Undergraduate Mathematics students' views of their pre-university mathematical preparation

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Introduction

This research took place during an extensive A level reform programme conducted by the UK government's Department for Education, in order to inform the redevelopment of the content and assessment used as part of optional Further Mathematics units. Whilst much research has been conducted on the transition between secondary and tertiary Mathematics in the UK and internationally, research has as yet not been conducted regarding students' perceptions of the usefulness of A level Mathematics and Further Mathematics. This study sought to answer the following research questions:

1. Which optional units in A level Mathematics and/or Further Mathematics did students find useful as preparation for their degree?
2. Did students believe that A level Mathematics and Further Mathematics were useful preparation for their degree?
3. Are there any areas in which A level Mathematics and/or Further Mathematics could be improved to suit the needs of future prospective Mathematics undergraduates?

Whilst this article reports on the responses of Mathematics undergraduates, the data was collected as part of a larger overarching project which sought the views of over 4,000 undergraduate Science and Social Science students regarding their perceptions of A level Mathematics as preparation for the mathematical demands of their degree (Darlington & Bowyer, 2016).

Mathematics degree courses

In recent years, the number of Mathematics undergraduates has increased substantially, both in absolute numbers and in terms of the proportion of Mathematics students of the whole full-time undergraduate population (see Figure 1). Numbers have increased from 13,188 (1.3% of all full-time undergraduates) in 1996/97 to 27,810 (2.1%) in 2014/15. The number of Mathematics graduates is important for economic development, and thus the need for a large number of mathematically-competent graduates continues to increase (Gago, 2004; Petocz & Reid, 2005; Wolf, 2002).

A levels

Advanced, or 'A' level qualifications in England and Wales are post-compulsory qualifications taken at the end of secondary schooling at age 18. A wide variety of subjects are available for students to choose from, with most studying three or four subjects over a two-year period. Students are then awarded separate grades for each subject. Whilst there are no compulsory A level subjects, A level Mathematics was the most popular subject in 2016, comprising 11.0% of all A levels taken (Joint Council for Qualifications [JCQ], 2016). An A level is also available

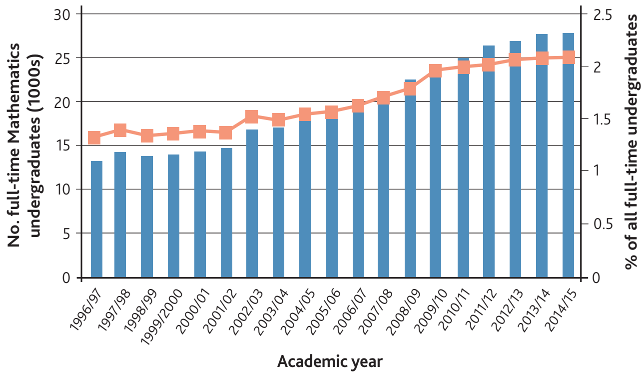

Figure 1: Full-time students of Mathematics degrees in the United Kingdom, 1996-2015 *Data from the Higher Education Statistics Agency (1998-2016).*

in Further Mathematics, which is one of the fastest-growing subjects in terms of uptake.

At the time of writing, A levels are examined at the end of a two-year course, with separate assessment for each unit which makes up each A level. Students are able to be examined at the end of the first year of the course, earning an Advanced Subsidiary 'AS' level. Students are generally required to achieve certain grades in A levels to be accepted onto a university degree course.

A level Mathematics and Further Mathematics: pre-September 2017

A level Mathematics comprises six equally weighted units (modules), which are individually assessed in 90 minute examinations. Four of these units are compulsory Pure Mathematics units: 'Core Pure Mathematics'. Two optional units may then be chosen from three Applied Mathematics strands, each of which contains up to five sequential units: Mechanics (M1-M5), Statistics (S1-S4), and Decision Mathematics (D1-D2). The two optional units may be chosen from the same strand (e.g., Statistics 1 + Statistics 2) or from a mixture of two (i.e., M1 + S1, M1 + D1, S1 +D1). Consequently, there are six routes through A level Mathematics. AS level Mathematics comprises Core Pure Mathematics 1 (C1) and Core Pure Mathematics 2 (C2) and one of S1, D1 and M1.

A level Further Mathematics comprises three compulsory Further Pure (FP) Mathematics units, and three optional units. The optional units may be selected from any combination of FP and Applied options from the three strands which are available as part of A level Mathematics. Students may not take units in AS or A level Further Mathematics which they have already taken as part of A level Mathematics.

Whilst Further Mathematics has been found to be the most (relatively) demanding A level subject (Hillman, 2014), high numbers of students achieve high grades in the subject – in 2016, 56.2% of A level Further Mathematics students achieved at least an A grade (JCQ, 2016).

A level Mathematics and Further Mathematics: post-September 2017

The A level system is currently undergoing a general reform programme with specific changes also planned to take place within certain subjects. From September 2017, there will be no optionality in A level Mathematics. All students will study Pure Mathematics, Mechanics and Statistics content. This change will mean that university admissions tutors will know that all students who have taken A level Mathematics will definitely have studied certain content. At present, for example, students embarking upon Engineering degrees (where prior study of Mechanics is beneficial) may have studied up to five units from the Mechanics strand, depending on the units they studied and whether or not they studied Further Mathematics.

Decision Mathematics will no longer be available for study as part of A level Mathematics, though it may be available through the study of Further Mathematics. Half of Further Mathematics' content will be compulsory Pure Mathematics material, with the remaining half optional. Optional content will be decided by the awarding bodies of the examinations, and is likely to involve innovative new units which might not necessarily follow the currently Decision-Mechanics-Statistics structure.

A level Mathematics as preparation for undergraduate mathematics

Apart from A level Mathematics and Further Mathematics, the most popular A level subjects taken by new Mathematics undergraduates are predominantly in the Sciences or Computing. These A levels also have mathematical components (see Table 1).

Table 1: Top 10 A level subjects taken by 2011's new undergraduate mathematicians (Vidal Rodeiro & Sutch, 2013)

Rank	*Subject*	*% students*
1	Mathematics	68.2
2	Physics	32.4
3	Further Mathematics	30.9
4	Chemistry	19.1
5	Information Communication Technology	12.4
6	General Studies	11.9
7	Computing	11.5
8	Biology	11.4
9	Economics	10.0
10	Business Studies	9.3

Note: This is the most recently available data.

As well as a rising number of entrants to undergraduate Mathematics, an increasing number of students are taking post-compulsory Mathematics qualifications. Mathematics is the most popular subject at A level, with 92,163 candidates in 2016 (JCQ, 2016). Additionally, Further Mathematics is the fastest growing A level, with the number of students taking this qualification more than doubling from 5,720 candidates in 2004 to 15,257 in 2016 (JCQ, 2016). The increasing numbers of students choosing to study post-compulsory Mathematics has been attributed to the changing economic climate. That is, students realise that Mathematics has a high exchange value in the workplace and in Higher Education, and therefore they study it in order to increase their future job prospects.

Mathematics requirements for undergraduate courses in the UK

Students are required to achieve high grades in A level Mathematics in order to study the subject at most universities, and increasing numbers of universities are making Further Mathematics a compulsory entry requirement. In fact, the increases in the proportion of new undergraduates who have taken A level Further Mathematics mean that some universities are now changing the structure and content of the first year of their degrees to accommodate the changing mathematical backgrounds of their students (Searle, 2014).

Students in the Mathematical Sciences are more likely to achieve top grades at A level than students in other subject areas, with 8.1% of them achieving three or more A* grades at A level (Vidal Rodeiro & Zanini, 2015), a figure which is much higher than in other degree subjects. Similarly, a greater proportion of Mathematical Sciences students graduate with a First-class degree result (32%) than other subjects (Vidal Rodeiro & Zanini, 2015).

The Mathematics problem

The preparedness of British undergraduate mathematicians for the demands of university study has been of concern since the 1990s. The term 'Mathematics problem' is used widely to describe anxieties regarding the relatively small number of students choosing to study the subject at tertiary level, not just in the UK but on an international scale. This has sometimes been attributed to an increased number of students having negative experiences of Mathematics at school (Smith, 2004). Furthermore, once students advance to undergraduate study, many fail to succeed in this new environment – the Mathematical Sciences had the highest drop-out rate (24.0%) of all disciplines in 2014/15 (Higher Education Statistics Agency [HESA], 2016a).

Savage (2003) reported that this phenomenon occurred despite many Mathematics students achieving good grades at A level, something which is essential for students to be accepted onto Mathematics degree courses. According to Savage (2003), incoming students were lacking in three areas:

1. They were unable to fluently and consistently perform algebraic manipulations and simplifications
2. Their analytical powers were weak in instances where they were required to solve multi-step problems
3. They were ignorant of the nature of Mathematics and, more specifically, undergraduate Mathematics.

Concerns have been raised over the past few decades that new students arrive at university with insufficient Mathematics knowledge (ACME, 2011; Williams, 2011). The skills taught at school are often considered by universities to be an insufficient basis for further study in Mathematics, and the gap between secondary and tertiary is widely researched and debated, with Tall (1991) describing it as a shift "from *describing* to *defining*, from *convincing* to *proving* in a logical manner based on those definitions" (p.20). The mathematical competency of incoming undergraduates has been found by Smith (2004) to be decreasing over time, with scores on a diagnostic test for new students decreasing with each cohort.

This frequently manifests itself in students' difficulties with mathematical proof. Selden (2012) called the new emphasis on proof at the undergraduate level a 'major hurdle' for newcomers, with much of it centred on mathematical analysis.

Criticisms of the current secondary curriculum and assessment regarding the preparedness of new undergraduate mathematicians have resulted in the evolution of the university Mathematics curriculum. The apparent discrepancy between what students actually know post-A level and what their lecturers expect them to know when they begin university study "will, at the very least, impair the quality of their education and, at worst, may prove too difficult for them to bridge" (Lawson, 1997, p.151).

As the content (and purpose) of A level Mathematics has continued to change throughout the decades, universities have made a number of concessions to change. Consequently, diagnostic testing is now used in many Mathematics departments across the UK (Edwards, 1996; LTSN MathsTEAM, 2003; Williams, Hernandez-Martinez, & Harris, 2010), with many universities conceding that "the idea that the final year of school should fit the students for the first year of mathematics is no longer automatic" (Baumslag, 2000, p.6).

The impact of mathematical backgrounds on degree performance

In a study of the secondary-tertiary Mathematics interface, Kajander and Lovric (2005) found that students' school experiences often shape study approaches at undergraduate level. These stemmed from their beliefs about Mathematics which were that Mathematics is a rule-based subject requiring the learner to memorise facts and algorithms (Anderson, Austin, Barnard, & Jagger, 1998; Crawford, Gordon, Nicholas, & Prosser, 1994, 1998a, 1998b).

The difference between secondary and undergraduate Mathematics in the UK has been outlined by Darlington (2014), who used the Mathematical Assessment Task Hierarchy (Smith et al., 1996) to compare the types of skills required to answer questions in A level Mathematics and Further Mathematics, admissions tests, and undergraduate Mathematics examinations. This analysis revealed A level to be dominated by the routine use of procedures, but undergraduate examinations to emphasise proofs and interpretations.

Indeed, Gueudet (2008) argues that, at school, "students just have to produce results. At university, they seem to have an increasing responsibility towards the knowledge taught" (p.240). This takes the form of applying what they have been taught in a creative fashion which should ultimately allow them to construct proofs of mathematical statements and conjectures; however, many have a "(false) belief that, given sufficient time and study, there will be an algorithm that will solve any given problem" (Ervynck, 1991, p.52). Students' ability to apply what they have learnt at school in terms of their mathematical understanding, learning approaches and conceptions of Mathematics to the undergraduate setting is essential in their success with the subject at tertiary level (Wood, 2001). Consequently, many students report experiencing a 'bump' in their educational path (Perrenet & Taconis, 2009).

Method

An online questionnaire was developed in order to gain an insight into Mathematics undergraduates' perceptions of their mathematical preparedness. Only students who had taken A level Further Mathematics were eligible to take part, as students were asked specifically to reflect on how well A level Mathematics and Further Mathematics had prepared them. They must also have completed at least one year of degree study in order that they could reflect on their experiences so far.

Students from different universities are subjected to different admissions requirements, and study different content, receive different types of teaching, and are subjected to different examination and assessment systems. The questionnaire was publicised to all UK universities offering single honours Mathematics degrees in the hope that participation could be gathered from a wide cross-section of the student population.

The questionnaire was developed by the authors in conjunction with A level Mathematics qualification specialists, and piloted with three recent graduates of STEMM and Social Science degrees who had taken A level Mathematics and Further Mathematics to ascertain whether the questions were appropriate, effective and clear. Minor changes were made in response to the piloters' feedback. The questionnaire sought to survey participants regarding:

- Mathematical background
- Current study and performance
- Perceptions of mathematical preparedness.

Results

The results of statistical testing in this article all refer to chi-squared tests, or Fisher's exact test, where a chi-squared test could not be performed.

Sample

After data cleaning, 928 students participated in the study.

Gender: The sample consisted of 35.6% female and 63.4% male participants. This is representative of the ratio of males to females studying Mathematics at university; in the 2014/15 academic year, 62% of undergraduates in the Mathematical Sciences at British universities were male (HESA, 2016b).

Study institution: Participants came from 42 different universities, with a median of 65 per university. Most participants studied at universities in England (91.9%), with 2.3% in Scotland, 2.1% in Northern Ireland and 3.7% in Wales.

Degree programmes: Students participating in the questionnaire studied one of 46 different specific degrees, including joint honours degrees (see Table 2). Most participants were in their second year of study (50.8%), with 34.5% in their third and 14.7% in their fourth.

Table 2: Degree courses studied

	Degree	*No. participants*	*% participants*
Single honours[a]		690	75.1
Joint honours	Physics	43	4.7
	Economics	40	4.4
	Operational Research	32	3.5
	Computer Science	30	3.3
	Statistics	29	3.2
	Finance	16	1.7
	Philosophy	11	1.2
	Modern Foreign languages	8	0.9
	Business Studies	7	0.8
	Other	13	1.4
Total		**919**	**100**

[a] including degrees such as 'Mathematical Sciences' and 'Mathematical Studies'.

Table 3: Participants A level Mathematics and AS/A level Further Mathematics grades

Grade	*% Students*						
	A level Mathematics			*AS level Further Mathematics*		*A level Further Mathematics*	
	Participants (n=927)	*All Mathematics & Computer Science undergraduates (2011)*	*All candidates (2016)*	*Participants (n=112)*	*All candidates (2016)*	*Participants (n=788)*	*All candidates (2016)*
A*	72.5	34.7	17.5	N/A	N/A	53.4	28.7
A	22.3	29.1	24.3	60.7	53.8	27.4	27.5
B	4.3	17.3	22.3	21.4	16.7	12.4	20.6
C	0.6	10.3	16.1	8.0	11.6	4.9	11.3
D	0.2	5.8	10.8	4.5	7.5	1.6	6.5
E	0.0	2.7	6.1	3.6	5.0	0.1	3.5
Fail	0.0	0.1	2.9	1.8	5.4	0.0	1.9

Additional data from the JCQ (2016) and Vidal Rodeiro (2012).

Academic performance: The majority (87.2%) of participants had taken the full Mathematics A level, and 12.8% the AS level. Only 41.7% of participants were required to have taken AS or A level Further Mathematics to be accepted onto their course.

In both Mathematics and Further Mathematics, most participants achieved an A*. Table 3 shows that the average participant was therefore a higher attainer than the average A level Mathematics or Further Mathematics student in 2016. This is particularly the case for A level Mathematics.

Students who were required to have taken Further Mathematics to be accepted on their course were more likely (*p<.001*) to have achieved an A* in A level Mathematics (86.1%) and Further Mathematics (61.8%) than those who were not (53.7% and 24.9%, respectively).

Significantly more males were awarded higher grades in A level Mathematics than women were (*p<.001*), with 61.8% of women achieving an A* compared to 78.7% of men. However, the proportions of each gender achieving grade B or lower were very similar.

Most participants were awarded their final Mathematics qualification in 2013 (42.1%) or 2012 (34.3%), with some finishing their A levels as far back as 2006.

Of the 928 participants, 916 (98.7%) recalled taking at least one Mechanics unit, 902 (97.2%) a Statistics unit, and 676 (72.8%) a Decision Mathematics unit (see Figure 2). Most of those who took Decision Mathematics only took D1. However, most of those who studied Mechanics or Statistics took two units from those strands. Most participants studied Further Pure Mathematics up to FP3, reflecting that the majority had taken the full A level in Further Mathematics.

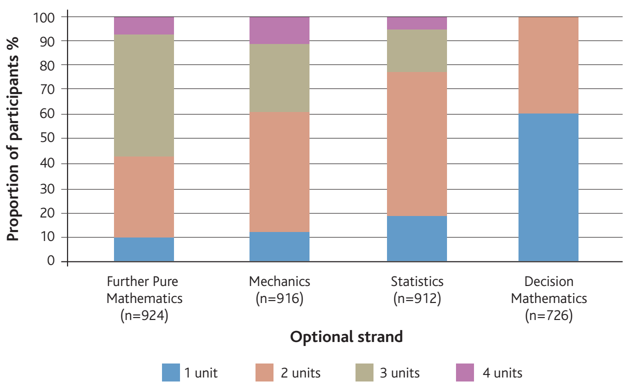

Figure 2: Number of optional units studied

Men studied significantly more Mechanics units than women (*p<.001*), though the majority of both male and female participants reported that they had studied two Mechanics units. Nearly 33% of men and 20.8% of women took three Mechanics units, with 11.1% of men taking four units compared to only 2.1% of women. Furthermore, 19.9% of women only studied M1, compared to only 8.2% of men.

Experiences of non-compulsory A level units

Participants were asked to comment on the relative utility of the non-compulsory units that they studied as part of Mathematics and Further Mathematics as preparation for university Mathematics (see Figure 3).

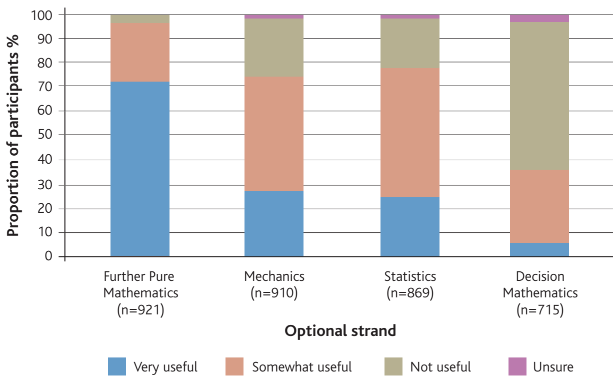

Figure 3: Students' views of the utility of optional units

Further Pure Mathematics, two units of which is compulsory in AS level Further Mathematics and three units for A level Further Mathematics, were described most positively. Overall, of the four strands participants were questioned about, these units were described most commonly as very useful preparation for their degree (73.0% participants). Only 22 of the 921 (2.4%) who answered this question described it as not useful.

Similar proportions of participants found Mechanics and Statistics units to be very or somewhat useful as preparation for their undergraduate courses (74.5% and 77.4%, respectively). However, the proportion of participants who reported that Decision Mathematics units were useful preparation was much smaller, with 60.2% reporting that this strand was not useful. Conversely, only 23.5% of participants who took Mechanics and 28.4% of those who took Statistics described them as not useful.

Significantly more women than men (*p<.001*) perceived Statistics to be useful for their degree, with 39.8% of the former describing it as very useful but only 22.0% of the latter agreeing.

Motivations for studying Further Mathematics

Students were asked to indicate how influential certain factors were in their decisions to study Further Mathematics (see Figure 4) using statements which had been used in an earlier study regarding the uptake of A level Mathematics (Qualifications and Curriculum Authority, 2007).

The vast majority of participants (89.5%) reported that they were influenced in their decision to take Further Mathematics to some extent by their perception that they had coped well with GCSE Mathematics.

Of the 15 options given to participants, the three factors most influencing their decision to study Further Mathematics were:

1. **Enjoyment of school Mathematics:** 85.0% of participants reported that this influenced them a lot in their decision-making.
2. **Being better at Mathematics than at other subjects:** 95.5% of participants reported that this influenced them either a lot or a little in their decision to study Further Mathematics.
3. **Thinking of studying for a Mathematics or Mathematics-related degree:** Only 5.0% of participants reported that this had no bearing on their decision to study Further Mathematics.

Factors which had little impact on participants' decisions to choose Further Mathematics included encouragement by parents, their school Mathematics departments' results and the subject choices of their peers.

Experiences of Further Mathematics

Participants were asked to indicate their relative agreement with 10 statements regarding their experiences of studying Further Mathematics. The data indicates that participants were generally positive (see Table 4).

Responses indicate that participants generally enjoyed Further Mathematics and were glad that they had taken it. Encouragingly, considering the sample and their A level performance (see Table 3), more than 60% of participants indicated agreement with the statement 'I found Further Maths challenging', although only 36.2% reported that Further Mathematics was their most difficult A level.

However, there were significant gender differences in these aspects, with women much more likely to have found Further Mathematics challenging than men (*p<.001*) and more likely to agree that it was their most difficult A level (*p=.006*). The requirement of Further Mathematics for university entry also affected responses, with 27.2% of those who were not required to have taken it strongly agreeing that Further Mathematics was their most difficult A level, compared to only 15.5% of those who were required to have taken it.

Additionally, whether or not the participant was required to have taken Further Mathematics to be accepted onto their current degree course impacted responses. Only 54.1% of such students agreed or strongly agreed that Further Mathematics was challenging, compared to 84.3% of participants who were not required to have taken Further Mathematics.

Most participants (79.9%) reported that there was some overlap between what they had studied at A level and what they were taught in the first year of their degree. Perhaps indicating that universities tailor their courses well for the entry requirements they make of their students, only 30.3% of participants who were required to have taken Further Mathematics strongly agreed that they were taught Further Mathematics material in their first year of university, compared to 48.7% of those who were not required to have taken Further Mathematics.

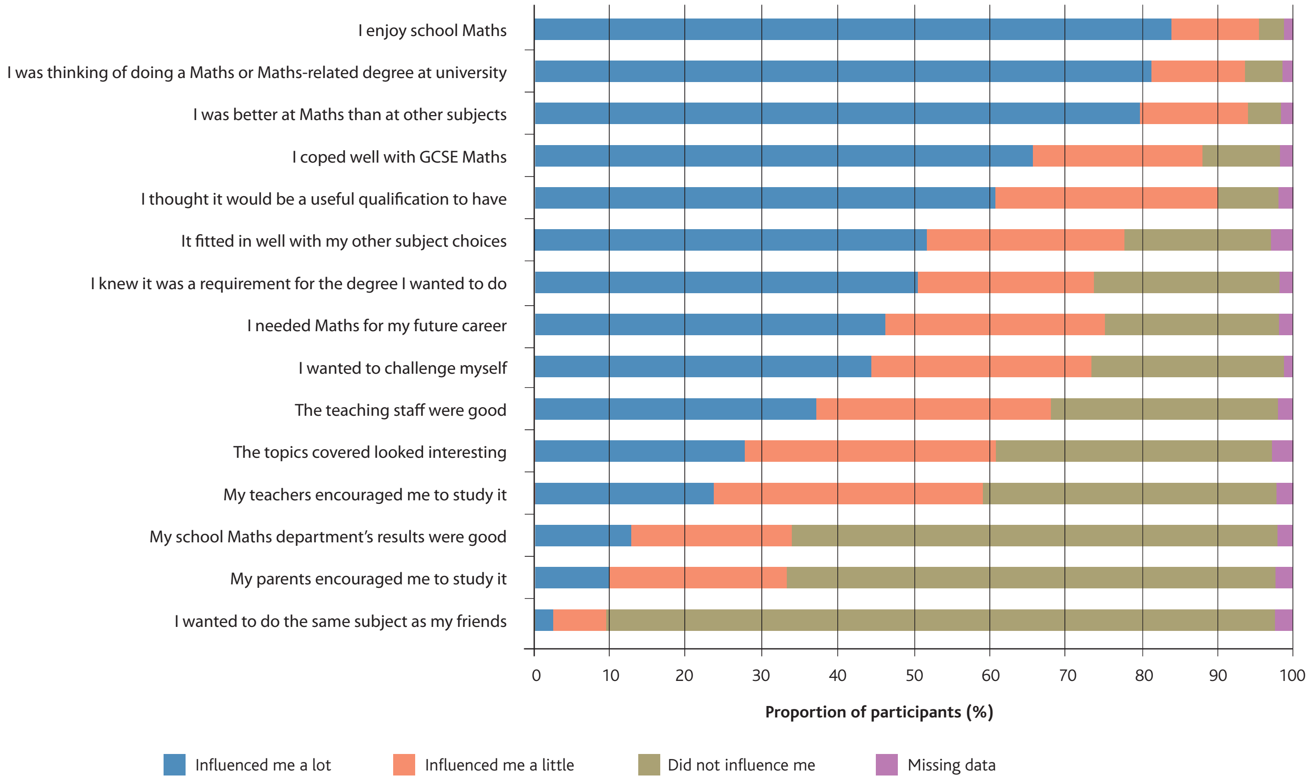

Figure 4: Students' motivations for studying Further Mathematics (*N*=928)

Table 4: Participants' experiences of studying Further Mathematics

	Strongly agree	*Agree*	*Neither agree nor disagree*	*Disagree*	*Strongly disagree*	*Unsure*
			No. participants (%)			
Further Maths was my most difficult A level	188 (20.4%)	143 (15.5%)	128 (13.9%)	299 (32.5%)	156 (16.9%)	7 (0.8%)
I'm glad I did Further Maths	673 (73.2%)	222 (24.1%)	19 (2.1%)	4 (0.4%)	2 (0.2%)	0 (0.0%)
I enjoyed Further Maths	492 (53.3%)	359 (38.9%)	50 (5.4%)	19 (2.1%)	3 (0.3%)	0 (0.0%)
In my first year at university, we were taught material that I had learned in Further Maths	349 (37.7%)	386 (41.7%)	76 (8.2%)	77 (8.3%)	32 (3.5%)	5 (0.5%)
Most people on my university course studied Further Maths	456 (49.5%)	193 (21.0%)	100 (10.9%)	96 (10.4%)	23 (2.5%)	53 (5.8%)
I found Further Maths challenging	211 (22.8%)	407 (44.0%)	143 (15.5%)	120 (13.0%)	43 (4.6%)	1 (0.1%)
Studying Maths and Further Maths was sufficient preparation for my degree	266 (28.9%)	351 (38.2%)	94 (10.2%)	137 (14.9%)	69 (7.5%)	2 (0.2%)
In my first year at university, we were taught material that I had learned in Further Maths	349 (37.7%)	386 (41.7%)	76 (8.2%)	77 (8.3%)	32 (3.5%)	5 (0.5%)

A levels as preparation for Mathematics degrees

Participants were asked how well they thought that A level Mathematics and Further Mathematics prepared them for their degrees. In the case of both Mathematics and Further Mathematics, most students believed that these papers were good preparation for studying undergraduate Mathematics (see Table 5).

Table 5: Students' views of the A levels as preparation for their degree

	A level Mathematics		*AS or A level Further Mathematics*	
	N	*%*	*N*	*%*
Good preparation	558	61.9	699	76.0
Neither good preparation nor bad preparation	262	29.0	178	19.3
Bad preparation	82	9.1	43	4.7
Total	**902**	**100.0**	**920**	**100.0**

The majority of participants described A level Mathematics and Further Mathematics as good preparation for their degree. Similar proportions of participants reported this regardless of whether they had taken Further Mathematics to AS or A level (77.2% and 75.5%, respectively).

Participants' responses also appeared to be influenced by whether or not they were required to have taken Further Mathematics to be accepted onto their course. Nearly three-quarters (72.5%) of those who were not required to have taken Further Mathematics described A level Mathematics as good preparation for their degree, compared to just over half (54.7%) of those who were required to have taken Further Mathematics ($p<.001$). Similarly, those who were not required to have taken Further Mathematics were more positive about Further Mathematics as preparation for their degree ($p<.001$), with 86.5% of them describing it as good preparation compared to 68.6% of those who were required to have taken it.

Improvements to Mathematics and Further Mathematics

Participants were asked to respond to two open-ended questions: The first question, about the ways in which A level Mathematics and/or Further Mathematics could have provided better preparation for tertiary study, received 746 responses. These responses were analysed and coded using MaxQDA. The predominant themes were depth and understanding, (perceived) difficulty, content, examinations, and Applied modules. Examples from participants' comments are given to illustrate these points.

Depth and understanding: The majority of comments indicated that students would have liked more depth in both A levels. Most participants reported that A levels did not go into sufficient depth in core areas such as algebra and calculus, therefore providing insufficient preparation for undergraduate Mathematics. A smaller proportion of participants proposed that increased depth into those areas most useful for university study could be achieved by reducing the breadth of topics. For example:

> *Depth. It's like studying to be a pilot by only flying in a simulator.*

Related to depth, although often discussed separately, was the concept of mathematical understanding. Most participants who discussed understanding in their responses indicated that the perceived lack of depth at A level hindered their understanding of mathematical concepts. They believed that this led to A level students applying particular mathematical methods with little understanding of why these methods were necessary, or the mathematical justification for doing so.

Difficulty: A substantial number of participants commented on the overall difficulty of both A levels (comments referring to the perceived difficulty of examinations were coded separately). The majority of these participants reported that A level Mathematics should be made more difficult. However, a smaller group of participants suggested that A level Mathematics was currently appropriately difficult, recognising that it is a 'service subject' for multiple degree areas and thus increasing the difficulty may reduce its accessibility.

The idea that A level Mathematics is taken by a wide range of students was reflected in participants' specific comments about Further Mathematics. Most proposed that Further Mathematics should be made more difficult, and around half of them specifically referred to the idea that students taking Further Mathematics are likely to study mathematical subjects at degree level. They proposed that Further Mathematics should cover Pure Mathematics in more depth as preparation for undergraduate Mathematics, and that the high mathematical ability of Further Mathematics candidates would enable them to cope with a higher level of difficulty.

Content: Many participants reported perceiving a lack of Pure Mathematics content in both A levels. This was the most commonly mentioned issue. The majority of these responses focused on the perceived disconnect between A level content (especially Further Mathematics) and university Mathematics. Most participants commented that there was a lack of proof and rigorous formal argument at A level, which they felt had left them poorly prepared for university study. For example:

> *It would have been very useful to start learning the mindset of a mathematician before coming to university; I've spoken to several freshers this year who are on the Maths course and all of them have said that this was the biggest and most challenging difference to school Mathematics.*

For a minority of participants, this perceived disconnect had caused concern about their choice of university course and led to a negative experience during the transition to undergraduate Mathematics:

> *More content related to topics in university – it seems like a whole different subject in university. I sometimes regret choosing it. I was so good at Maths at [A] level, I found it so easy, it came to me so fast and I loved it. At uni it's completely different and I dread going to class. I wish I had've been better prepared, [A] level does not do this!*

Examination questions: The most commonly cited overall issue (other than Pure Mathematics and proof) was the style of examination questions in both A levels. The majority of these responses described A level questions as predictable, repetitive and formulaic. This was closely related to the issue of mathematical understanding, with a similar proportion of students suggesting that it was possible to be successful in A level examinations by regurgitating known methods, without a real understanding of the actual Mathematics. The most common suggestions for improvements to rectify this perceived problem were less structural scaffolding, the use of a wider variety of contexts, and an increase in the similarity to university examinations.

Applied units: For Statistics units, the majority of participants suggested an increase in probability content and greater depth overall, which they indicated would help students' understanding of statistical theory. For Mechanics, students' opinions were split. Around one-third of participants who commented on Mechanics reported that they felt these units were too difficult and too calculation-focused, whilst approximately the same proportion perceived there to be a disconnect between Mechanics at A level and the Mechanics studied at university. Participants' comments about Decision Mathematics units reflected the negative opinions given about this strand in other data in this study.

> *The Statistics course is bad as it focuses too much on the rote application of statistical methods, which are easy to learn from scratch when their application is required, and too little on the underlying probabilistic theory and development of Statistics. Mechanics at A level feels unrelated to Mechanics at degree level, maybe the concept of calculus in Mechanics should be introduced in A level.*

Additionally, although they were in the minority of all responses, participants who mentioned Applied units in their responses often argued that both Statistics and Mechanics content should be made compulsory at A level, to ensure common grounding in both. This suggests that the introduction of compulsory applied content in the reformed A level Mathematics will be welcomed by prospective undergraduate Mathematics students.

Additional topics: There were 691 participants' responses to an open-ended question regarding additional topics that they believed should have been included in either A level Mathematics or Further Mathematics (see Table 6).

Whilst some of these topics are already covered at A level (for example, proof by induction is examined as part of Further Pure Mathematics, and the more advanced Mechanics units introduce some vector calculus), participants may not have been able to study these units at A level due to what was available for them to study at their school at the time. The most commonly suggested area for inclusion was Pure Mathematics, particularly proof, analysis, logic and group theory.

Limitations

It was not possible to state a response rate for the questionnaire because we cannot be certain how many students were contacted. There were no guarantees that (1) the questionnaire was actually sent by the departments we contacted to their students (though many departments replied to say they agreed or declined to do this), or (2) the method used to reach students was successful.

Therefore, there was potential for self-selection in terms of the departments which agreed to pass on details of the study, and in terms of the students who decided to take part once they were contacted by their department. Various factors may have played a part in certain students deciding to take part or not, including:

- Frequency with which students receive survey requests
- Time available to complete the survey
- Personal beliefs – students who felt particularly strongly either negatively or positively may have been more likely to take part
- Encouragement or presentation of the survey in communications from students' departments.

Nonetheless, a large number of students from a large number of universities took part in the study, suggesting that the methods used were effective. However, given so many of the students who took part were required to have taken Further Mathematics to be accepted onto their course, and because this study does not include responses from

Table 6: Suggested topics for inclusion at A level

	Topic	
Pure Mathematics	Proof	Proof by induction Proof by contradiction Proof by counterexample
	Analysis	
	Logic	
	Group theory	
	Number theory	
	Set theory	
Linear algebra	Matrices	Inverse matrices 3x3 matrices Operations on matrices Eigenvalues and eigenvectors Gaussian elimination
Calculus	Differentiation	Partial differentiation Higher order differential equations Second order differential equations
	Integration	Multiple integration Integration by parts
	Limits	
Mechanics	Vectors	Cross product Vector calculus 3D Vectors Vector spaces
	Kinematics	
	Circular motion	
	Quantum mechanics	
Statistics and probability	Moment generating functions	
	Expectation and variance	
	Probability theory	
Series and sequences	Fourier series	
	Convergence	
	Summation of series	
	Taylor series	
Other	Hyperbolic functions	
	Notation	
	Financial Mathematics	

students who did not take AS or A level Further Mathematics, caution must be taken when interpreting the results.

Statistical testing was conducted to ascertain whether there were any differences between students who were and were not required to have taken Further Mathematics to be accepted onto their course. This found that those who were not required to have taken Further Mathematics were often more positive about their perceptions of its usefulness in preparing them for university than those who were required to have taken it. Additionally, 90.8% of them agreed or strongly agreed that they were taught material in their first year of university that they had learned in Further Mathematics. Only 71.7% of those who were required to have taken Further Mathematics responded in the same way. These findings suggest that universities succeed in tailoring their courses to their students' mathematical backgrounds, as well as suggesting that Further Mathematics is useful preparation for undergraduate Mathematics, regardless of university entry requirements.

Further research regarding the experiences of Mathematics students who did not take Further Mathematics would be valuable. The views of students who took alternative qualifications such as the International Baccalaureate, the Cambridge Pre-U or Cambridge International A levels would also give a useful insight into students' perceptions of their mathematical preparation. Additionally, research into the mathematical backgrounds of those students who drop out of Mathematics degrees may also give an insight into the impact of students' preparation on persistence.

Discussion and conclusion

It is not currently clear how A level reform will affect the preparation of prospective undergraduate mathematicians. A shift from modular examination throughout the two years has the potential to result in a reduction in the number of students who take Further Mathematics. Until recently, many students would study four subjects in the first year of A level study, after which they would stop studying one subject and receive an AS level qualification (Gill, 2013). Without positive feedback from examination results in the first year, students may not wish to risk taking a subject they are unsure about.

The introduction of compulsory Statistics and Mechanics content in A level Mathematics will certainly be a welcome change, and will go some way in reducing the variability in students' Applied Mathematics backgrounds. However, there will be little change to the Pure Mathematics content in A level Mathematics and Further Mathematics. Topics such as matrices and complex numbers will remain in Further Mathematics rather than being moved into A level Mathematics, and there have not been any substantial changes to the proof and formal Mathematics content, both topics which new undergraduate mathematicians traditionally struggle with.

Students who participated in this study were positive about these experiences of post-compulsory Mathematics. In particular, participants valued the additional benefits of A level Further Mathematics to A level Mathematics, with 76% agreeing that it had been good preparation for their degree. Additionally, students' views of non-compulsory units suggest that Further Pure Mathematics units are by far the most beneficial in terms of the preparation they offer, and that prospective Mathematics undergraduates would benefit from a mixed background in Mechanics and Statistics, with both strands receiving a reasonable reception. Decision Mathematics appears to have had very little benefit for future undergraduate mathematicians.

Nonetheless, many students reported shortcomings in A level Mathematics and Further Mathematics, both in terms of its assessment and its content. It appears that the difficulties which students have traditionally faced with the secondary-tertiary Mathematics transition have not changed (e.g., emphasis on proof). It can only be hoped that redevelopments of the qualifications will tackle this issue, and that more students will take Further Mathematics in order to have the opportunity to study more of the useful, advanced topics prior to going to university.

Consequently, secondary teachers and careers advisers should ensure that students receive well-informed advice about useful A levels to study in preparation for certain degree subjects. Further Mathematics should be a subject which students aim to take if they are considering pursuing the subject at university; hence it is important for schools to take advantage of initiatives such as the Further Mathematics Support Programme (2016), a government-funded project aiming to facilitate and promote the teaching of Further Mathematics in all secondary schools.

Importantly, more universities should consider either introducing Further Mathematics as a requirement for admission to their Mathematics courses or strongly recommending its study. Although universities might be reluctant to make it compulsory due to reasons relating to courses' accessibility, they should note that participants were enthusiastic about its study; most reported that they enjoyed studying it and were glad that they had done so. Prospective undergraduates should be made more aware of the views of current undergraduates regarding the usefulness of the A levels that they took at school, as well as the views of admissions tutors.

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Question selection and volatility in schools' Mathematics GCSE results

Cara Crawford Mosaic Data Science (The study was completed when the author was based in the Research Division)

Introduction

Exam-setters face a common problem: how to condense a year or more's worth of learning into a couple of hours of test-taking. In the end, they make choices, and some topics receive more coverage in examinations than others. As a result, students may do better on one version of the test than they would do on a hypothetical alternative. In other words, for students, there is always a bit of luck involved.

But what about schools? Certainly individual students have different strengths and weaknesses within a topic area. However, there is less reason to think that the choice of test questions would have a large impact on an entire school's[1] results. Schools have recently expressed concern that test scores vary considerably from year-to-year (Headmasters' and Headmistresses' Conference [HMC], 2012), and previous research has suggested that the questions selected for a test may have small influences on candidates' grades (Benton, 2013a, 2014). If schools are not large enough to be insulated from small question-related effects on their students' grades (because each student has a non-negligible effect on the school's performance), it is possible that question-level influences on students' achievement translate to increased variability in school-level outcomes.

This research estimated the extent to which volatility in schools' scores may be attributable to changes in the selection of questions on question papers by comparing candidates' performance on two halves of the same assessment. Once student grades had been calculated for each half-test, these were aggregated within each school to form school-level outcomes for each half-test (e.g., percentage of students with a grade of C or above). Comparing the variation in schools' outcomes for their students' performance on two parts of a single test should give us some idea of the amount of variation in actual year-to-year results that could be due to changes in test questions.

Data

Data was obtained from 54,167 students who took OCR's GCSE Mathematics B (J567) qualification in the June 2014 exam session. This was chosen because it had the largest entry of any OCR GCSE and also because it consisted of a large number of questions, leaving plenty of scope for looking at variations between them. The assessment was fully linear and consisted of two written question papers. Candidates could either enter for the two Foundation Tier papers (Papers 1 and 2), covering simpler material, or for the two Higher Tier papers (Papers 3 and 4), covering upper-level material. About 56 per cent (30,310 students) were entered for the Foundation Tier (Papers 1 and 2).

All four papers had a maximum possible mark of 100, and qualification grades were based on the sum of the marks achieved on the two completed question papers. This meant that the two papers had an equal impact on final grades for the qualification.

Table 1 shows the breakdown of items (part-questions) and questions across the papers for both tiers (e.g., on Paper 1, 59 item-level marks were combined into 20 question-level marks).

Table 1: Questions and items on OCR's GCSE Mathematics B (J567), June 2014

Foundation Tier	Paper 1	59 items	20 questions
	Paper 2	65 items	23 questions
Higher Tier	Paper 3	48 items	21 questions
	Paper 4	46 items	19 questions

Methods

Overview

This research compared how the same candidates performed on two halves of a single full-length assessment. First, question papers were split by tier, with all Higher Tier questions from Papers 3 and 4 in one set and all Foundation Tier questions from Papers 1 and 2 in a second set. Within each set, questions were split into two subgroups that were as similar as possible. Candidates' marks were calculated for both subgroups of questions completed, and then mapped onto the same mark scale as the complete qualification so that grade boundaries could be set for the subgroups, and subgroup marks could be converted into grades. Each subgroup of grades in one tier was then paired with a subgroup of grades in the other tier, resulting in two combined sets of half-qualification grades. Within each school, the percentage of students achieving grades A*-C and A*-A was calculated for each half-qualification, yielding two pairs of scores for each school. Finally, school-level outcomes on the two half-qualifications were compared.

1. In this article the term 'school' is used for ease of communication instead of the more generic 'centre'. The vast majority of GCSE candidates are in schools.