Mathematical Linguine
Everything is Entangled

by **Dominika Majewska**

When you think of the practice of mathematics, remembering your learning experiences, what do you automatically think of? What memories, topics and mistakes spring to mind? Are the pictures you see about numbers? Facts? Symbols? Or perhaps people? Emotions? Words?

My colleague, Lucy Rycroft-Smith, appeared on a copywriting podcast last year as a mathematical expert. One of the questions she was asked – "Is everyone just a words person or a numbers person?" – was typical of the way the world seems to segregate those two modes of communication. Here's a typical quote from a business article on the subject: "Left brain, right brain, we all operate more on one side of our cranium than the other. Some of us are more adept at handling complex mathematical equations whilst others of us are more proficient at articulating things verbally" (Lofaro, n.d.). In short, you either like alphabetti spaghetti, or number spaghetti. You have to pick one.

Of course, this dichotomy – either numbers or words – is likely to fall apart under scrutiny, as most oversimplifications do. In fact, Keith Devlin's book, *The Math Gene* (2000), explores the hypothesis that the way our brains evolved to handle language produced the ability to think mathematically too; that "the feature of our brains that enables us to use language is the same feature that makes it possible for us to do mathematics." More than this, the two are intimately related: they both involve making plans, predictions, adaptation, abstraction, solving problems – and both are intensely human in their expression.

"A mathematician is someone for whom mathematics is a soap opera," Devlin suggests. "Mathematics is not about numbers, but about life. It is about the world in which we live. It is about ideas. And far from being dull and sterile, as it is so often portrayed, it is full of creativity" (Devlin, 2000).

Of course, mathematics and non-symbolic language interact in important ways too. While it is true that some mathematical problems can be expressed in algebra alone, words are very often the method of delivery for mathematical ideas, questions and thoughts, especially for school-aged pupils. As you answer the questions below, taken from the UKMT Junior Mathematical Challenge, 2019, it is worth considering the part language plays in the process. Do you think you could still solve the problem if the question were written in a language you speak only a little of? When you noodled around with the problem, did you write or speak any words out loud?

1. *Sam has eaten three-quarters of the grapes. What is the ratio of the number of grapes that remain to the number Sam has eaten?*

   A 1 : 3; B 1 : 4; C 1 : 5; D 1 : 6; E 1 : 7

2. *The shortest street in the UK, Ebenezer Place in Wick, is 2.06 m long. The Trans-Canada Highway, one of the world's longest roads, is approximately 7821 km in length. Approximately, how many times longer than the street is the highway?*

   A 4 000 000; B 400 000; C 40 000; D 4000; E 400

3. *All four L-shapes shown in the diagram below are to be placed in the 4 by 4 grid so that all sixteen cells are covered and there is no overlap. Each piece can be rotated or reflected before being placed and the black dot is visible from both sides.*

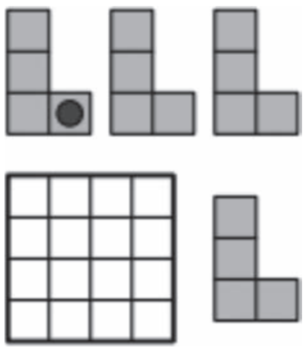

Figure 1: An example of a mathematical question from the Junior Mathematical Challenge (UKMT, 2019)

   How many of the 16 cells of the grid could contain the black dot?

   A 4; B 7; C 8; D 12; E 16

How much language skill did you need to work through these problems? What aspects of language did you need? Did anything in the language make the problem harder to understand?

Obviously, you need to be able to read in the English language to be able to interpret these questions. But that is not enough: you also need to be able to read with comprehension and relate the quantities, understanding the relationships outlined and their correspondence with any diagrams. You may also need some linguistic perseverance, too – I had to read and parse the last question several times before it became clear what it meant.

Research also supports the significant role of language and vocabulary in mathematics. Mathematical vocabulary has been defined as "those words that label mathematical concepts (e.g. hexagon, dividend, and numerator)" (Monroe & Orme, 2002, p. 140). Riccomini et al. (2015) suggest that developing mathematical language is crucial in teaching mathematics to children and that it continues throughout one's mathematical education journey. Research also suggests that one's ability to use vocabulary to describe, justify and communicate mathematically are important in the development of mathematical proficiency (Seethaler et al., 2011, as cited in Riccomini et al., 2015). In fact, mathematical proficiency refers to "the ability to communicate and reason through written and spoken language" (Riccomini et al., 2015, p. 236). Understanding of mathematical vocabulary allows us to understand ideas, instructions and communicate concepts; therefore language is vital in enabling students to access mathematics (Monroe, 1998, as cited in Riccomini et al., 2015).

Communicating mathematically can be difficult even for students who appear to be performing well. The ability to communicate ideas and comprehend presented ideas in mathematics requires a number of factors to be in place, including: a knowledge of vocabulary, flexibility, fluency with numbers, words, symbols, mathematical images and diagrams, and good comprehension skills (Riccomini et al., 2015, p. 237). Research has suggested at least 11 categories of difficulty that students could face when learning mathematical vocabulary and language. These include:

a. meanings being context-dependent (e.g. the possibility that foot could mean 12 inches or the bottom of the bed)

b. mathematical meanings being more precise (e.g. product meaning the solution to a multiplication problem or the product of a company)

c. terms being specific to mathematical contexts (e.g. polygon, parallelogram, imaginary number)

d. terms having multiple meanings (e.g. side of a triangle or side of a cube)

e. technical meanings that are specific to the discipline (e.g. cone as in the shape or food)

f. everyday homonyms (e.g. pi and pie)

g. terms that are related yet different (e.g. circumference and perimeter)

h. challenges with translated words (e.g. mesa vs table)

i. irregularities in spellings (e.g. obelus vs obeli)

j. concepts being verbalised in different ways (e.g. 15 minutes past or quarter past)

k. students and teachers using informal vocabulary instead of mathematical terms (e.g. diamond vs rhombus) (Rubenstein & Thompson, 2002 as cited in Riccomini et al., 2015, p. 238).

It has been suggested that a key step in supporting learners to understand and use mathematical language effectively is for teachers to understand the difficulties that learners may experience when dealing with mathematical vocabulary (Monroe & Orme, 2002). By unpicking learners' needs in relation to language, educators can start to address the difficulties and misconceptions around language that their students may have (Riccomini et al., 2015).

Why does this matter? Since the very beginning of the Cambridge Mathematics project, the Cambridge Maths team has read widely. An important finding was that some sources of mathematical terminology were inaccurate, unclear or promoted misconceptions. Since language is so important in building mathematical understanding, what influence could inaccurate mathematical definitions have on the development of mathematical competency? One of the Cambridge Maths design principles is that early experiences matter. What if children who are beginning to develop their mathematical thinking are not clear on the meanings of mathematical words – or learn to associate meanings with them that later turn out to be incorrect, requiring time and effort to resolve? How could this affect mathematical understanding and future learning? Additionally, the team noted many instances of frustration where different stakeholders in maths education used words differently, and communication across the maths education community was fraught with language barriers and misunderstandings, often caused by using the same words to mean very different things.

With all of the above in mind, the Cambridge Maths team decided to develop the *CM Define It* app – a survey tool for professionals who work in maths education, including teachers, researchers, lecturers, and resource and curriculum designers. From October 2019 to December 2020, the app presented users with a specific mathematical key term (such as *area, circle, number line*) and up to five definitions of the key term taken – with permission – from international sources, such as books, glossaries and pedagogical support documents. The sources are aimed at a wide range of learners, covering primary years through to higher education. Each week users were asked to choose the group(s) of learners they work with (for instance a university lecturer may have chosen the "advanced learners" group whilst someone who works with young children may have selected the "beginner learners" category). Users were asked to rate the definitions on a five-star scale, whilst thinking about the definitions in relation to the group(s) they work with. Then app users had the option to provide further justification of the ratings they gave to the definitions, such as how technically accurate the definitions were, whether they emphasised key points and whether they added to or clarified users' own understanding (Majewska, 2019). Users were also presented with a quirky mathematical term (as a "reward") for rating the definitions and are then given the definition of the unusual word if they provided further information about the ratings they assigned. To give you some idea of these, some of the quirky words used were "syzygy", "beard-second" and "eierlegende Wollmilchsau." We had a lot of fun working on these!

Figure 2: A screen from the CM Define It app

The app will not solve all the problems outlined above, but it's a step in the right direction. It will allow the Cambridge Maths team to collect information such as which definitions are rated highest and lowest and why, and whether certain sources were preferred by certain professionals working with specific groups of learners. We want to get an insight into what the mathematics education community perceives to be a good definition and what makes for a less successful definition of a mathematical key word. Ultimately, we want these insights to inform the glossary that is attached to the Cambridge Mathematics Framework which maps mathematical ideas from age 3-19. The exploratory data collected through the suvey app btween October 2019 and December 2020 is the first step in informing the glossary.

We consider there would be enormous power in creating such a large and interconnected web of glossary items,

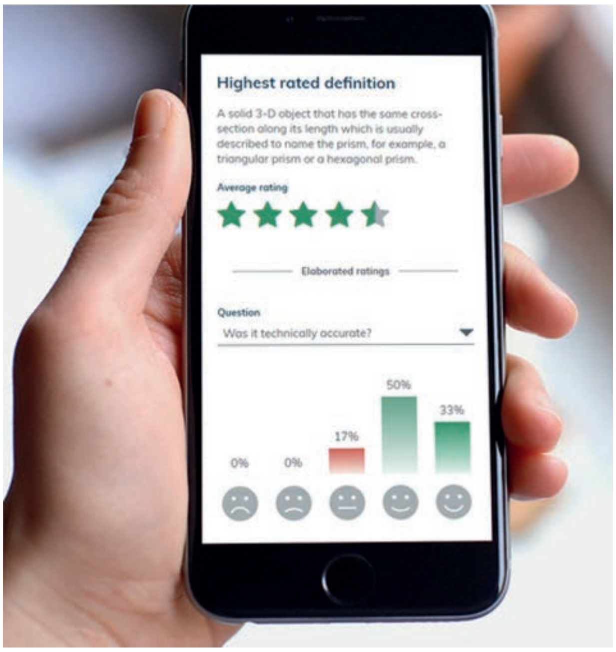

Figure 3: Another screen from the CM Define It app

semantically linked throughout the Framework – provided the definitions are of good quality and usable across the age range, which is a huge undertaking.

To see an example of the key term "number line" and its connections in the Cambridge Mathematics Framework, please see Figure 4 below. Although the *CM Define It* survey app is no longer available, if you are interested in a summary of the findings from this project, please visit www.cambridgemaths.org/research/framework-documentation/view/cm-define-it-a-summary/, where you will find a discussion of some initial findings. For information about the development of the survey app and its journey, visit www.cambridgemaths.org/research/framework-documentation/view/methodology-glossary-app/ and https://www.cambridgemaths.org/research/framework-documentation/view/glossary-app-the-development-and-pilot-phase-of-cm-define-it/. And if you are interested in keeping up with the latest developments in the Cambridge Maths Framework, join our newsletter list at www.cambridgemaths.org/newsletter/.

Got a comment or feedback you'd like to share? You can tweet us **@CambridgeMaths.**

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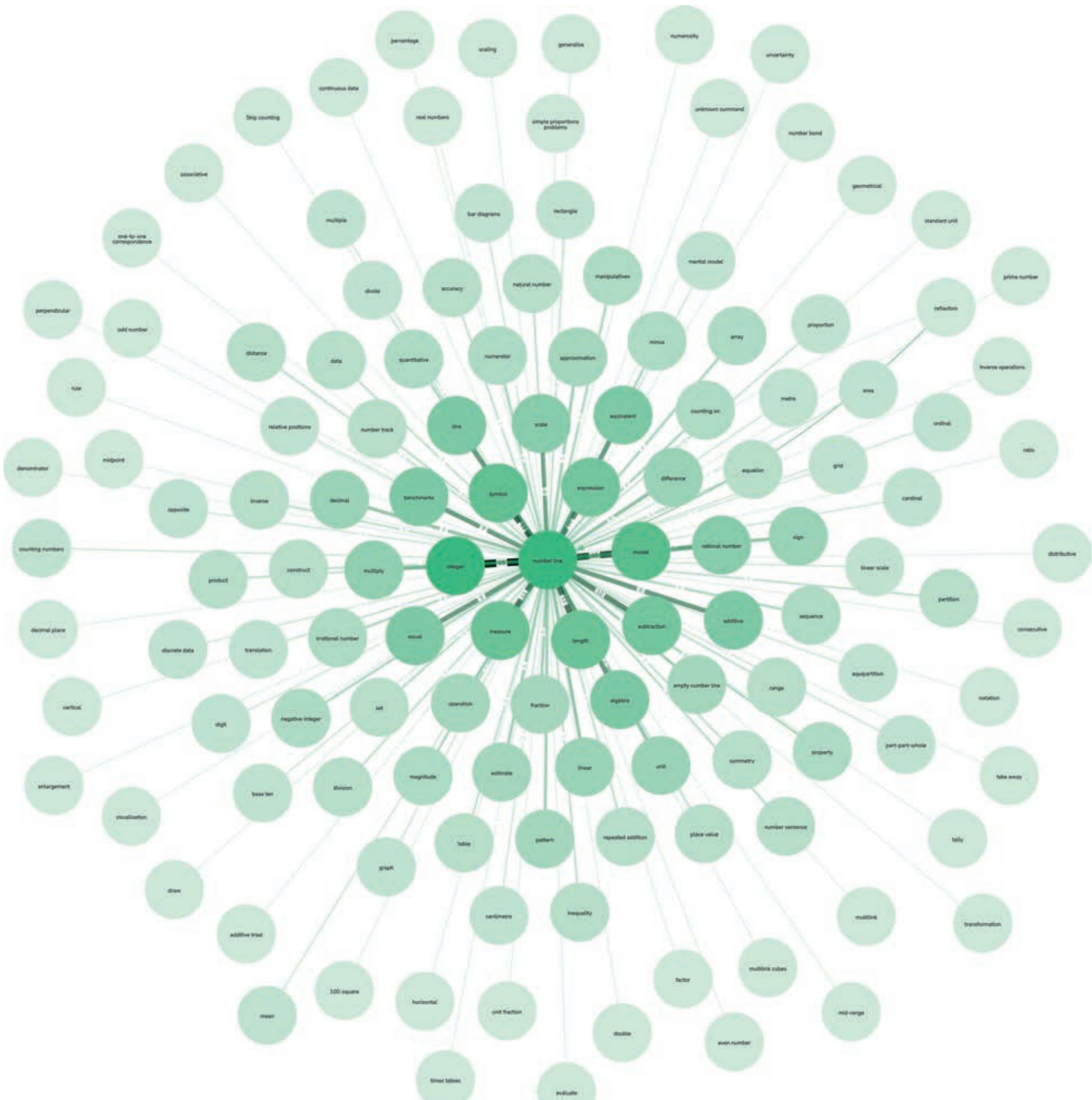

Figure 4: An example of the key glossary term “number line” and its connections to other terms in the Cambridge Mathematics Framework.

proofing every document reporting on the *CM Define It* survey app.

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