Cambridge Assessment

Other jurisdictions' use of technology in Mathematics curricula

Research Report

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Executive Summary

In order to assist with Cambridge Mathematics' framework development, the curriculum documents for 5 different jurisdictions (England, Victoria, Finland, Singapore, and Ontario) were analysed to identify references to the use of technology in the teaching and learning of Mathematics.

Generally, England's curriculum documents were much vaguer than those of other jurisdictions in terms of specific references to technology. Singapore and Ontario curricula gave more specific examples of technology than other jurisdictions. The technologies referred to by the curriculum documents analysed are summarised in Table 1.

Table 1 – Specific technologies referenced by jurisdiction

Technology	England			Singapore	Ontario	Victoria	Finland
	Key Stage 3	GCSE	Functional Skills				
AlgeDisc™				✓			
AlgeBar™				✓			
AlgeTools™				✓			
BBC Bitesize			✓				
Computer algebra systems					✓		
Databases					✓		
Dynamic geometry software				✓	✓		
Dynamic mathematics software							✓
Dynamic statistical software					✓		✓
E-STAT					✓		
Excel				✓			
Fathom					✓		
Geogebra		✓					
Geometer's Sketchpad				✓	✓		
Graphing calculator					✓		
Graphing software				✓	✓	✓	
Graphmatica				✓			
NRICH		✓					
Presentation software					✓		
Simulations					✓		
Spreadsheet		✓	✓	✓	✓		✓
Statistics Canada					✓		
Symbolic computation software							✓
Word processing software					✓		✓

Now that this phase of work has been complete, it may be extended to include other jurisdictions which are of interest to Cambridge Mathematics.

1. Research aims

In order to assist Cambridge Maths in the development of their framework, this project sought to answer the following research questions:

1. Which jurisdictions state explicitly in their official secondary mathematics curriculum and assessment documentation that:
    a. Summative assessment includes students' use of technology in mathematics?
    b. Technology should be used in mathematics education?
2. What guidance is given when jurisdictions do stipulate (a) or (b) above?

The research outlined in this report focused on secondary education – defined as being for students aged 12 onwards, up to the end of compulsory mathematics education, to include any school-leaving examinations (or equivalent) such as GCSEs. Only mathematics qualifications were examined.

Cambridge Maths selected a number of jurisdictions of interest after consultation of Elliott's (2016) method for identifying high performing jurisdictions. Cambridge Maths has ranked them by interest, meaning that researchers will focus on examining documents from each jurisdiction in turn. This report is based on the first phase of research of the top five jurisdictions.

The jurisdictions of interest are:

1. England
2. Victoria, Australia
3. Finland
4. Singapore
5. Ontario, Canada
6. Estonia
7. Taiwan
8. Massachusetts, USA
9. New Zealand
10. Shanghai.

Further phases of research can investigate jurisdictions 6-10.

Additionally, further phases might include exploring the learning and assessment of younger pupils (e.g. primary education), or the mathematics content in other subjects (e.g. sciences).

Throughout this report, 'technology' refers to digital tools through which learners can engage, either directly or via demonstration, with mathematical content or processes in a way that has a potential to impact on their learning. This includes computers, tablets and graphing calculators.

2. Method

Curriculum documents from each jurisdiction were found online, after other jurisdictions' equivalent levels of schooling to GCSE (and similar) were identified. All free resources and documentation relating to the curriculum and assessment specifications were downloaded and thoroughly analysed, looking for any references to technology. It was possible to read all of the documents entirely without having to resort to searching for key terms.

The Finnish curriculum was not available online in English, and so the translated version was ordered in a hard copy.

Tables in Section 3 summarise any references to technology within the curriculum/specification and are stated verbatim from the curriculum document. Where possible, the sections/topics under which they fall in the document are also given in order to provide a sense of the topic areas in Mathematics in which teaching and learning may be enhanced through the use of technology.

3. Education systems

A summary of the different education systems in the countries of interest is given in Table 2. Information has been given up to the end of compulsory schooling. It should be noted that not all countries of interest have examinations at the end of compulsory schooling, and therefore the analysis of curricula which were conducted for this section were be based only on curricula rather than qualification specifications in most instances.

Table 2 – Summary of education systems

Country	Age compulsory schooling ends	Age compulsory maths study ends	Name of school leaving qualification (or certificate)	School-leaving mathematics qualification						
				Compulsory?	No. exams	Total hours of exams	Paper names	Calculators permitted?		
								All papers	Some papers	None
England	16	16	GCSE	Yes	3	4.5	Paper 1 Paper 2 Paper 3		✓	
Singapore[*]	15	15	O-level	Yes	2	4.5 (2+2.5)	Paper 1 Paper 2	✓		
			Normal (Academic) Level	Yes	2	4 (2+2)	Paper 1 Paper 2	✓		
			Normal (Technical) Level	Yes	2	3 (1.5+1.5)	Paper 1 Paper 2	✓		
Finland	16	16[†]	Basic education certificate	Final assessment at the end of the course. Set by teachers but nationally comparable.						
Ontario, Canada	18	17[‡]	Ontario secondary school diploma	No mathematics leaving exam. Credit awarded as follows: 70% for evaluation throughout the course, 30% for a final evaluation that is not necessarily an examination.						
Victoria,	16	16	N/A	None. Compulsory education finishes at the end of Year 10 (age 16), although						

[*] Students are streamed based on their score in the Primary School Leaving Examination (PSLE). The Express Stream is for the most able students, who take O-levels after four years of secondary education. The Normal (Academic) stream, abbreviated to N(A), takes the middle ability students who sit N(A) levels after four years, and then may take O-levels after an additional year of education. The Normal (Technical) stream, N(T), takes the lowest ability students who take N(T) levels after four years of secondary education. These students may also take N(A) levels in some subjects. Students are able to move between academic and technical level study, depending on their performance.

[††] Mathematics courses are also compulsory in upper secondary education and form part of the core subjects in vocational and technical uppers secondary education courses.

[‡] Students musts take four Mathematics credits in secondary school, one of which must be taken in year 11 or 12 (age 17 or 18)

Country	Age compulsory schooling ends	Age compulsory maths study ends	Name of school leaving qualification (or certificate)	School-leaving mathematics qualification						
				Compulsory?	No. exams	Total hours of exams	Paper names	Calculators permitted?		
								All papers	Some papers	None
Australia				students must stay in some form of employment, education or training until they are 17. NAPLAN is taken in Year 9, and the VCE in Year 12 (at the end of upper secondary).						

In the following sections, mentions of the use of technology in the upper secondary Mathematics curriculum are summarised for each jurisdiction. Where possible, references to technology are given by Mathematics topic area so that the reader can see which areas of the curriculum tend to have technology associated with their teaching and learning in each jurisdiction.

Some colour coding has been used to draw the attention of the reader to the technology referenced in each curriculum document:

- **RED** – Vague mentions of technology
  Examples: technology, software
- **GREEN** – Software types and generic tools
  Examples: Spreadsheets, calculators, dynamic geometry software
- **BLUE** – Specific programs
  Geogebra, Geometer's Sketchpad

3.1 England

3.1.1 Key Stage 3

At Key Stage 3, the Department for Education (2013b) states that:

> ***Calculators*** *should not be used as a substitute for good written and mental arithmetic. In secondary schools, teachers should use their judgement about when* ***ICT*** *tools should be used.*
>
> (p. 2)

Only two mentions of technology are made in the guidance for Key Stage 3 Mathematics (see Table 3).

Table 3 – References to the use of technology in Key Stage 3 Mathematics in England

Topic	Pupils should be taught to…	Page
Number	Use a **calculator** and other **technologies** to calculate results accurately and then interpret them appropriately	6
Geometry and measures	Derive and illustrate properties of triangles, quadrilaterals, circles and other plane figures using appropriate language and **technologies**	8

Source: Department for Education (2013b)

3.1.2 GCSE

Only one reference is made to the use of technology in the Department for Education's GCSE Mathematics subject content (see Table 4).

Table 4 – References to the use of technology in GCSE Mathematics in England

Topic		Student requirement	Page
Number	Measures and accuracy	estimate answers; check calculations using approximation and estimation, including answers obtained using **technology**	5

Source: Department for Education (2013a)

Calculators are allowed in two of the three equally-weighted examinations students take in order to be awarded a GCSE.

AQA (2014) specifies that mentions of technology in the specification "implies **calculators** and, perhaps, **spreadsheets**" (p. 6).

Supporting resources and materials by OCR make some references to the use of technology in teaching and learning GCSE Mathematics content. For example, they recommend **dynamic graphing software** when teaching students about equations of straight lines (OCR, 2017), **Geogebra** and **NRICH** activities including one which has an online protractor (OCR, 2016b). None of the other GCSE awarding bodies specifically reference any technology in their free online support resources. Their paid-for services may make specific references; however, they were not available to the researcher for this project.

3.1.3 Functional Skills

Functional Skills assessments are done 'on-demand' and are available as paper-based tests or computer-based tests.

Ofqual states that Functional Skills qualifications must assess Mathematics across three interrelated process skills:

1. Representing (selecting the Mathematics and information to model a situation)
2. Analysing (processing and using Mathematics)
3. Interpreting (interpreting and communicating the results of the analysis)

As part of 'representing', learners must "decide on methods, operations and tools, including **information and communication technology** (ICT), to use in a situation" (p. 2). Only two specific references to ICT are mentioned in the curriculum (see Table 5).

Table 5 – References to the use of technology in Functional Skills Mathematics in England

Level	Skill standards	Coverage and range	Page
2	Representing Analysing Interpreting	Collect and represent discrete and continuous data, using **ICT** where appropriate	7
2	Representing Analysing Interpreting	Use and interpret statistical measures, tables and diagrams, for discrete and continuous data, using **ICT** where appropriate	7-8

Source: Ofqual (2011)

Edexcel supply some teacher's notes for certain topics in their Functional Skills qualifications. Within those there are a handful of specific references to technology, suggesting the use of **spreadsheets** for some tasks (Edexcel, 2008a, 2008b, 2008c). OCR's teacher support materials recommend **BBC Bitesize** (OCR, 2012) and **spreadsheet** packages (OCR, 2009, 2010, 2016a).

Calculators are allowed in Functional Skills examinations.

3.2 Ontario, Canada

Ontario's Ministry of Education (2007) acknowledges the changing technological landscape in today's world and the consequences this will have for students' learning and lives.

> *The unprecedented changes that are taking place in today's world will profoundly affect the future of today's students to meet the demands of the world in which they will live, students will need to adapt to changing conditions and to learn independently. They will require the ability to use technology effectively and the skills for processing large amounts of quantitative information.*
>
> (p. 4)

> *Operations that were an essential part of a procedures-focused curriculum for decades can now be accomplished quickly and effectively using technology, so that students can now solve problems that were previously too time-consuming to attempt, and can focus on underlying concepts.*
>
> (p. 5)

They describe, generally, situations which might call for the use of technology. For example:

> *Students can use **calculators** and **computers** to perform operations, make graphs, manipulate algebraic expressions, and organize and display data that are lengthier or more complex than those addressed in curriculum expectations suited to a paper-and-pencil approach. Students can also use **calculators** and **computers** in various ways to investigate number and graphing patterns, geometric relationships, and different representations; to simulate situations; and to extend problem solving.*
>
> (p. 19)

> *Students' use of the tools should not be laborious or restricted to inputting and learning algorithmic steps. For example, when using **spreadsheets** and **statistical software** (e.g. **Fathom**), teachers could supply students with prepared data sets, and when using **dynamic geometry software** (e.g. **The Geometer's Sketchpad**), they could use pre-made sketches to that students' work with the software would be focused on manipulation of the data or the sketch, not on the inputting of data or the designing of the sketch.*
>
> (p. 20)

> *Students, working individually or in groups, can use **Internet websites** to gain access to **Statistics Canada**, mathematics organizations, and other valuable sources of mathematical information around the world.*
>
> (p. 20)

> *Useful **ICT** tools include **simulations**, multimedia resources, **databases**, sites that give access to large amounts of statistical data, and computer-assisted learning modules. Applications such as databases, **spreadsheets**, **dynamic geometry software**, **dynamic statistical software**, **computer algebra systems** (CAS), **word-processing software**, and **presentation software** can be used to support various methods of inquiry in mathematics.*
>
> (p. 37)

Analysis was conducted of Ontario's Grade 9/10 and Grade 11 Mathematics curricula. Table 6 outlines areas in the curriculum which refer to use of technology in Mathematics in Grade 9/10, and Table 7 outlines references to technology in Grade 11. Headings are named and organised to reflect the layout of Ontario's curriculum document.

Table 6 – References to the use of technology in Grade 9/10 Mathematics in Ontario, Canada

Grade	Course	Route	Section		Skill	Page
9	Principles of Mathematics	Academic	Number sense and algebra	Manipulating expressions solving equations	Simplify numerical expressions involving integers and rational numbers, with and without the use of **technology**	30
9	Foundations of Mathematics	Applied				39
9	Principles of Mathematics	Academic	Linear relations	Using data management to investigate relationships	Design and carry out an investigation or experiment involving relationships between two variables, including the collection and organisation of data, using appropriate methods, equipment, and/or **technology** (e.g. surveying; using measuring tools, scientific probes, the **Internet**) and techniques (e.g. making tables, drawing graphs)	32
9	Foundations of Mathematics	Applied				41
9	Principles of Mathematics	Academic	Analytic geometry	Investigating properties of slope	Identify, through investigation with **technology**, the geometric significance of m and b in the equation $= mx + b$	34
9	Principles of Mathematics	Academic	Analytic geometry	Investigating properties of slope	Identify, through investigation, properties of the slopes of lines and line segments (e.g. direction, positive or negative rate of change, steepness, parallelism, perpendicularity), using **graphing technology** to facilitate investigations, where appropriate	34
10	Foundations of Mathematics	Applied	Modelling linear relations	Manipulating and solving algebraic equations	Identify, through investigation, properties of the slopes of lines and line segments (e.g. direction, positive or negative rate of change, steepness, parallelism), using **graphing technology** to facilitate investigations, where appropriate	56
10	Foundations of Mathematics	Applied	Modelling linear relations	Solving and interpreting systems of linear equations	determine graphically the point of intersection of two linear relations (e.g. using graph paper, using **technology**)	56
10	Principles of Mathematics	Academic	Quadratic relations of the form $= ax^2 +$	Identifying characteristics of quadratic relations	Collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and **technology** (e.g. concrete materials, scientific probes, **graphing calculators**), or from secondary	47
10	Foundations of	Applied				58

Grade	Course	Route	Section		Skill	Page
	Mathematics		$bx + c$		sources (e.g. the **Internet**, **Statistics Canada**); graph the data and draw a curve of best fit, if appropriate, with or without the use of **technology**	
10	Principles of Mathematics	Academic	Quadratic relations of the form $= ax^2 + bx + c$	Identifying characteristics of quadratic relations	Determine, through investigation using **technology**, that a quadratic relation of the form $y = ax^2 + bx + c$ $(a \neq 0)$ can be graphically represented as a parabola, and determine that the table of values yields a constant second difference	47
10	Foundations of Mathematics	Applied				58
10	Foundations of Mathematics	Applied	Quadratic relations of the form $= ax^2 + bx + c$	Identifying characteristics of quadratic relations	Identify the key features of a graph of a parabola (i.e. the equation of the axis of symmetry, the coordinates of the vertex, the y-intercept, the zeroes, and the maximum or minimum value), using a given graph or a graph generated with **technology** from its equation, and use the appropriate terminology to describe the features	58
10	Foundations of Mathematics	Applied	Quadratic relations of the form $= ax^2 + bx + c$	Identifying characteristics of quadratic relations	Compare, through investigation using **technology**, the graphical representations of a quadratic relation in the form $y = x^2 + bc + c$ and the same relation in the factored form $= (x - r)(x - s)$ (i.e. the graphs are the same), and describe the connections between each algebraic representation and the graph	58-59
10	Principles of Mathematics	Academic	Quadratic relations of the form $= ax^2 + bx + c$	Investigating the basic properties of quadratic relations	Compare, through investigation using **technology**, the features of the graph of $y = x^2$ and the graph of $y = 2^x$, and determine the meaning of a negative exponent and of zero as an exponent	47
10	Principles of Mathematics	Academic	Quadratic relations of the form $= ax^2 + bx + c$	Relating the graph of $= x^2$ and its transformations	Identify, through investigation using **technology**, the effect on the graph of $y = x^2$ of transformations (i.e. translations, reflections in the x-axis, vertical stretches or compressions) by considering separately each parameter a, h and k	47
10	Principles of Mathematics	Academic	Quadratic relations of the form	Solving quadratic equations	Interpret real and non-real roots of quadratic equations, through investigations using **graphing technology**, and relate the roots to the x-intercepts of the corresponding	48

Grade	Course	Route	Section		Skill	Page
			$= ax^2 + bx + c$		relations	
10	Principles of Mathematics	Academic	Quadratic relations of the form $= ax^2 + bx + c$	Solving problems involving quadratic relations	Solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of **technology**	48
10	Foundations of Mathematics	Applied	Quadratic relations of the form $= ax^2 + bx + c$	Solving problems by interpreting graphs of quadratic relations	Solve problems involving a quadratic relation by interpreting a given graph or graph generated with **technology** from its equation	59

Source: Ontario Ministry of Education (2005)

Table 7 – References to the use of technology in Grade 11 Mathematics in Ontario, Canada

Course	Section		Skill	Page
Functions *University Preparation*	Characteristics of functions	Representing functions	determine the numeric or graphical representation of the inverse of a linear or quadratic function, given the numeric, graphical or algebraic representation of the function, and make connections, through investigation using a variety of tools (e.g. **graphing technology**, Mira, tracing paper), between the graph of a function and the graph of its inverse	45
Functions *University Preparation*	Characteristics of functions	Representing functions	determine, using function notation when appropriate, the algebraic representation of the inverse of a linear or quadratic function, given the algebraic representation of the function, and make connections, through investigation using a variety of tools (e.g. **graphing technology**, Mira, tracing paper), between the algebraic representations of a function and its inverse	46
Functions *University Preparation*	Characteristics of functions	Representing functions	determine, through investigation using **technology**, and describe the roles of the parameters a, k, d and c in functions of the form $y = a\,f\big(k(x-d)\big) + c$ in terms of the transformations on the graphs of $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$ and $f(x) = \frac{1}{x}$	46
Functions *University Preparation*	Characteristics of functions	Determining equivalent algebraic expressions	verify, through investigation with and without **technology**, that $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, $a > 0$, $b > 0$, and use this relationship to simplify radicals and radical expressions obtained by adding, subtracting, and multiplying	47
Functions *University Preparation*	Exponential functions	Representing exponential functions	graph, with and without **technology**, an exponential relation, given its equation in the form $y = a^x$ $(a > 0,$ $a \neq 1)$, define this relation as the function $f(x) = a^x$, and explain why it is a function	48
Functions *University*	Exponential functions	Connecting graphs and equations of	determine, through investigation using **technology**, and describe the roles of the parameters a, k, d and c in	49

Course	Section		Skill	Page
Preparation		exponential functions	functions of the form $y = a\,f\big(k(x-d)\big)+c$ in terms of the transformations on the graphs of $f(x)=a^x$ ($a>0$, $a\neq 1$) (i.e. translations; reflections in the axes; vertical and horizontal stretches and compressions)	
Functions *University Preparation*	Exponential functions	Connecting graphs and equations of exponential functions	determine, through investigation using **technology**, that the equation of a given exponential function can be expressed using different bases, and explain the connections between the equivalent forms in a variety of ways (e.g. comparing graphs; using transformations; using the exponent laws)	49
Functions *University Preparation*	Exponential functions	Solving problems involving exponential functions	collect data that can be modelled as an exponential function, through investigation with and without **technology**, from primary sources, using a variety of tools (e.g. concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g. websites such as **Statistics Canada**, **E-STAT**), and graph the data	49
Functions *University Preparation*	Discrete functions	Solving problems involving financial applications	make and describe connections between simple interest, arithmetic sequences, and linear growth, through investigation with **technology** (e.g. use a **spreadsheet** or **graphing calculator** to make simple interest calculations, determine first differences in the amounts over time, and graph amount versus time)	51
Functions *University Preparation*	Discrete functions	Solving problems involving financial applications	make and describe connections between compound interest, geometric sequences, and exponential growth, through investigation with **technology** (e.g. use a **spreadsheet** to make compound interest calculations, determine finite differences in the amounts over time, and graph amount versus time)	51
Functions *University Preparation*	Discrete functions	Solving problems involving financial applications	determine, through investigation using **technology** (e.g. **scientific calculator**; the TVM solver in a **graphing calculator**; **online tools**), and describe strategies for calculating the number of compounding periods, n,	51

Course	Section		Skill	Page
			using the compound interest formula in the form $A = P(1 + i)^n$, and solve related problems	
Functions *University Preparation*	Discrete functions	Solving problems involving financial applications	explain the meaning of the term *annuity*, and determine the relationships between ordinary annuities, geometric series, and exponential growth, through investigation with **technology** in situations where the compounding period and the payment period are the same (e.g. use a **spreadsheet** to determine and graph the future value of an ordinary annuity for varying numbers of compounding periods; investigate how the contributions of each payment to the future value of an ordinary annuity are related to the terms of a geometric series)	52
Functions *University Preparation*	Discrete functions	Solving problems involving financial applications	determine, through investigation using **technology** (e.g. the TVM Solver in a **graphing calculator**; **online tools**) the effects of changing the conditions (i.e. the payments, the frequency of the payments, the interest rate, the compounding period) of ordinary simple annuities (e.g. long-term savings plans, loans)	52
Functions *University Preparation*	Discrete functions	Solving problems involving financial applications	solve problems, using **technology** (e.g. **scientific calculator**, **spreadsheet**, **graphing calculator**), that involve the amount, the present value, and the regular payment of an ordinary simple annuity (e.g. calculate the total interest paid over the life of a loan, using a **spreadsheet**, and compare the total interest with the original principal of a loan)	52
Functions *University Preparation*	Trigonometric functions	Connecting graphs and equations of sinusoidal functions	make connections between the sine ratio and the sine function and between the cosine ratio and the cosine function by graphing the relationship between angles from $0°$ to $360°$ and the corresponding sine ratios or cosine ratios, with or without **technology** (e.g. by generating a table of values using a **calculator**; by unwrapping the unit circle), defining this relationship as the function $f(x) = \sin x$ or $f(x) = \cos x$, and explaining	54

Course	Section		Skill	Page
			why the relationship is a function	
Functions *University Preparation*	Trigonometric functions	Connecting graphs and equations of sinusoidal functions	determine, through investigation using **technology**, and describe the roles of the parameters a, k, d, and c in functions of the form $y = a\, f\big(k(x-d)\big) + c$ in terms of transformations on the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ with angles expressed in degrees, and describe these roles in terms of transformations on the graphs of $f(x) = \sin x$ and $f(x) = \cos x$ (i.e. translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes)	54
Functions *University Preparation*	Trigonometric functions	Solving problems involving sinusoidal functions	collect data that can be modelled as a sinusoidal function (e.g. voltage in an AC circuit, sound waves) through investigation with and without **technology**, from primary sources, using a variety of tools (e.g. concrete materials; measurement tools such as motion sensors), or from secondary sources (e.g. websites such as **Statistics Canada**, **E-STAT**), and graph the data	54
Mathematics for Collect Technology *College Preparation*	Trigonometric functions	Solving problems involving sinusoidal functions	pose problems based on applications involving a sinusoidal function, and solve these and other such problems by using a given graph or a graph generated with **technology**, in degree mode, from a table of values or from its equation	131
Functions and Applications *University/College Preparation*	Quadratic Functions	Solving quadratic equations	explore the algebraic development of the quadratic formula (e.g. given the algebraic development, connect the steps to a numerical example; follow a demonstration of the algebraic development, with **technology**, such as **computer algebra systems**, or without **technology**), and apply the formula to solve quadratic equations, using **technology**	59
Functions and Applications *University/College*	Quadratic Functions	Connecting graphs and equations of quadratic functions	determine, through investigation using **technology**, and describe the roles of a, h, and k in quadratic functions	60

Course	Section		Skill	Page
Preparation			of the form $f(x) = a(x-h)^2 + k$ in terms of transformations on the graph of $f(x) = x^2$ (i.e. translations; reflections in the x-axis; vertical stretches and compressions to and from the x-axis)	
Foundations for College Mathematics *College Preparation*	Mathematical Models	Connecting graphs and equations of quadratic relations		69
Functions and Applications *University/College Preparation*	Quadratic Functions	Connecting graphs and equations of quadratic functions	express the equation of a quadratic function in the standard form $f(x) = ax^2 + bx + c$, given the vertex form $f(x) = a(x-h)^2 + k$, and verify, using **graphing technology**, that these forms are equivalent representations	60
Foundations for College Mathematics *College Preparation*	Mathematical Models	Connecting graphs and equations of quadratic relations		69
Foundations and Applications *University/College Preparation*	Quadratic Functions	Connecting graphs and equations of quadratic functions	express the equation of a quadratic function in the vertex form $f(x) = a(x-h)^2 + k$, given the standard form $f(x) = ax^2 + bx + c$, by completing the square, including cases where $\frac{b}{a}$ is a simple rational number (e.g. $\frac{1}{2}$, 0.75), and verify, using **graphing technology**, that these forms are equivalent representations	60
Foundations and Applications *University/College Preparation*	Quadratic Functions	Solving problems involving quadratic functions	collect data that can be modelled as a quadratic function, through investigation with and without **technology**, from primary sources, using a variety of tools (e.g. concrete materials; measurement tools such as measuring tapes, electronic probes, motion sensors), or from secondary sources (e.g. websites such as **Statistics Canada**, **E-STAT**), and graph the data	61
Foundations and Applications *University/College Preparation*	Quadratic Functions	Solving problems involving quadratic functions	determine, through investigation using a variety of strategies (e.g. applying properties of quadratic functions such as the x-intercepts and the vertex; using transformations), the equation of the quadratic function that best models a suitable data set graphed on a scatter plot, and compare this equation to the equation	61

Course	Section		Skill	Page
			of a curve of best fit generated with **technology** (e.g. **graphing software**, **graphing calculator**)	
Foundations and Applications *University/College Preparation*	Exponential functions	Connecting graphs and equations of exponential functions	determine, through investigation using a variety of tools (e.g. **calculator**, paper and pencil, **graphing technology**) and strategies (e.g. patterning, finding values from a graph, interpreting the exponent laws), the value of a power with a rational exponent	62
Foundations and Applications *University/College Preparation*	Exponential functions	Connecting graphs and equations of exponential functions	determine, through investigation using **technology**, the roles of the parameters a, k, d, and c in functions of the form $y = af(k(x - d)) + c$ and describe these roles in terms of transformations on the graph of $f(x) = a^x$ ($a > 0$, $a \neq 1$) (i.e. translations, reflections in the axes, vertical and horizontal stretches and compressions to and from the x- and y-axes)	66
Foundations and Applications *University/College Preparation*	Exponential functions	Connecting graphs and equations of exponential functions	evaluate, with and without **technology**, numerical expressions containing integer and rational exponents and rational bases	62
Foundations and Applications *University/College Preparation*	Exponential functions	Solving financial problems involving exponential functions	explain the meaning of the term annuity, through investigation of numerical and graphical representations using **technology**	63
Foundations and Applications *University/College Preparation*	Trigonometric functions	Applying the sine law and cosine law in acute triangles	verify, through investigation using **technology** (e.g. **dynamic geometry software**, **spreadsheet**), the sine law and the cosine law (e.g. compare, using dynamic geometry software, the ratios $\frac{a}{\sin A}$, $\frac{b}{\sin B}$ and $\frac{c}{\sin C}$ in triangle ABC while dragging one of the vertices)	65
Foundations and Applications *University/College Preparation*	Trigonometric functions	Connecting graphs and equations of sine functions	make connections, through investigation with **technology**, between changes in a real-world situation that can be modelled using a periodic function and transformations of the corresponding graph	66
Foundations and	Trigonometric functions	Connecting graphs	determine, through investigation using **technology**, and	66

Course	Section		Skill	Page
Applications *University/College Preparation*		and equations of sine functions	describe the roles of the parameters a, c and d in functions in the form $f(x) = a\sin x$, $f(x) = \sin x + c$, and $f(x) = \sin(x - d)$ in terms of transformations on the graph of $f(x) = \sin x$ with angles expressed in degrees (i.e. translations; reflections in the x-axis; vertical stretches and compressions to and from the x-axis)	
Foundations and Applications *University/College Preparation*	Trigonometric functions	Solving problems involving sine functions	collect data that can be modelled as a sine function (e.g. voltage in an AC circuit, sound waves), through investigation with and without **technology**, from primary sources, using a variety of tools (e.g. concrete materials; measurement tools such as motion sensors), or from secondary sources (e.g. websites such as **Statistics Canada** and **E-STAT**) and graph the data	66
Foundations for College Mathematics *College Preparation*	Mathematical models	connecting graphs and equations of quadratic relations	determine, through investigation using a variety of tools and strategies (e.g. graphing with **technology**; looking for patterns in tables of values), and describe the meaning of negative exponents and of zero as an exponent	70
Foundations for College Mathematics *College Preparation*	Mathematical models	Solving problems involving exponential relations	collect data that can be modelled as an exponential relation, through investigation with and without **technology**, from primary sources, using a variety of tools (e.g. concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g. websites such as **Statistics Canada**, **E-STAT**), and graph the data	70
Foundations for College Mathematics *College Preparation*	Mathematical models	Solving problems involving exponential relations	describe some characteristics of exponential relations arising from real-world applications (e.g. bacterial growth, drug absorption) by using tables of values (e.g. to show a constant ratio, or multiplicative growth or decay) and graphs (e.g. to show, with **technology**, that there is no maximum or minimum value)	70
Foundations for College	Mathematical models	Solving problems involving	pose problems involving exponential relations arising from a variety of real-world applications (e.g. population	70

Course	Section		Skill	Page
Mathematics *College Preparation*		exponential relations	growth, radioactive decay, compound interest), and solve these and other such problems by using a given graph or a graph generated with **technology** from a given table of values or a given equation	
Functions and Applications *University/College Preparation*	Mathematical models	Connecting graphs and equations of quadratic relations	determine, through investigation using **technology**, the roles of a, h, and k in quadratic relations of the form $= a(x-h)^2 + k$, and describe these roles in terms of transformations on the graph of $= x^2$ (i.e. translations; reflections in the x-axis; vertical stretches and compressions to and from the x-axis)	60
Foundations for College Mathematics *College Preparation*	Personal finance	Solving problems involving compound interest	determine, through investigation using **technology**, the compound interest for a given investment, using repeated calculations of simple interest, and compare, using a table of values and graphs, the simple and compound interest earned for a given principal (i.e. investment) and a fixed interest rate over time	71
Foundations for College Mathematics *College Preparation*	Personal finance	Solving problems involving compound interest	determine, through investigation using **technology** (e.g. a TVM Solver in a **graphing calculator** or on a **website**), the effect on the future value of a compound interest investment or loan of changing the total length of time, the interest rate or the compounding period	71
Foundations for College Mathematics *College Preparation*	Personal finance	Comparing financial services	gather, interpret, and compare information about current credit card interest rates and regulations, and determine, through investigation using **technology**, the effects of delayed payments on a credit card balance	72
Foundations for College Mathematics *College Preparation*	Transportation and travel	Owning and operating a vehicle	solve problems, using **technology** (e.g. **calculator**, **spreadsheet**), that involve the fixed costs (e.g. licence fee, insurance) and variable costs (e.g. maintenance, fuel) of owning and operating a vehicle	72 83
Foundations for College Mathematics *College Preparation*	Data management	Working with one-variable data	identify different types of one-variable data (i.e. categorical, discrete, continuous) and represent the data, with and without **technology** in appropriate graphical forms (e.g. histograms, bar graphs, circle	74

Course	Section		Skill	Page
			graphs, pictographs)	
Foundations for College Mathematics *College Preparation*	Data management	Working with one-variable data	calculate, using formulas and/or **technology** (e.g. **dynamic statistical software**, **spreadsheet**, **graphing calculator**), and interpret measures of central tendency (i.e. mean, median, mode) and measures of spread (i.e. range, standard deviation)	74
Foundations for College Mathematics *College Preparation*	Data management	Applying probability	determine, through investigation using class-generated data and **technology**-based simulation models (e.g. using a random number generator on a **spreadsheet** or on a **graphing calculator**), the tendency of experimental probability to approach theoretical probability as the number of trials in an experiment increases	75
Mathematics for Work and Everyday Life *Workplace Preparation*	Earning and purchasing	Earning	solve problems, using **technology** (e.g. **calculator**, **spreadsheet**), and make decisions involving different remuneration methods and schedules	79
Mathematics for Work and Everyday Life *Workplace Preparation*	Earning and purchasing	Purchasing	calculate discounts, sale prices, and after-tax costs, using **technology**	80
Mathematics for Work and Everyday Life *Workplace Preparation*	Saving, investing, and borrowing	Saving and investing	determine, through investigation using **technology** (e.g. **calculator**, **spreadsheet**), the effect on simple interest of changes in the principal, interest rate, or time, and solve problems involving applications of simple interest	81
Mathematics for Work and Everyday Life *Workplace Preparation*	Saving, investing, and borrowing	Saving and investing	determine, through investigation using **technology**, the compound interest for a given investment, using repeated calculations of simple interest for no more than 6 compounding periods	81

Course	Section		Skill	Page
Mathematics for Work and Everyday Life *Workplace Preparation*	Saving, investing, and borrowing	Saving and investing	determine, through investigation using **technology** (e.g. a TVM Solver in a **graphing calculator** or on a **website**), the effect on the future value of a compound interest investment of changing the total length of time, the interest rate or the compounding period	82
Mathematics for Work and Everyday Life *Workplace Preparation*	Saving, investing, and borrowing	Saving and investing	solve problems, using **technology**, that involve applications of compound interest to saving and investing	82
Mathematics for Work and Everyday Life *Workplace Preparation*	Saving, investing, and borrowing	Borrowing	calculate, using **technology** (e.g. **calculator**, **spreadsheet**), the total interest paid over the life of a personal loan, given the principal, the length of the loan, and the periodic payments, and use the calculations to justify the choice of a personal loan	82

Source: Ontario Ministry of Education (2007)

3.3 Singapore

One of the 'teaching principles' for O-level Mathematics is that "teaching should connect learning to the real world, harness ICT tools and emphasise 21st century competencies" (Ministry of Education Singapore, 2012b, p. 32). There are extensive references to how teachers can support students' learning in Mathematics through the use of ICT, much more so than in the documents analysed from other jurisdictions. For example:

> *The curriculum must engage the 21st century learners, who are digital natives comfortable with the use of technologies and who work and think differently. The learning of mathematics must take into cognisance the new generation of learners, the innovations in pedagogies as well as the affordances of* ***technologies****.*
>
> (p. 2)

> *To develop a deep understanding of mathematical concepts, and to make sense of various mathematical ideas as well as their connections and applications, students should be exposed to a variety of learning experiences including hands-on activities, and use of* ***technological aids*** *to help them relate abstract mathematical concepts with concrete experiences.*
>
> (p. 15)

> *Teachers should consider affordances of* ***ICT*** *to help students learn.* ***ICT*** *tools can help students understand mathematical concepts through visualisations, simulations and representations. They can also support exploration and experimentation and extend the range of problems accessible to students.*
>
> (p. 22)

> *Students should be given opportunities to work in groups and use* ***ICT*** *tools for modelling tasks.* ***ICT*** *tools empower students to work on problems which would otherwise require more advanced mathematics or computations that are too tedious and repetitive.*
>
> (p. 31)

Analysis was conducted of Singapore's Ministry of Education Mathematics syllabus documents for all of the Normal courses (Academic and Technical) as well as Additional Mathematics for the sake of completeness.

Unlike many other curricula or specifications, the Singapore O-level specification details many specific possible uses of technology, including naming specific programs and software in the relevant contexts. Table 8 outlines areas in the syllabus which refer to use of technology in Mathematics.

Table 8 – References to the use of technology in secondary Mathematics in Singapore

Secondary level	Learning experience	Statement	Page reference		
			Academic	Technical	Additional
1	Algebraic expressions and formulae	Use **spreadsheets**, e.g. **Microsoft Excel**, to • explore the concept of variables and evaluate algebraic expressions • compare and examine the differences between pairs of expressions	35	42	
1	Algebraic expressions and formulae	Use algebra discs or the **AlgeDisc™** application in **AlgeTools™** to make sense of and interpret linear expressions with integral coefficients	35	42	
1	Algebraic expressions and formulae	Use the **AlgeDisc™** application in **AlgeTools™** to construct and simplify linear expressions with integral coefficiences.	57		
1, 2	Algebraic expressions and formulae	Use the **AlgeBar™** application in **AlgeTools™** to formulate linear expressions (with integral coefficients) with pictorial representations.		42	
1	Angles, triangles and polygons	Use **GSP** or other **dynamic geometry software** to explore a given type of quadrilateral (e.g. parallelogram) to discover its properties	38		
1	Angles, triangles and polygons	Use **GSP** or other **dynamic geometry software** to construct and study the properties of the perpendicular bisector of a line segment and the bisector of an angle	38		
1	Angles, triangles and quadrilaterals	Use **GSP** or other **dynamic geometry software** to discover the relationships of angles formed by two parallel lines and a transversal.		43	
1	Data analysis	Work collaboratively on a task to present data using an appropriate statistical representation (including the use of **software**)	59		
1	Data analysis	Carry out a statistical project which involves data collection, representation and interpretation, involving the use of a **spreadsheet** such as **Microsoft Excel** to tabulate and represent data		44	

Secondary level	Learning experience	Statement	Page reference		
			Academic	Technical	Additional
1	Equations and inequalities	Use the virtual balance in **AlgeTools™** to explore the concepts of equation, and to construct, simplify and solve linear equations with integral coefficients.	57		
1	Equations and inequalities	Use the **AlgeBar™** application (for whole numbers) in **AlgeTools™** to formulate linear equations to solve problems (Students can draw models to help them formulate the equations.)	57		
1	Functions and graphs	Use a **spreadsheet** or **graphing software** to study how the graph of $y = ax + b$ changes when either a or b varies	36		
1	Mensuration	Use **GSP** or other **dynamic geometry software** to explore the properties of triangles, parallelograms, trapeziums and circles.		43	
1	Number and algebra	Use the **AlgeDisc™** application in **AlgeTools™** to construct and simplify linear expressions with integral coefficients.	36		
1	Numbers and their operations	Use algebra discs or the **AlgeDisc™** application in **AlgeTools™** to make sense of addition, subtraction and multiplication involving negative integers and develop proficiency in the 4 operations of integers.	34	41	
1	Percentage	Use the **AlgeBar™** application in **AlgeTools™** to formulate linear equations to solve problems. (Students can draw models to help them formulate equations.)	35		
1	Ratio and proportion	Use the **AlgeBar™** application in **AlgeTools™** to formulate linear equations to solve problems. (Students can draw models to help them formulate equations.)	34		
1	Ratio and proportion	Use the **AlgeBar™** application in **AlgeTools™** to express the ratio of 2 or 3 quantities in pictorial form.		41	
1	Symmetry	Explore and create symmetric figures and patterns, including with the use of **ICT**.		43	
2	Algebraic expressions and formulae	Use the **AlgeDisc™** application in **AlgeTools™**, to factorise a quadratic expression of the form $ax^2 + bx + c$ into two linear factors where a, b and c are integers	60		
2	Algebraic expressions and	Use the **AlgeDisc™** application in **AlgeTools™** to construct linear expressions with integral coefficients, and simplify the expressions		45	

Secondary level	Learning experience	Statement	Page reference		
			Academic	Technical	Additional
	formulae	by collecting like terms and removing brackets.			
2	Angles, triangles and polygons	Use **GSP** or other **dynamic geometry software** to explore a given type of quadrilateral (e.g. parallelogram) to discover its properties	38		
2	Angles, triangles and polygons	Use **GSP** or other **dynamic geometry software** to construct and study the properties of the perpendicular bisector of a line segment and the bisector of an angle	38		
2	Angles, triangles and quadrilaterals	Construct triangles given specific measurements of angles and sides (e.g. 2 sides and 1 angle) using a variety of tools including **ICT**.		46	
2	Angles, triangles and quadrilaterals	Use **GSP** or other **dynamic software** to construct and study the properties of the perpendicular bisector of a line segment and the bisector of an angle.		46	
2	Congruence and similarity	Use **GSP** or other **dynamic software** to draw, make measurements (of lengths, angles and areas) and explore the effects of translation, rotation, reflection and enlargement on the shape and size of a figure.		46	
2	Data analysis	Use a **spreadsheet** such as **Microsoft Excel** to show how the mean, mode and median are affected by changing data values.		47	
2	Equations and inequalities	Use the **AlgeBar™** application (for whole numbers) in **AlgeTools™** to formulate linear equations to solve problems (Students can draw models to help them formulate the equations.)	57		
2	Equations and inequalities	Use **Graphmatica**, **applets** or other **software** to draw the graph of $ax + by = c$ (a straight line), check that the coordinates of a point on the straight line satisfy the equation, and explain why the solution of a pair of simultaneous linear equations is the point of intersection of two straight lines.	61		
2	Functions and graphs	Use a **spreadsheet** or **graphing software** to study how the graph of $y = ax^2 + bx + c$ changes when either a, b or c varies	41		
2	Functions and graphs	Use a **spreadsheet** such as **Microsoft Excel** to produce a table of input and output for a given function describing the relationship in a real-life context, e.g. phone bill = basic subscription charge +		45	

Secondary level	Learning experience	Statement	Page reference		
			Academic	Technical	Additional
		utilisation charge			
2	Functions and graphs	Use a **spreadsheet** or **graphing software** to study how the graph of $y = ax + b$ changes when either a or b varies		45	
2	Rate and speed	Use a **spreadsheet** such as **Microsoft Excel** to compare the effects of simple interest and compound interest.		45	
2	Probability	Compare and discuss the experimental and theoretical values of probability using **computer simulations**.	43		
2	Pythagoras' theorem	Use drawings or **GSP** (or **dynamic geometry software**) to explore the validity/invalidity of the theorem on different triangles and hence its use in showing if a triangle is right-angled.		47	
3 and 4	Coordinate geometry	Use **GSP** or other **dynamic geometry software** to explore and describe the gradients of straight lines, including the gradient of a vertical line as undefined, and to investigate how the signs of $_2 - \ _1$ and $x_2 - x_1$ affect the sign of the gradient of a striaght line.	68		
3 and 4	Coordinate geometry	Use **GSP** or other **dynamic geometry software** to explore and describe the gradients of straight lines, including the gradient of a vertical line as undefined, and to investigate how the signs of $_2 - \ _1$ and $x_2 - x_1$ affect the sign of the gradient of a straight line.	48		
3 and 4	Data analysis	Use a **spreadsheet** such as **Microsoft Excel** to construct a cumulative frequency diagram, and use it to estimate quartiles and percentages.		51	
3 and 4	Equations and inequalities	Use a **graphing software** to investigate how the positions of the graph $y = ax^2 + bx + c$ vary within the sign of $b^2 - 4ac$, and describe the graph when $b^2 - 4ac < 0$.			36
3 and 4	Equations and inequalities	Use **graphing software** to investigate the relationship between the number of points of intersection and the nature of solutions of a pair of simultaneous equations, one linear and one quadratic.			36, 52
3 and 4	Functions and graphs	Use **Graphmatica** or other **graphing software** to explore the characteristics of various functions	44		

Secondary level	Learning experience	Statement	Page reference		
			Academic	Technical	Additional
3 and 4	Functions and graphs	Use **Graphmatica** or other **graphing software** to explore the characteristics of various functions	65		
3 and 4	Functions and graphs	Use a **spreadsheet** or **graphing software** to study how the graph of $y = ax^2 + bx + c$ changes when either a, b or c varies		50	
3 and 4	Functions and graphs	Use **ICT** (e.g. **Graphmatica**) to draw the graph of $ax + by = c$ (a straight line) and check that the coordinates of a point on the straight line satisfies the equation, and draw and describe the lines $x = a$ and $y = b$, and their gradients.		50	
3 and 4	Geometry and trigonometry	Use **graphing software** to investigate the graph of $^2 = kx$ when k varies.			42, 57
3 and 4	Properties of circles	Use **GSP** or other **dynamic geometry software** to explore the properties of circles, and use geometrical terms correctly for effective communication	47		
3 and 4	Polynomials and partial fractions	Use **graphing software** to investigate the graph of a cubic polynomial and discuss i. the linear factors of the polynomial and the number of real roots; and ii. the number of real roots of the related cubic equation, with reference to the points of intersection with the x-axis.			37, 53-54
3 and 4	Power, exponential, logarithmic and modulus functions	Use **graphing software** to explore the characteristics of various functions.			38
3 and 4	Power, exponential, logarithmic and modulus functions	Use **graphing software** to display real-world data graphically and match it with an appropriate function.			38
3 and 4	Solutions of equations	Use **ICT** (e.g. **Graphmatica**) to draw the graphs of a pair of simultaneous linear equations, and explain why the solution is the point of intersection of the two straight lines.		50	

Secondary level	Learning experience	Statement	Page reference		
			Academic	Technical	Additional
3 and 4	Trigonometric functions, identities and equations	Use a **graphing software** to display the graphs of trigonometric functions and discuss their behaviours, and investigate how a graph (e.g. $y = a\sin bx + c$) changes when a, b or c varies.			40, 55

Sources: Ministry of Education Singapore (2012a, 2012b, 2012c)

3.4 Finland

Analysis was conducted of Finland's national curriculum for upper secondary students. The curriculum states that Finnish education generally encourages students to use ICT diversely, and gives some examples of instances when Mathematics study would benefit from the use of technology:

> *The student develops skills in using computer programs as a tool for learning and exploring mathematics as well as in solving problems. The studies in mathematics include utilising, for example,* ***dynamic mathematics software****,* ***symbolic computation software****,* ***statistical software****,* ***spreadsheets****,* ***text processing****, and, when possible,* ***digital sources****. It is also important to assess the usefulness of the aids and limitations of their use.*
>
> (Finnish National Board of Education, 2016, p. 138)

Table 9 outlines areas in the curriculum which refer to use of technology in Mathematics. Headings are named and organised to reflect the layout of Finland's curriculum document. No specific technology or software is mentioned anywhere in the document. Instead, references are made to the objective for students to be able to use 'technical tools' in certain areas of Mathematics.

Table 9 – References to the use of technology in upper secondary Mathematics in Finland

Course	Topic	Student objective	Page
Compulsory	Numbers and number sequences	is able to use **technical tools** in examining the graph of a function and number sequences as well as in solving application problems related to number sequences	140
Compulsory	Polynomial functions and equations	is able to use **technical tools** in examining polynomial functions and in solving application problems related to polynomial equations, polynomial inequalities and polynomial functions	140
Compulsory	Geometry	is able to use **technical tools** to examine figures and objects and in solving application problems related to geometry	141, 146
Compulsory	Vectors	is able to use **technical tools** in examining vectors and in solving application problems related to straight lines and planes	141
Compulsory	Analytical geometry	is able to use **technical tools** in examining an equation of a set of points as well as solving equations, systems of equation, absolute value equations, and inequalities in problem-solving assignments	142
Compulsory	Derivative	is able to use **technical tools** in examining limits, continuity, and derivatives, solving rational equations and inequalities, and determining the derivatives of polynomial and rational functions in problem-solving assignments	142
Compulsory	Trigonometric functions	is able to use **technical tools** in examining trigonometric functions, solving trigonometric equations, and determining the derivatives of trigonometric functions in problem-solving assignments	142
Compulsory	Radical and logarithmic functions	is able to use **technical tools** to examine radical, exponential, and logarithmic functions and to solve radical, exponential and logarithmic equations as well as determining radical, exponential and logarithmic functions in application problems	143
Compulsory	Integral calculus	is able to use **technical tools** to examine the properties of functions and determine integral functions as well as calculate the definite integral in application problems	143
Compulsory	Probability and statistics	is able to use **technical tools** to acquire, process, and examine digital data as well as determine the distribution parameters and calculating probabilities with the help of the distribution and parameters provided	144
Compulsory	Mathematical models	is able to use **technical tools** to examine polynomial and exponential functions as well as to solve polynomial and exponential equations in connection with problem-solving assignments	147
Compulsory	Statistics and probability	is able to use **technical tools** to acquire, process, and examine digital data as well as to determine the parameters of discrete distributions and probability calculation	147

Course	Topic	Student objective	Page
Compulsory	Commercial mathematics	is able to use **technical tools** to make calculations and solve problems in problem-solving assignments	147
National specialisation	Number theory and mathematical proofs	is able to use **technical tools** to examine the properties of numbers	144
National specialisation	Algorithms in mathematics	is able to use **technical tools** to examine algorithms and for arithmetic operations	145
National specialisation	Advanced differential and integral calculus	is able to use **technical tools** to examine the properties of the function and to calculate derivatives with a given variable as well as to calculate integrals, the limits of number sequences and the sums of series in application tasks	145
National specialisation	Mathematical analysis	is able to use **technical tools** to examine the continuity of a function and to define the extreme values of a bounded interval in application tasks	148
National specialisation	Statistics and probability II	is able to use **technical tools** to acquire, process, and examine digital data, determine the expected value and average deviation of probability distributions, calculate probabilities with the given distribution and parameters, and calculate confidence intervals	148

Source: Finnish National Board of Education (2016)

3.5 Victoria, Australia

The Victorian Certificate of Education (VCE) Mathematics curriculum states that teachers should incorporate technology throughout each unit of study. That said, references to technology throughout the curriculum only refer to 'technology' and are not specific, such that all could be interpreted only as the use of a calculator, when there are possibilities for other forms of technology to be used. Moreover, one of the key skills in the Mathematics curriculum is to "use **technology** effectively for accurate, reliable and efficient calculation" (Victorian Curriculum & Assessment Authority, 2015, p. 14) which seems to imply calculator use or, at most, spreadsheet use for more complex calculations.

Analysis was conducted of two documents: the Australian Curriculum, which spans up to school year 10, and Victoria's Certificate of Mathematics Study Design document. Both have been analysed because, whilst the VCE is a school-leaving examination, there is no compulsory school-leaving examination.

Tables 10 and 11 outline areas in the syllabuses which refer to use of technology in Mathematics in the Australian Curriculum and VCE, respectively. References are nearly entirely vague, mostly referring to 'technology' or 'digital technologies'.

Table 10 – References to the use of technology in the F-10 curriculum in Australia

Year	Topic		Skill
7	Number and algebra	Real numbers	Multiple and divide fractions and decimals using efficient written strategies and **digital technologies**
	Number and algebra	Real numbers	Express one quantity as a fraction of another, with and without the use of **digital technologies**
	Number and algebra	Real numbers	Find percentages of quantities and express one quantity as a percentage of another, with and without **digital technologies**
	Number and algebra	Money and financial mathematics	Investigate and calculate 'best buys', with and without **digital technologies**
8	Number and algebra	Number and place value	Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate **digital technologies**
	Number and algebra	Real numbers	Solve problems involving the use of percentages, including percentage increases and decreases, with and without **digital technologies**
	Number and algebra	Real numbers	Solve a range of problems involving rates and ratios, with and without **digital technologies**
	Number and algebra	Money and financial mathematics	Solve problems involving profit and loss, with and without **digital technologies**
	Number and algebra	Linear and non-linear relationships	Plot linear relationships on the Cartesian plane with and without the use of **digital technologies**
9	Number and algebra	Linear and non-linear relationships	Find the distance between two points located on the Cartesian plane using a range of strategies, including **graphing software**
	Number and algebra	Linear and non-linear relationships	Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies, including **graphing software**
	Number and algebra	Linear and non-linear relationships	Graph simple non-linear relations with and without the use of **digital technologies** and solve simple related equations
10	Number and	Money and	Connect the compound interest formula to repeated applications of simple interest using

Year	Topic		Skill
	algebra	financial mathematics	appropriate **digital technologies**
	Number and algebra	Linear and non-linear relationships	Solve linear simultaneous equations, using algebraic and graphical techniques, including using **digital technology**
	Number and algebra	Linear and non-linear relationships	Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using **digital technology** as appropriate
10A	Measurement and geometry	Pythagoras and geometry	Use the unit circle to define trigonometric functions, and graph them with and without the use of **digital technologies**
	Statistics and probability	Data representation and interpretation	Use **information technologies** to investigate bivariate numerical data sets. Where appropriate use a straight line to describe the relationship allowing for variation

Source: Australian Curriculum (2017)

Table 11 – References to the use of technology in the Mathematics VCE in Victoria, Australia

Unit	Area of study	Topic		Key skill/knowledge	Page
General Mathematics 1 and 2	2	Arithmetic and Number	Computation and practical arithmetic	effective use of **technology** for computation	19
General Mathematics 1 and 2	2	Arithmetic and Number	Matrices	matrix addition, subtraction, multiplication by a scalar, and matrix multiplication including determining the power of a square matrix using **technology** as applicable	19
General Mathematics 1 and 2	2	Arithmetic and Number	Computation and practical arithmetic	use **technology** effectively for computation	24
General Mathematics 1 and 2	3	Discrete mathematics	Matrices	add and subtract matrices, multiply a matrix by a scalar or another matrix, raise a matrix to a power and determine its inverse, using **technology** as applicable	24
General Mathematics 1 and 2	6	Statistics	Investigating relationships between 2 numerical variables	estimate the value of the correlation coefficient r from a scatterplot and calculate its value from the data using **technology**	28
General Mathematics 1 and 2	6	Statistics		the difference between exact numerical and approximate numerical answers when using **technology** to perform computation, and rounding to a given number of decimal places or significant figures	29
General Mathematics 1 and 2	6	Statistics		similarities and differences between formal mathematical expressions and their representation by **technology**	29
General Mathematics 1 and 2	6	Statistics		the selection of an appropriate functionality of **technology** in a variety of mathematical contexts	29
General Mathematics 1 and 2	6	Statistics		distinguish between exact and approximate presentations of mathematical results produced by **technology**, and interpret these results to a specified degree of accuracy	29
General Mathematics	6	Statistics		use **technology** to carry out numerical, graphical and symbolic computation as applicable	29

Unit	Area of study	Topic		Key skill/knowledge	Page
1 and 2					
General Mathematics 1 and 2	6	Statistics		produce results using a **technology** which identifies examples or counter-examples for propositions	29
General Mathematics 1 and 2	6	Statistics		produce tables of values, families of graphs and collections of other results using **technology**, which support general analysis in problem-solving, investigative and modelling contexts	29
General Mathematics 1 and 2	6	Statistics		specify the similarities and differences between formal mathematical expressions and their representation by **technology**	29
General Mathematics 1 and 2	6	Statistics		select an appropriate functionality of technology in a variety of mathematical contexts and provide a rationale for these selections	29
General Mathematics 1 and 2	6	Statistics		relate the results from a particular **technology** application to the nature of a particular mathematical task (investigative, problem solving or modelling) and verify these results	29
General Mathematics 1 and 2	6	Statistics		specify the process used to develop a solution to a problem using **technology** and communicate the key stages of mathematical reasoning (formation, solution, interpretation) used in this process	29
Specialist Mathematics 1 and 2	2	Arithmetic and Number	Number systems and recursion	sequences and series as maps between the natural numbers and the real numbers, the use of **technology** to generate sequences and series and their graphs, and sequences generated by recursion, including arithmetic and geometric sequences	43
Specialist Mathematics 1 and 2	4	Geometry, measurement and trigonometry	Geometry in the plane and proof	complete geometric constructions using compass and straight edge and dynamic geometry **technology**	49
Further Mathematics 3 and 4	1	Core	Data analysis	least squares line of best fit $= a + bx$ where x represents the explanatory variable and y represents the response	55

Unit	Area of study	Topic		Key skill/knowledge	Page
				variable; the determination of the coefficients a and b using **technology**, and the formulas $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$	
Further Mathematics 3 and 4	1	Core	Recursion and financial modelling	use of **technology** with financial modelling functionality to solve problems involving reducing balance loans, such as repaying a personal loan or a mortgage, including the impact of a change in interest rate on repayment amount, time to repay the loan, total interest paid and the total cost of the loan	56
Further Mathematics 3 and 4	1	Core	Recursion and financial modelling	use of **technology** to solve problems involving annuities including determining the amount to be invested in an annuity to provide a regular income paid, for example, monthly, quarterly	56
Further Mathematics 3 and 4	1	Core	Recursion and financial modelling	use of **technology** with financial modelling functionality to solve problems involving annuity investments, including determining the future value of an investment after a number of compounding periods, the number of compounding periods for the investment to exceed a given value and the interest rate or payment amount needed for an investment to exceed a given value in a given time	56
Specialist Mathematics 3 and 4	3	Functions and graphs	Calculus	numeric and symbolic integration using **technology**	83
Specialist Mathematics 3 and 4	3	Functions and graphs	Calculus	formulation of differential equations from contexts in, for example, physics, chemistry, biology and economics, in situations where rates are involved (including some differential equations whose analytic solutions are not required, but can be solved numerically using **technology**)	83

Source: Victorian Curriculum & Assessment Authority (2015)

4. Summary

The data collected and presented in this report give an insight into the areas of Mathematics which other jurisdictions recommend the use of technology, as well as the technology that is used or suggested in different topic areas. In most cases, references to technology were just examples and as such there might be other topic areas which could benefit from students' use of technology, but that were not specifically referenced in the documents analysed. Examples of software which could be used were given by most of the jurisdictions; these were usually the most popular software packages available for certain areas of Mathematics, but others are available and some teachers might find that they are more suitable for their students than the stated examples.

Table 12 summarises some of the salient points from the analysis of curriculum documents in the previous section.

Table 12 – Use of technology across jurisdictions analysed

	Technology in the curriculum					Computing applications mentioned		Is technology used in terminal examinations?					Is technology used in controlled assessment?				
	Required	Recommended	Optional	No mention	Discouraged	General software	Specific applications	Compulsory	Permitted*	Not permitted	No mention	N/A	Compulsory	Optional	Not permitted	No mention	N/A
ENGLAND																	
GCSE																	
AQA				✓					✓								✓
Edexcel				✓					✓								✓
OCR			✓			Spreadsheets Dynamic graphing software	GeoGebra NRICH		✓								✓
Functional Skills																	
AQA	✓					ICT			✓								✓
Edexcel	✓					ICT Spreadsheet	Excel		✓								✓
NCFE	✓					ICT			✓								✓
OCR	✓					ICT Spreadsheet	NRICH BBC Bitesize		✓								✓

	Technology in the curriculum					Computing applications mentioned		Is technology used in terminal examinations?					Is technology used in controlled assessment?				
	Required	Recommended	Optional	No mention	Discouraged	General software	Specific applications	Compulsory	Permitted*	Not permitted	No mention	N/A	Compulsory	Optional	Not permitted	No mention	N/A
SINGAPORE																	
Academic	✓					Computer simulations Dynamic geometry software Graphing software Software Spreadsheets	AlgeBar AlgeDisc AlgeTools Excel Graphmatica GSP		✓								✓
Technical	✓					Dynamic geometry software ICT Spreadsheets	AlgeDisc AlgeTools Excel Graphmatica GSP		✓								✓
Additional Mathematics	✓					Graphing software			✓								✓

	Technology in the curriculum					Computing applications mentioned		Is technology used in terminal examinations?					Is technology used in controlled assessment?				
	Required	Recommended	Optional	No mention	Discouraged	General software	Specific applications	Compulsory	Permitted*	Not permitted	No mention	N/A	Compulsory	Optional	Not permitted	No mention	N/A
CANADA (ONTARIO)																	
Grade 11	✓					Graphing technology Spreadsheet Graphing calculator Scientific calculator Online tools Computer algebra systems Dynamic geometry software Dynamic statistical software						✓					✓
AUSTRALIA (VICTORIA)																	
Australian Curriculum	✓					Digital technologies Graphing software						✓					✓
VCE	✓					Technology			✓								✓
FINLAND																	
Upper secondary curriculum	✓					Technical tools						✓					✓

*In all cases examined, this referred to the use of a scientific calculator.

Table 13 summarises references to graphing calculators in the curricula analysed.

Table 13 – Use of graphing calculators in curricula and assessment

Jurisdiction	Are graphing calculators part of curriculum guidance?		Are graphing calculators part of guidance for assessment?	
	Yes	No	Yes	No
ENGLAND				
GCSE				
AQA		✓		✓
Edexcel		✓		✓
OCR		✓	✓	
Functional Skills				
AQA		✓		✓
Edexcel		✓		✓
NCFE		✓		✓
OCR		✓		✓
SINGAPORE				
Express Course Normal		✓		✓
Normal Technical		✓	✓*	
Additional Mathematics		✓		✓
CANADA (ONTARIO)				
Grade 11	✓			
AUSTRALIA (VICTORIA)				
Australian Curriculum	✓*			✓
VCE		✓		✓
FINLAND				
Upper secondary curriculum		✓		✓

*Note that reference is not made to *graphing calculators*, but *graphing software*.

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