

Educational Standards over time: Has Mathematics Education in England improved?

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- Increasing Competence and Confidence in Algebraic and Multiplicative Structures (ICCAMS)
- ESRC Targeted Initiative on Science and Mathematics Education (TISME)



International Comparisons in Mathematics Education: Three studies



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Just before Christmas the most comprehensive survey of global educational achievement ever conducted showed just how daunting the challenge is. ... But we haven't been progressing relative to our competitors; we've been retreating. In the last ten years we have plummeted in the rankings: from 4th to 16th for science, 7th to 25th for literacy and 8th to 28th for maths.

(Michael Gove, National Curriculum Review launch, 21/1/11)

The "standards" debate

- "School leaving exam" (GCE/GCSE) A*-C:
23% (early 1980s) → 58% (2012)
BUT considerable slippage in standards
(Coe, 2008)
- TIMSS: Since 1995
484 → 542 (Grade 4)
498 → 507 (Grade 8)
- PISA: Since 2003
508 → 493

Tom & Kookaburra (1)

- Partner responsible for UK bank audit
- What debt would Kookaburra have to write off if reconstructed?
- What is the bank's liability?
- Cash flow statement on Kookaburra's interim accounts

(Dawes, 2007)

Tom & Kookaburra (2)

Total creditors of	7.4bn	
Core debt	4.8	
Buffer Debt	<u>1.5</u>	
That's	6.3	Long term debt
	7.4	creditors
	1.1	Current liabilities

"So we have got 6.5bn debt ..."

Tom & Kookaburra (3)

Calculates interest on 6.5bn ...

6.5×0.05 ... on a calculator [0.325]

"That's approximately 350 million to 400 million interest every year"

Tom & Kookaburra (4)

"320m net [financial charges] plus 386 [m] of operating expenses ... we need turnover of 700m at least before we can repay debt" [mentally]

Net cash flow $194 + 140 = 334$ [calculator]

"Cash flow is paying interest but not repaying capital"

Tom & Kookaburra (5)

Writes: $x/90 + x \times 0.05 = 350$

"I am trying to get a feel of where turnover needs to go if we are to repay debt or alternatively if we are to write off to put on an even footing"

Tries 3.25 [half of 6.5bn debt]:
 $3.25/90 + 3.25 \times 0.05 = 199$ [calculator]

Surprised: "I thought they needed half the debt to make it. ... So we might have to write off 25%"

Tom's mathematics

- Approximation and estimation
- Calculation mainly with a calculator
 - Some mental calculation
 - No pencil and paper methods
- Multiplicative reasoning
- Algebra used to 'model'
 - No manipulation

ICCAMS: Increasing Confidence and Competence in Algebra and Multiplicative Structures

Phase 1 (2008/11):

- Nationally representative surveys of attainment in Algebra, Ratio, Decimals & Fractions
- Ages 12, 13, 14
- Stratified random sample of schools
- Comparison to 1970s Concepts in Secondary Mathematics & Science (CSMS) (Hart et al, 1984)
- Tests administered in mathematics lessons in 2008 & 2009
- Sample: 7000

Phase 2 (2008/11):

- Design experiment

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Sample (Y9 element)

	Algebra	Number	Ratio
2008/9	1647	1661	1595
1976/7	961	247	767

Methods

CSMS (1970s)

- Items: Diagnostic interviews / theory
- Levels 0 → 4:
 - Items theoretically constructed
 - Empirically derived (Rasch-like process)
 - Based on “best-performing” items

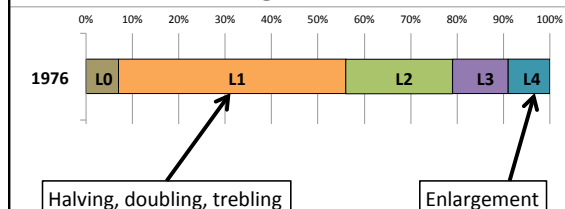
ICCAMS

- Levels & items: Reality check / Rasch
- Differences: Bootstrap & Simulation

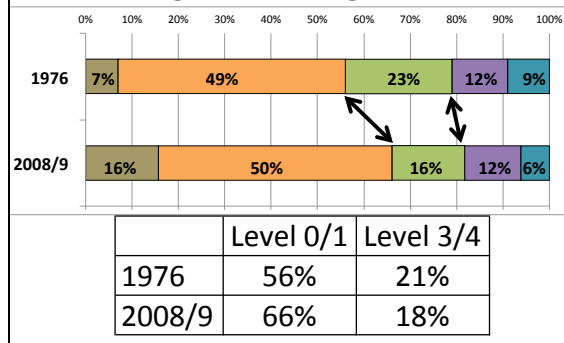
What does ICCAMS test?

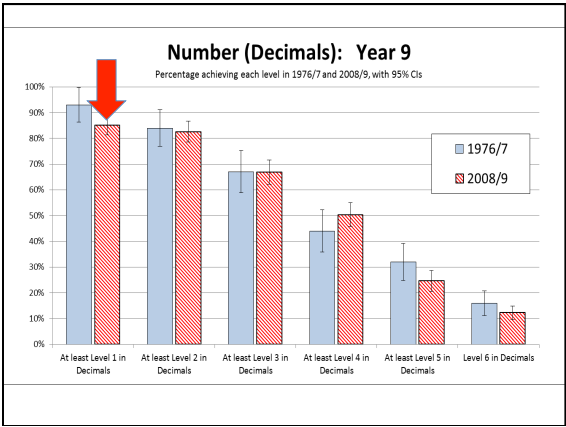
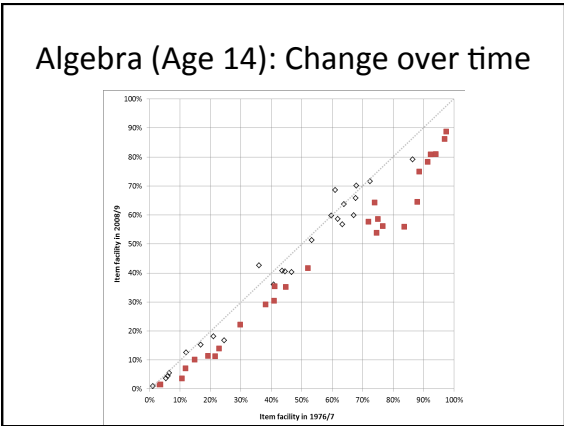
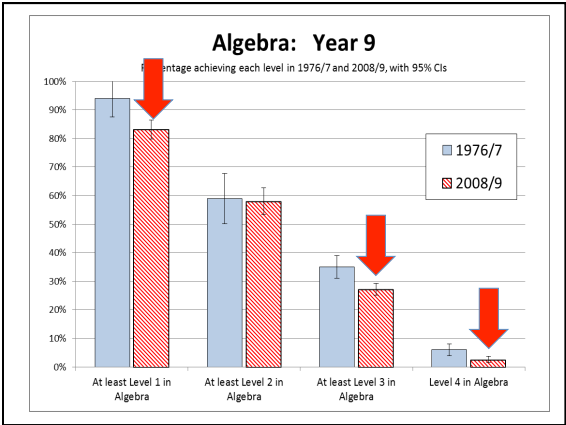
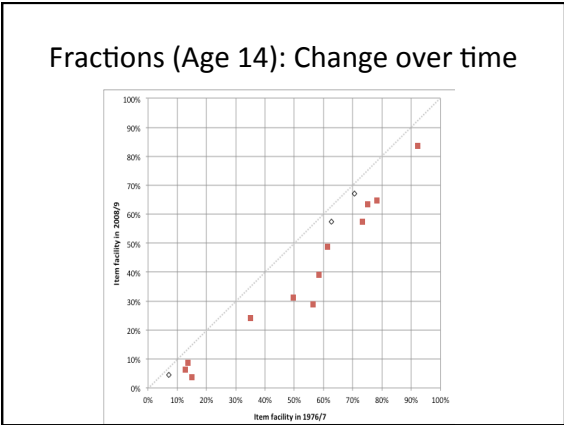
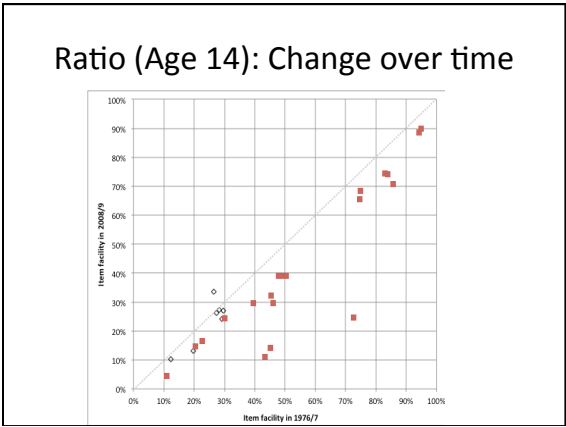
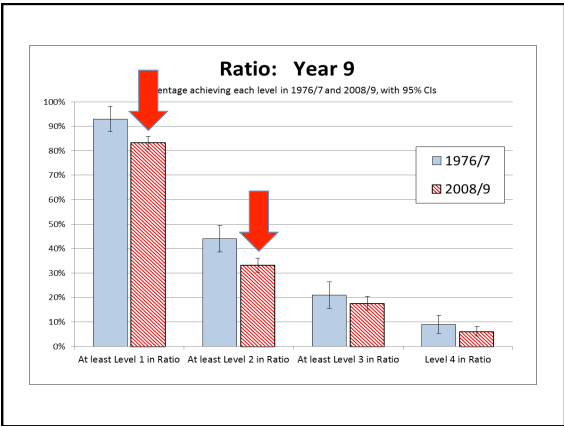
- Understanding:
 - Problems ... recognisably connected to the mathematics curriculum but which would require the child to use methods which were not obviously ‘rules’.” (Hart & Johnson, 1983)
 - Excessive calculation avoided
 - Reveal the strategies used by children
- Algebra: Generalised number & variable
- Ratio: Ratio, particularly additive /multiplicative thinking
- Decimals: Measurement & multiplicative aspects

Ratio (Age 14): 1976

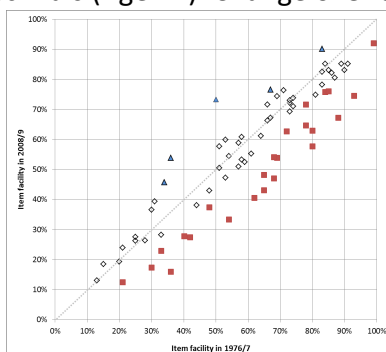


Ratio (Age 14): Change over time

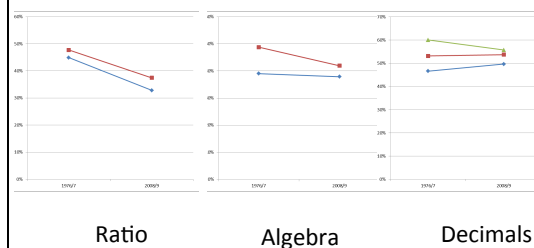




Decimals (Age 14): Change over time



Gains across KS3



A widening gap in attainment?

Algebra

	Percentiles						
	5th	10th	25th	50th	75th	90th	95th
Y7 --> Y8	0.09	0.18	0.35	0.44	0.35	0.53	0.35
Y8 --> Y9	0.00	0.08	0.17	0.25	0.25	0.25	0.25

Effect sizes estimates (Cohen's *d*) of annual growth in learning in algebra across the attainment range

Change in attainment since 1970s

- Algebra, ratio & fractions
 - No improvement
- Number
 - Slight improvement a middle of attainment range
- All
 - Decrease in proportion of highest attainers
 - Increase in proportion of lowest attainers
- Similar picture at Year 8

Dangers of "Cherry-picking"

- What "works" somewhere does not necessarily "work" elsewhere
 - "disorderly classrooms": Korea ↓ - US ↑
 - "disciplined environment": Japan ↑ - Brazil ↓
 - Technology: Brazil ↑ - Norway - Japan ↓

PD in mathematics education

	Maths Content	Maths Pedagogy	Maths Curric.	ICT & maths	Problem solving	Maths Assess.
England	60	73	62	48	53	51
HK	70	68	71	51	49	63
Singapore	67	79	55	68	48	58
Finland	9	21	6	16	8	5
Int. Ave	55	58	52	48	43	47

Why do countries attain highly?

All we can safely say (*we hope*) is that students do experience different types of instructional arrangements cross-nationally and the influence of these arrangements generically appears weak relative to such matters as prior learning and the contents of learning opportunities during the course of study.

(Burstein / IEA / SIMS, 1992, p. 278)

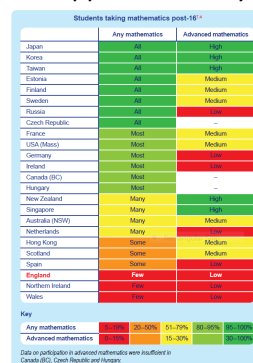
Textbooks

- *Slightly* better textbooks in *some* other systems
 - Variation of examples and non-examples
 - Development and trialing/piloting:
 - Fong Ho Kheong (Singapore: KCL PhD)
- Current English textbooks worse than the 1970s
 - (Hodgen, Küchemann & Brown, 2010)

% of students whose teachers give mathematics tests ...

	Every 2 weeks	About once a month	A few times a year or less
England	9	31	60
Finland	1	44	55
HK	56	39	5
Japan	15	44	41
Singapore	39	51	10
Int. Ave	45	40	15

Participation in upper secondary mathematics



- The 30 year comparison raises serious questions about England mathematics education
 - International comparisons need to be treated with caution
- BUT ...
- We could improve our textbooks
 - Whilst we have high stakes tests, we do relatively little assessment
 - ICCAMS Intervention based around formative assessment doubles the annual rate of learning
 - Don't stop mathematics at 16
 - Mathematics education needs to be appropriate

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