, but its perceived difficulty meant that students were reluctant to take the qualification. Consequently, universities were presented with a much smaller pool of STEMM applicants who had taken A level Mathematics, and were thus forced to take mathematically weaker candidates. Additionally, ACME (2011) argues that the low proportion of students taking post-compulsory mathematics means that many are not prepared for the transition to either higher education or employment. However, they argue that this problem is exacerbated by many universities refusing to explicitly require that applicants have taken A level Mathematics or Further Mathematics, particularly in subjects such as bioscience, chemistry, economics, accounting and psychology. Consequently, many students begin university having not studied any mathematics for at least two years. This inevitably leads to cohorts who both do not expect, and are not prepared for, the volume and type of mathematics entailed at undergraduate level.

Preparedness for undergraduate study in mathematical subjects has thus been reported to be a problem in bioscience (Koenig, 2011; Shallcross & Yates, 2014); chemistry (Gadd, 2000; Shallcross & Yates, 2014); computer science (Boyle, Carter, & Clark, 2002; Gordon, 2004; Porkess, 2013); economics (Dawson, 2014; Economics Network, 2010, 2011, 2012); engineering (Lee, Harrison, & Robinson, 2006; Perkin, Pell, & Croft, 2007; The Institute of Physics, 2011); medicine (Freeman, Collier, Staniforth, & Smith, 2008; Koenig, 2006; MacDougall, 2009); physics (Barham, 2012; The Institute of Physics, 2011); and psychology (Bhakta, Wood, & Lawson, 2010; Field, 2014; Mulhern & Wylie, 2006).

2. The current study

Given the substantial forthcoming changes to A level Mathematics and Further Mathematics, as well as the wide range of degree subjects potentially affected, we were interested in current undergraduates' transitions to university study in mathematical subjects. We were specifically interested in lecturers' and admissions tutors' perceptions of students' mathematical capabilities upon beginning university study, as well as their perceptions of existing A levels in Mathematics and Further Mathematics in terms of preparing students for their course. This formed part of a larger study investigating students' experiences of post-compulsory mathematics and the transition to higher education (see Darlington & Bowyer, 2016).

3. Method

3.1 Sampling

Semi-structured interviews were conducted with a selection of admissions tutors and lecturers across a variety of subject areas. Participants were invited to be interviewed regarding their opinions of A level Mathematics and Further Mathematics and the mathematical competency of their undergraduates.

University departments in STEMM and the Social Sciences were contacted via email to request participation in the larger study. As part of this contact, we asked admissions tutors and lecturers with responsibility for first-year teaching to participate in a telephone or face-to-face interview. Some academics were not able to participate in an interview but instead provided written comments.

In total, 30 lecturers or admissions tutors across 10 subjects either participated in an interview or gave written comments (see Table 1). Six interviews were conducted face-to-face, eighteen were by telephone, and seven academics provided written comments. The university mission group of each participant is also included in order to provide a general idea of the range of institutions included.

Table 3: Interview participants

Subject	No. Participants	University Mission Group
Biology	3	All Russell Group
Chemistry	2	All Russell Group
Computer science	4	1 Russell Group, 3 unaffiliated
Economics	4	2 Russell Group, 2 unaffiliated
Engineering	4	3 Russell Group, 1 unaffiliated
Mathematics	3	1 Million+, 2 Russell Group
Medicine (and related)	2	All Russell Group
Physics	3	2 Russell Group, 1 unaffiliated
Psychology	4	1 Million+, 1 University Alliance, 2 unaffiliated
Veterinary Medicine	1	Russell Group

3.2 Procedure

During interviews, participants were asked about:

- the mathematical preparedness of their undergraduates and any areas of mathematics that their students find particularly difficult;
- their perceptions of A level Mathematics and Further Mathematics; and
- any particular areas of mathematics (either specific concepts or skills) that would be useful as preparation for their course.

3.3 Analysis

As we were looking for a general overview of the above three issues, interviews were not transcribed and coded. Instead, a concise summary of each interview was written, incorporating the participant's overall message, the areas they perceive undergraduates to most struggle with, and any additional areas of mathematics that would provide useful preparation for undergraduate study in their subject. These summaries were sent to participants to confirm that they were an accurate representation of their beliefs and to edit as necessary.

4. Findings

4.1 Overall perceptions

Despite the wide range of subjects and the differing mathematical backgrounds of participants' teaching cohorts, there were several common findings regarding perceptions of A level Mathematics and the mathematical preparedness of undergraduates. Throughout all subjects an overarching theme emerged: nearly all participants reported that new undergraduates struggle to apply familiar mathematical concepts in unfamiliar contexts. This manifests itself as a difficulty with 'translating' between mathematics and their particular discipline, such as when modelling a practical engineering or economics problem.

Overall, the following themes emerged from participants' responses. Subject specific summaries follow.

- There does not appear to be any serious concern about the *content* of A level Mathematics. The majority of participants were broadly happy with both the existing and proposed content for A level Mathematics and Further Mathematics.
- However, there appears to be a problem with *skills*: across all undergraduate subjects participants reported that students find it difficult to apply their mathematical knowledge in unfamiliar contexts. In particular, students find it difficult to select and apply mathematical methods that they already know when confronted with unfamiliar problems or contexts.
- Lecturers find that students can also struggle to either understand the relevance of mathematics to their subject (especially in biology or psychology), or find it difficult to 'translate' a problem between mathematics and their discipline (particularly in economics and physics).
- These problems were frequently perceived to be a result of either the modular structure of A level or assessment at A level. Some participants reported that the modularisation of content meant that students perceived different areas of mathematics to be rigidly separate. Students could therefore find it difficult to translate particular skills or concepts to other areas of mathematics.
- Regarding examination at A level, some participants believed that A level promoted rote learning. Some believed that students were taught to associate particular mathematical methods with a certain type of examination question, making it difficult for students to apply these methods in unfamiliar contexts at undergraduate level.
- Where there were specific areas of content that students struggle with, these tended to be relatively basic in nature. Common difficulties were algebraic manipulation, arithmetic and dilutions/concentrations in biology and medicine.
- However, proof, mathematical arguments and the presentation of formal mathematics were cited as a particular difficulty for students in undergraduate mathematics.
- Across the sciences, references were frequently made to science A levels and the perceived lack of mathematics in these qualifications. Participants in engineering and physics made particular reference to the alleged 'demathematisation' of A level Physics, which was found to cause difficulties at undergraduate level as students struggled to apply calculus to physical problems.

4.2 Biology

Three biology lecturers participated in interviews. All were from Russell Group universities. None of their courses required students to have taken A level Mathematics. All three participants perceived students to have difficulties with applying mathematics in appropriate biological contexts. All three also reported that students struggle with relatively basic areas of mathematics such as units, dilutions and arithmetic. One also raised the issue that biology students need to be able to critically read and assess research articles, but new undergraduates are seemingly unable to do this. Whilst this is not currently an A level Mathematics criterion, the ability to understand and criticise the statistical methods used in biological research is an essential skill.

Dean of Education, Russell Group university	
Overall message	A level Mathematics is 'fantastic', in principle, but students seem to "*leave numbers behind*". Students struggle with simple mathematical concepts, such as dilutions and concentrations, and manipulation of numbers. The problem seems to be arithmetic rather than more advanced mathematical concepts.
Specific areas of difficulty	Manipulation of numbers; mental arithmetic; scientific notation; dilutions and concentrations; logarithms.
Additional useful content for university study	• Not necessarily advanced statistical concepts as they will learn this at university, but statistics in the context of scientific experiments. • Confidence intervals • Arithmetic
Head of Biology, Russell Group university	
Overall message	• Students struggling with selecting and applying mathematics in biological contexts rather than with "*doing the Maths*".
Specific areas of difficulty	• Linking mathematics to a biological context • Modelling a problem • Sometimes there can be confusion over units
Additional useful content for university study	• Use of technology (e.g. excel and data sets) • Multi-step problems which are posed without guidance • Curve sketching
Lecturer, Russell Group university	
Overall message	• Students can 'do mathematics' but struggle to select and apply mathematics in unfamiliar contexts. • They regard mathematics as "*an esoteric arcane art*" and do not understand that they are using mathematics to draw out a conclusion that is biologically interesting.
Specific areas of difficulty	• Unit conversion, proportionality, dilution series (i.e. simple division) and regression • The necessity to critically read data and research • Students have not seen any real-world data sets • Applying mathematics to specific contexts
Additional useful content for university study	• Knowledge of the exponential function

4.3 Chemistry

Two chemistry lecturers with responsibility for first year teaching at Russell Group universities participated. A level Mathematics was not a prerequisite qualification for either undergraduate course. One commented that the major problems encountered by their students were related to their mathematical fluency, supporting a range of research in this area. However, the other participant appeared to share the views of lecturers in other subjects, citing the application of mathematics to chemical contexts as the predominant issue.

Lecturer, Russell Group university	
Overall message	• Students have not practised mathematics in a chemistry context before and struggle with "*converting [their] mathematics ability into chemistry*". • The content of A levels is fine; however, it is the way of examining it that is the problem in both mathematics and chemistry. Student can get A*s by memorising the syllabuses, rather than understanding the concepts.
Specific areas of difficulty	• Rearranging complex equations • "*symbol blindness*"
Additional useful content for university study	• Choice of questions in A level examinations would be preferable. • Current applied units: Mechanics is most useful, Statistics is not, and Decision Mathematics units can be useful in the type of thinking they promote.
Lecturer, Russell Group university	
Overall message	• Less than one third of students at this institution have A level Mathematics and students struggle with basic mathematical concepts. • Students' level of mathematical preparedness is decreasing significantly over time. • "*Frankly speaking only students that have Further Maths can understand **basic** Maths*". • The proposed changes to the A level look very positive and would go some way to rectifying the existing problems.
Specific areas of difficulty	• Fractions, conversion of numbers, logarithms.
Additional useful content for university study	• Basic things about functions. • How to get an angular momentum vector from a determinant using the position and linear momentum vectors.

4.4 Computer science

Four computer science lecturers took part, coming from a range of universities. Two universities require a grade A in A level Mathematics, and one requires students to offer an A level in at least one of Mathematics, Physics or Computing. As with other subjects, interviewees' comments suggested that students struggle to independently apply mathematics that they already know to problems in computer science. All four also commented on students' struggles with proof by induction, and the relevance of proof and formal mathematics to success in undergraduate computer science. Decision Mathematics was described as the most useful strand of applied mathematics at A level, which may have implications for students' preparedness for future study in computer science in future, as this content will no longer form part of A level Mathematics.

Director of undergraduate studies, unaffiliated university	
Overall message	• Students "*aren't prepared to think*" and struggle when they come across questions that do not have a single right answer or method. • The content of A level Mathematics is fine but the assessment has become more formulaic with increased hint-giving in exams, especially with calculus.
Specific areas of difficulty	• Set theory • Proof by induction • Students struggle to select and apply methods: they do not have "*a box of 5 tools and look at a question and go, 'Ooh, this is a 'Type 2 tool' kind of question'*".
Additional useful content for university study	• Decision Mathematics units are useful and this institution looks favourably on students who have taken them. • It would almost be better that proof by induction was not taught at A level as not all students study it, and so the university continues to reteach it from scratch. • Relating to ALCAB's decision not to use computer algebra tools at A level: believes that this is the right decision for now but wonders, "*how long can we hold that line*?"
Director of undergraduate studies, unaffiliated university	
Overall message	• This institution has very low numbers of students with A level Mathematics. • Students without A level Mathematics tend to struggle with basic algebraic manipulation. Those with A level Mathematics appear to have forgotten how to do mathematics "*involving numbers*".
Specific areas of difficulty	N/A
Additional useful content for university study	• Basic algebra • Functions • Vectors • Logic • Set theory • Proof by induction • The current Decision Mathematics units seem to promote the right way of thinking for computer science, even if the content is not great.

Lecturer, unaffiliated university	
Overall message	• The problem that students have is one of skills, rather than knowledge. • A level Mathematics is a good indicator of likely success on the course.
Specific areas of difficulty	• Rigour and the notion of proof, particularly proof by induction and contradiction. • Students who did not take, or did not do well in, A level Mathematics tend to struggle with programming.
Additional useful content for university study	• Current applied units at A level: Decision Mathematics units, along with parts of Further Pure units in Further Mathematics, are of particular relevance for computer science. • The best preparation mathematically is an understanding of the notion of proof and formal definition.
Admissions tutor, Russell Group university	
Overall message	• The current A level modular system "*boxes up*" the mathematics too much and is too compartmentalised, meaning that students aren't used to selecting mathematics to solve problems and are taught to 'pattern match' methods to particular questions in exams. • Many students aren't very confident, and want to be told which 'tool in their tool box' to use when solving a problem. • There has been very little Decision Mathematics in A level Computing until recently, so A level Mathematics has been more useful in terms of university preparation. However, they recognise that Computing teaching in schools is changing.
Specific areas of difficulty	• Abstract mathematics • Proof by induction • Set theory • Students find it "*difficult to abstract over variables.*" • Weaker students want things to be "*more pinned down.*"
Additional useful content for university study	• How to approach proofs. • Logical basis of approaching problems. • Current applied units at A level: Decision Mathematics is very useful; Statistics also reasonably useful; Mechanics is "*neither here nor there*".

4.5 Economics

Four economics lecturers were interviewed, all from a range of universities. Two universities require students to achieve an A or A* grade in A level Mathematics; the others had no mathematics entry requirement. The lecturers were mostly negative about the current (unreformed) A level Mathematics, and again commented that students were not capable of thinking mathematically for themselves. Furthermore, some lecturers reported that even students who had achieved top grades in A level Mathematics struggled with the mathematical component of economics. Statistics and pure mathematics were cited as the most useful areas of mathematics for students to have studied before going to university to read economics.

Lecturer, unaffiliated university	
Overall message	• A levels are too structured and cover too much content, meaning that students have not had time to fully understand the more advanced concepts and rely on rote learning. They then struggle with unfamiliar contexts and questions at university level. Even students who do very well at A level "*are lost*" when shown how calculus and statistics are used in economics. • Would have liked to see less content in the new A level specification, with more time for the conceptual topics.
Specific areas of difficulty	• Performing calculations with letters replacing numbers. • Sketching graphs of functions and visualising how the derivatives dy/dx and d^2y/dx^2 relate to slopes and convexity • A reliance on rote learning at A level.
Additional useful content for university study	• Econometrics, Time Series and Stochastic processes. • Would like to see trigonometry replaced by more differentiation, integration and differential equations, especially containing *e* and log functions. • For A level Further Mathematics, matrix algebra is extremely useful. • There is no use in complex numbers or Mechanics units for economics.
Lecturer, unaffiliated university	
Overall message	• Students find it difficult to translate between mathematics and economics: a good majority can 'do mathematics' but find it difficult to apply it. Students are often surprised by the volume and difficulty of mathematics in undergraduate economics. • Taking A level Mathematics makes students better prepared for economics as they have studied some calculus, whereas it would be nice if A level economics contained more mathematics.
Specific areas of difficulty	• Students seem to be "*marginally worse at stats than in other areas of maths.*"
Additional useful content for university study	• 'Mathematics for Economics' type units at A level could be useful because they could act as a signal that economics involves mathematics, and also could encourage uptake of AS or A level Further Mathematics by students wanting to go on to read economics. • Would "*strongly disagree*" that students need to have A level Mathematic s in order to study economics but it helps manage students' expectations regarding how mathematical the subject will be.

<table>
<tr><th colspan="2">Deputy head of department, Russell Group university</th></tr>
<tr><td>Overall message</td><td>Students struggle to apply mathematics in unfamiliar contexts and find it challenging to “translate backwards and forwards” between mathematics and economics.
Statistical knowledge is useful in terms of preparation for economics but pure mathematics is by far the most important, especially calculus.</td></tr>
<tr><td>Specific areas of difficulty</td><td>Algebraic manipulation (biggest weakness).
Interpreting equations in applied contexts.
Expressing economics problems in mathematical terms.</td></tr>
<tr><td>Additional useful content for university study</td><td>Current A level units: Statistics are the most useful applied units for Economics.
Statistics – “understanding something about hypothesis testing”
Basic probability
Calculus</td></tr>
<tr><th colspan="2">Admissions tutor, Russell Group university</th></tr>
<tr><td>Overall message</td><td>Getting an A* at A level means “getting a lot of easy things right” and does not mean that a student is necessarily mathematically competent.
Compared to the international students on the course, A level Mathematics is the worst qualification in terms of preparing students for undergraduate economics.
Students do not necessarily struggle with the content, but are surprised about the way that mathematics is taught at university and expect model answers.</td></tr>
<tr><td>Specific areas of difficulty</td><td>There is an 8-10% fail rate at the end of Year 1. This is believed to be a problem with A level as an A* in Mathematics is compulsory for all students, but some students still struggle significantly. Making it harder to get an A* would mean that these students would get a lower grade and thus not be admitted onto the course in the first place. This would make it easier to differentiate between students before admission.</td></tr>
<tr><td>Additional useful content for university study</td><td>Single variable calculus
Matrix algebra
Mathematical fluency.
More challenging questions at A level, to differentiate between the students who get an A*.</td></tr>
</table>

4.6 Engineering

Four engineering lecturers were interviewed. Two of the lecturers were from the same Russell Group university. Two of the participating universities require an A* in A level Mathematics as well as an A grade in Further Mathematics if the student's school offers this qualification. Participants commented that they perceive students' conceptual knowledge to be poor and that students particularly struggle with applying mathematics in physical contexts. Mechanics was described as the most useful area of applied mathematics for students to have studied at A level. Furthermore, they all complained that A level Physics was not sufficiently mathematical.

Director of Engineering degrees, unaffiliated university	
Overall message	• Students have low mathematical resilience. They can rely on their knowledge of A level Mathematics to get through the first module at university but do significantly worse thereafter when introduced to new concepts. There isn't enough time to develop the conceptual knowledge at A level. • Students think that mathematics is a precise description of an engineering problem but it is not that simple. Engineering requires problem solving and whilst a certain proportion of marks at A level are supposed to come from this, this does not seem to be the case in practice.
Specific areas of difficulty	• Problem solving • Making basic assumptions • Basics such as algebraic manipulation
Additional useful content for university study	• There appears to be a disconnect between mathematics and Physics at A level, even though "*Physics is basically the application of maths*".
Director of studies, Russell Group university	
Overall message	• There appear to be two predominant problems: either students have seen content before but cannot apply it, or they have not had the opportunity to see and grapple with the content before. • Avoidance of mechanics is a problem but this is not students' fault. A level "*Maths hasn't been dumbed down*" and the content is actually useful. However, students who have not taken Further Mathematics are "*always running to catch up*". • A level Physics is also an entry requirement, but is a bigger problem than A level Mathematics: "*There is no calculus so you can't do any physics, which seems like a nonsense*" and contains "*sexy things*" at the expense of 'real' Physics.
Specific areas of difficulty	• Avoidance of Mechanics. • Even students who have taken Mechanics units and Further Mathematics still struggle with: mechanics and structures; thermodynamics and thermofluids; convolution and Fourier series; electrical applications.
Additional useful content for university study	• Basic Mechanics. Mechanics is very important in teaching how to set up a problem and also the practical side of engineering that students need. • Setting up optimisation problems. • A level Electronics is very good and in some ways better than A level Physics, but few schools teach it (or teach it well). • Matrices and calculus are very important, but are retaught in the first year.

Director of undergraduate education, Russell Group university	
Overall message	• The modular structure of A levels "*emphasise[s] "nuggetised" information at the expense of general understanding and a grasp of the nature of rigorous proof.*" • It also encourages rote learning, which synoptic testing does not do – although the change to linearity may decrease this problem. The skills of international students are better than A level students, although home students are better at problem solving. • The "*demathematisation*" of A level Physics is a core problem and students should not be able to study physics without mathematics; studying them separately is also not satisfactory as they learn the skill/concept in mathematics without applying it in physics. This means students know less overall but also do not know "*what physics is all about*".
Specific areas of difficulty	• Algebraic manipulation • "*Using maths to formulate, clarify and solve problems in the physical world.*"
Additional useful content for university study	• "*The roots of the key problems are deeper and lie in the early years of secondary school, and even primary school, so that adjustments at A level will always be limited to making modest differences.*" • The fundamental problem is the lack of mathematics in A level Physics.
Senior tutor and head of teaching, Russell Group university	
Overall message	• The overall problem is one of skills rather than content. Engineering is about applications of mathematics and students have problems "*picking the right tool from the tool box*". • Although the concepts of Mechanics are found in A level Physics, this A level is now taught "*without maths embedded in it*".
Specific areas of difficulty	• Complex numbers, statistics, differentiation and differential equations • Students cannot reason whether their answer to a problem would work in the real world.
Additional content	• Current units: Mechanics is most useful but students without any Mechanics background can catch up at university.

4.7 Mathematics

Four Mathematics lecturers participated in an interview; two were from the same Russell Group university and gave a joint interview. One university requires A* grades in both Mathematics and Further Mathematics, one requires A grades in both A levels, and one prefers, but does not require, A level Mathematics. In common with other subjects in this study, all three commented that their students were not able to think independently regarding the mathematics required to solve a problem. In particular, proof and formal mathematics were perceived to be the most challenging for undergraduates. Generally, they viewed Mechanics as the most useful applied strand for students to have specialised in before going to university.

Undergraduate course leader, Million+ university	
Overall message	• A level Mathematics in general is good preparation for mathematics at university. • However, students are good at techniques and content but struggle to select and apply the mathematics they do know in context. The problem is more global than specific topics at A level. Instead, students struggle with "*joined up thinking.*"
Specific areas of difficulty	• Matrix algebra in particular is underemphasised at A level and when it is taught, it is not motivated by an application. • Algebraic manipulation. • Students have forgotten the basics from GCSE.
Additional useful content for university study	• Matrix algebra • Logarithms
Head of department, Russell Group university	
Overall message	• Students are less prepared to think and apply mathematics in different contexts than in the past: they are "*more brittle*", not less bright. Lecturers want students to demonstrate a mathematical argument but students seem to seek templates. • There appears to very little proof at A level even though assessment objectives state there should be a set proportion of marks for proof.
Specific areas of difficulty	• Proof • Applications of mathematics in different contexts
Additional useful content for university study	• Number theory as part of Decision Mathematics • A level Physics for Mechanics • Logarithms for science degrees • Current A level Mathematics units: Mechanics is hardest; Statistics has become increasingly easy and the Statistics 1 unit is "*pathetic*". Students should tackle large data sets with technology. • Group theory would continue to be retaught from scratch at university even if this content were to be included in the reformed Further Mathematics.

Admissions tutor and university lecturer, Russell Group university	
Overall message	• A level Mathematics does not "*serve the good students well*", but appreciate that it is now more accessible and extension papers, such as STEP, can stretch the most able students. • In terms of the content, "*it would almost be better if students knew less, but knew it better*". Some students can do very complicated mathematics but struggle with the basics, and there is an issue of quality over quantity. It is important that students are able to derive things from first principles. • Students seem better than in the past but this is largely because of higher entrance requirements at this institution. International students are less good at calculus, but much better at proof, than A level students.
Specific areas of difficulty	• Proof • Analysis
Additional useful content for university study	• More logic • Mathematics as axioms • An expectation of formal mathematics • Current optional units: anyone wanting to study undergraduate mathematics should take Mechanics. Statistics is not very useful. Decision could perhaps be more useful if it contained more graph theory.

4.8 Medicine

Two lecturers gave an interview. They echoed participants elsewhere, and commented that their students found the application of mathematics difficult, rather than lacking a particular area of knowledge.

Admissions tutor, Russell Group university	
Overall message	• It is not necessarily the mathematics content that is the problem but rather that students struggle with applying mathematics in context independently. They do not see the importance of "*maths as a tool to describe something*". • "*Increasingly of the view that it's not maths A levels that need to change; it's the science A levels*" - it could be argued that the mathematics content needed for biological courses should be included in the relevant science A levels (Biology and Chemistry) rather than in A level Mathematics.
Specific areas of difficulty	• Using the symbolism of mathematics in applied contexts • Logarithms • Dilutions
Additional useful content for university study	• Logarithms • Things need to be taught in context otherwise they do not make sense, especially logarithms. • Other things should not be taught in context e.g. differentiation.

Admissions tutor, Russell Group university	
Overall message	• It has become increasingly difficult to teach students certain areas of mathematics over the past 10 years. Students struggle with areas that they should know from A level but appear not to. • In particular, students should know trigonometric identities and differential equations but struggle with them at university. A level content is reasonably comprehensive and well-structured but students need to be better at applying what they are taught. • Very few medicine students take A level Physics, when it is actually very useful. Physiology is like the engineering of the human body, and certain concepts are easier to explain when analogous to physics.
Specific areas of difficulty	• Trigonometric identities • Interpreting and applying statistics • Deriving equations from first principles • Differential equations
Additional useful content for university study	• Exponents and logarithms • Differential equations in applied contexts • Statistics should be very applied and focused on use in interpreting data collected from experiments. • Significance tests, analysis of variance, χ^2 test, non/parametric tests, confidence intervals.

4.9 Physics

Three Physics lecturers took part. All three universities required students to have taken A level Mathematics: two universities require an A grade and one required a B grade. Like the engineering lecturers, a dominant concern was that A level Physics did not contain sufficient mathematics content. The application of mathematics in unfamiliar contexts was also cited as a problem for their students.

Director of studies, Russell Group university	
Overall message	• Students are good with principles but are "*shaky on unfamiliar ground*". • The main problems are with visualisation and applying mathematics in unfamiliar contexts. Lecturers get the impression that there is less calculus in the A level than there used to be, because students struggle with it at university. • A level Physics has become "*demathematised*" and this seems odd.
Specific areas of difficulty	• Visualisation and applications • Calculus • Abstract mechanics problems (even when students have studied Mechanics at A level) • Proof • Notation
Additional useful content for university study	• Calculus • Applications of mathematics to physics
Senior tutor, unaffiliated university	
Overall message	• The main problems are students' mathematical fluency, rather than specific content or topics. Students struggle with selecting and applying the mathematics they already know, even in situations they should be familiar with (such as differentiating and integrating). • The problem with students not being able to apply mathematics would probably be best addressed by changes to A level Physics, rather than A level Mathematics.
Specific areas of difficulty	• Mathematical fluency
Additional useful content for university study	• Current A level units: Mechanics is most useful but Statistics still has some use later in the degree. • More multi-stage problems where the required steps are not clearly signposted in the question.

Head of department, Russell Group university	
Overall message	• Students find it difficult to translate physics problems into mathematics, especially calculus. • They also struggle with knowing what an appropriate level of precision is when giving numerical answers and drop units during calculations.
Specific areas of difficulty	• Vectors • Integration
Additional useful content for university study	• A basic understanding of statistics and probability would be very useful for laboratory classes and quantum mechanics (measures of central tendency and dispersion as well as the idea of an expectation value).

4.10 Psychology

Four Psychology lecturers took part in the study. A level Mathematics was not an entry requirement for any of the courses. Consequently, the participants spoke of their students lacking mathematical confidence as well as competence. They commented that students struggle with aspects of statistics as well as basic numeracy. Although it was considered beneficial to have a statistical background, some felt that much of the pure mathematics content in A level was not relevant to psychology at undergraduate level and may therefore not meet the needs of their students.

Admissions tutor, unaffiliated university	
Overall message	• A level Mathematics is not a requirement for this course and therefore students tend to struggle quite badly with the mathematical elements. Most will not have studied any mathematics since GCSE. The volume of mathematics in the course is a surprise for students. • A lot of A level Mathematics is not relevant for Psychology, although pure mathematics is good at developing logical reasoning and thinking arguments through.
Specific areas of difficulty	• Hypothesis testing • Significance levels
Additional useful content for university study	• Thinking through arguments and scientific reasoning.
University lecturer, University Alliance	
Overall message	• A majority of students on the course will not have studied A level Mathematics and therefore whilst students appear to be more mathematically prepared now, there is always a proportion who have not studied any mathematics beyond GCSE and who lack confidence as a result. • Students who are very proficient at mathematics can sometimes lack the ability to apply their knowledge to understand psychological issues and concepts.
Specific areas of difficulty	• Statistics • Numerical reasoning • Probability • *z*-scores
Additional useful content for university study	• Probability, statistics and correlation.

Senior lecturer, Unaffiliated university	
Overall message	• A level Mathematics is not that relevant for psychology: "*Students don't study psychology because they're interested in numbers and statistics!*" • Most students are new to statistics and research methods and therefore find it difficult, but as it is entirely new material they do rate it as one of their most enjoyable first year modules (according to their own surveys).
Specific areas of difficulty	• Central tendency, population parameters, normal distributions and standard deviation.
Additional useful content for university study	• Understand what statistical terms are meant to do. • Interpret the outcome of statistical tests in context. • Inferential statistics
Deputy head of department, Million+	
Overall message	• A mixed cohort in terms of prior mathematics qualifications causes difficulties with mathematical knowledge. • There is also some difficulty with their ability to interpret and understand mathematics to draw conclusions from psychological data. In particular, there is a difficulty in understanding that "*psychologists don't prove anything*" and what confidence intervals and significance actually mean.
Specific areas of difficulty	• Probability • Basic calculations (including multiplication of negative numbers)
Additional useful content for university study	• Co-ordinate geometry in the context of regression. • All statistics outlined in the new A level Mathematics. • It would be good if students knew the theory of statistics and probability – it is not so important that they have used it in context as they will do this later. • Introducing the differences between parametric and non-parametric tests. • Knowing when a test/technique is appropriate.

4.11 Veterinary medicine

One Veterinary Medicine lecturer took part in the study, from a university where a reasonable proportion of incoming students have taken A level Mathematics.

Director of teaching, Russell Group university	
Overall message	• The areas students struggle with are more arithmetic than difficult mathematical concepts. However, these are arithmetic skills that are crucial for veterinary medicine, such as calculating drug dosages. • After seeing students struggle with the mathematics content of the degree, it seems surprising that students have taken A level Mathematics and implies that "*Maths ain't what it used to be*". • As well as simple calculations like dilutions and unit conversions, a core skill is "*applications, applications, applications*", which is a "*higher level skill*".
Specific areas of difficulty	• Dilutions/concentrations • Unit conversions • Knowing which statistics to apply in certain circumstances
Additional useful content for university study	• Statistics: central tendency, *p* values, confidence intervals etc. to enable students to "*interpret scientific papers properly*". • Multi-step problems to help later on at university when studying Mathematics as part of fluid therapy, to calculate drug dosages (*"important when you're looking at a Great Dane one minute, and a Chihuahua the next!")* and reading ECGs. • Logs, calculus, area under a curve, rates of change. • Vets will also go into private practice so some financial Mathematics would also be an advantage. • Questions should be asked in context to aid understanding.

5. Discussion and conclusions

The findings of this study build on existing literature that suggests that lecturers perceive new undergraduates to struggle mathematically during the transition to higher education. Whilst recent studies have made similar conclusions (e.g. see ACME, 2011, and the Higher Education Academy's *Tackling Transition* series), this study covers the broadest range of university subjects since the seminal London Mathematical Society report, *Tackling the Mathematics Problem* (1995). Additionally, the interviews in this report differ from previous studies in that they specifically focused on the roles that A level Mathematics and Further Mathematics play in aiding the transition.

Despite the wide range of subjects and the differing mathematical backgrounds of participants' teaching cohorts, there were several common findings regarding perceptions of A level Mathematics and Further Mathematics and the mathematical preparedness of undergraduates. The primary issue is one of mathematical skills, rather than content: nearly all participants reported that whilst the content of A level qualifications was broadly appropriate, new undergraduates' main difficulty is applying familiar mathematical concepts in unfamiliar contexts. This manifests itself as a difficulty with 'translating' between mathematics and their particular discipline, such as when creating a mathematical model for a practical engineering or economics problem. Additionally, in some subjects that are traditionally considered to be less mathematical, such as biology and psychology, interviewees reported that students fail to see the relevance of mathematics to their degree, and thus struggle to see where and how to apply their mathematical knowledge. This is despite the fact that many reformed A levels in the sciences and social sciences have increased quantitative and mathematical components, meaning that students who have taken these A levels should be able to make connections between mathematics and their subject. Interviewees also reported that some undergraduates lack basic mathematical skills such as algebraic manipulation, arithmetic and the use of units. Mathematics and computer science undergraduates particularly struggle with proof and formal mathematics.

Whilst students' difficulties with applying mathematics in unfamiliar contexts is concerning, participants' comments that both the existing and proposed A level content is not in itself a problem is reassuring. Indeed, the existing applied mathematics strands in both A levels seem to enable the qualification to fulfil its role as a service subject by meeting the needs of various subjects, although there is concern that some of the existing Statistics and Decision Mathematics units are insufficiently challenging. Additionally, the flexibility of the modular system means that not all students are able to take the applied units that would most benefit them, for example in the case of the Engineering lecturer who reported that many students are unable to take Mechanics. A level reform will, to an extent, improve this situation: the introduction of prescribed content (including both Mechanics and Statistics) will ensure that all students study the same areas of mathematics and thus will reduce the variability in students' mathematical backgrounds. Furthermore, the transition to a linear system may reduce the tendency to compartmentalise different areas of mathematics and consequently enable students to link different areas of mathematics more effectively.

Nevertheless, it seems unlikely that A level reform alone will rectify perceived problems with students' ability to *apply* mathematics. Similarly, greater signalling from the university sector about the benefits of taking a post-compulsory mathematics qualification will help increase awareness about the volume of mathematics involved in STEMM and the Social Sciences at undergraduate level, but will not improve perceived problems regarding the skills of even those students who attain high grades in A level Mathematics. A greater use of context in mathematics teaching at A level would be helpful, allowing students to see practical applications of mathematical concepts. Additionally, a greater emphasis on mathematics in relevant A level subjects would perhaps be beneficial, and is a factor that has been addressed during qualification reform as subjects such as Economics and the Sciences now have more quantitative content. However, ensuring mathematics is embedded across the curriculum requires all teachers to have the necessary expertise, as well as an increase in the number of qualified mathematics teachers. Given that there is currently a national shortage of 5,500 mathematics teachers, and that one quarter of those teaching mathematics at secondary school are not mathematics specialists (ACME, 2014), this will be very difficult to implement in practice. As a result, teachers, especially non-specialists, should be made aware of the full range of resources and professional development that is available through non-profit organisations such as MEI and the Further Mathematics Support Programme.

A final point of consideration is the role of A level Mathematics. As previously mentioned, this qualification acts as a service subject for a wide range of degrees. The A level must therefore meet the needs of prospective undergraduate mathematics students, as well as those intending to study subjects such as psychology, biology or civil engineering. Whilst there are some commonalities in the type of mathematics students of all disciplines should know, individual subjects have very different mathematical needs. When using contextual examples, therefore, it is extremely difficult for teachers to find an example that is appropriate for all students. Furthermore, the profile of the A level cohort has significantly changed since the 1980s and the end of the selective school system. Rather than being the preserve of grammar school students, the A level is now taken by a considerably larger proportion of students today and caters to students of varying ability. Whilst it is to be expected that universities have very high expectations of their students, it is perhaps inevitable that a single qualification is unable to meet all of the needs of such a wide range of stakeholders.

Consequently, it is imperative that universities are aware of the range and content of the full spectrum of post-compulsory mathematics qualifications. For psychology and biology, for example, advising students to take one of the new Core Mathematics qualifications could be a sensible move, as some of these specifications allow students to study statistics and quantitative analysis without studying pure mathematics. Additionally, if an exam board includes additional mechanics content in the reformed AS and A level Further Mathematics, then admissions tutors in engineering or physics may wish to recommend that prospective students take it in order to improve their exposure to this area of mathematics. A level reform provides an opportune time for universities to fully consider the needs of their students and the demands of their course, as well as the advice that is given to prospective students.

Related publications

Other publications based on the overarching project from which this article is based include:

Darlington, E. (2015). *Students' perceptions of A-level Further Mathematics as preparation for undergraduate mathematics*. Proceedings of the British Society for Research into the Learning of Mathematics, *35*(2), 13-18. Durham University, Durham, UK.

Darlington, E. (2015). The Mathematics Needs of Higher Education. *Sigma Network Newsletter*, *9*.

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Darlington, E. & Bowyer, J. (2016). Accounting for students' mathematical preparedness for Finance and Business degrees. *Research Matters: A Cambridge Assessment Publication*, *22*, 9-16.

Darlington, E. & Bowyer, J. (in press). Students' views of, and motivations for, studying A-level Further Mathematics. *MSOR Connections*.

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Bowyer, J. & Darlington, E. (in press). "Should I take Further Mathematics?" Physics undergraduates' experiences of post-compulsory Mathematics. *Physics Education*.

Bowyer, J. & Darlington, E. (accepted). Mathematical struggles and ensuring success: post-compulsory mathematics as preparation for undergraduate bioscience. *Journal of Biological Education*.

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